AMTH251a: Applied Mathematics I
Fall 2004, Yale University

Lectures: MWF 10.30-11.20; BCT 105
Instructor: Ann B. Lee (ann.lee@yale.edu)
Room 502 in Watson Bldg; Phone: 2-1290
Office hours: Tu,Th 4-5 PM

Supplementary material (to be distributed)

In this class, I will cover ordinary differential equations and systems (7 weeks), Fourier series and orthogonal functions (4 weeks), and applications of the Fourier eigenfunction method to boundary value problems and partial differential equations (2 weeks). Emphasis is on mathematical techniques and concepts that are important in the engineering and physical sciences. By taking this course, a student will acquire a solid mathematical knowledge base for junior/senior classes in the applied sciences, as well as a proper intuition for later more theoretical courses, such as AMTH260, where rigorous proofs are constructed.

Prerequisites for the course are multivariable calculus and linear algebra; the latter could be taken concurrently. A natural continuation of the class is AMTH252b in the spring term which will include transforms, variational calculus, and more on partial differential equations.

Course Work and Grading Policy: There will be weekly homework assignments, one midterm and a final. The final grade will approximately be based 30% on the homework, 30% on the midterm and 40% on the final exam.
Course Schedule (Tentative)

First-order differential equations
Week of 9/1  Differential equations: Basic concepts and ideas.
            Geometrical meaning of $y' = f(x,y)$. Direction fields and solution curves.
Week of 9/6  Linear 1st order equations. Applications.
            Separable equations. Differences between linear and non-linear equations.
Week of 9/13 Substitution methods and exact equations.
            More applications, e.g. population models, stability.
            Review of linear algebra and complex numbers.

Linear Differential Equations of Higher Order.
Week of 9/20 Superposition principle for homogeneous equations. Concept of a basis.
            Second-order homogeneous equations with constant coefficients.
            Modeling: Free oscillations of a mass-spring system.
Week of 9/27 Inhomogeneous equations and the method of undetermined coefficients.
            Modeling: Forced oscillations and resonance.
            Method of variation of parameters and Green’s functions.

Systems of Differential Equations
Week of 10/4 Linear Systems with constant coefficients. Eigenvalues, eigenvectors.
            Stability and phase portraits.
Week of 10/11 Modeling: Normal modes in mechanics.
            Qualitative methods for non-linear systems. Examples in physics.

Fourier Series and Orthogonal Functions
Week of 10/18 Midterm (to be scheduled)
Week of 11/1 Finite and infinite-dimensional vector spaces.
            Orthogonal expansions in Hilbert spaces. Bessel’s and Parseval’s theorems.

Application of Fourier Series to Partial Differential Equations
Week of 11/15 Vibrating strings and the 1D wave equation.
            Heat flow and conduction in a rod.
            Steady-state temperature and Laplace’s equation.
Week of 11/29 Vibrating rectangular membrane and the use of double Fourier series.
            Introduction to other orthogonal functions. Review and wrap-up.

12/17/04 Final exam