36-401 Homework #3 Due September 27, 2001

1. If $\mathbf{A}$ and $\mathbf{B}$ are both 8×3 matrices, are the following operations possible? If so, what are the dimensions of the result? Assume that all square matrices are nonsingular.

   (a) $\mathbf{AB}$

   (b) $\mathbf{BA}$

   (c) $\mathbf{AB}'$

   (d) $\mathbf{A}^{-1}$

   (e) $(\mathbf{A}'\mathbf{A})^{-1}$

2. Given the matrix

$$\mathbf{X} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

   use S-plus to compute $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{X}'$ where, for a matrix $\mathbf{A}$, $\mathbf{A}'$ = transpose of $\mathbf{A}$ and $\mathbf{A}^{-1}$ = inverse of $\mathbf{A}$. Note: you will need the following commands for this problem:

   - `t(X)` = transpose of $\mathbf{X}$,
   - `solve(X)` = inverse of $\mathbf{X}$, and
   - `X %*% Y` = matrix multiplication of conformable matrices, $\mathbf{X}$ and $\mathbf{Y}$.

   Furthermore, note that a command such as `x <- c(2,3,4,45,3,4)` can be treated as a row or a column vector in most S-plus expressions.

3. Refer to the data set, "Prestige.dat" (used previously in homework #2) for the following problems:

   (a) Create a subset of the prestige dataset named `p.sub <- prestige[,-c(5,6)]`. Use `p.sub` and the S-plus command, `cor(p.sub)`, to create the correlation matrix.

   (b) Using your result from part (a), perform a simple regression on each of the variables in relation to occupational prestige.

   (c) Perform a multiple regression on all variables to occupational prestige.

   (d) Write one paragraph each, interpreting your results from parts (a) - (c). Remember: use some of the principles of style and design discussed in the readings from 9/18/01.

4. Read the scenario described at

   http://www.maths.uq.edu.au/~gks/data/oz/medals.html,

   and save the associated data set to a file in your directory (you can locate the data by pressing "Data File" in the top right-hand corner). Create a matrix $\mathbf{X}$ by reading the data,

```
medals <- read.table("medals.txt",header=T)
```

and combining the necessary columns with the command `X <- cbind(1,as.matrix(medals[,-1]))`. Find the following:

(a) The matrix of regression coefficients, **B** (use Equation A3.3 on page 339 of RwG).

(b) The hat matrix, **H**, and leverage for each case, h_i (use Equation A3.4 on page 339 of RwG).

(c) The variance-covariance matrix of coefficients, **S** (use Equation A3.5 on page 339 of RwG).

5. Show mathematically that $\mathrm{E}(b_1) = \beta_1$, i.e. b_1 is an *unbiased* estimate of β_1. Be sure to explain and justify your steps! Hint: recall from 9/18/01 class that

$$\frac{\sum(X_i - \bar{X})c}{\sum(X_i - \bar{X})^2} = 0,$$

for any constant c.