Classical Multi-level and Bayesian Approaches to Population Size Estimation Using Multiple Lists*

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Introduction

Let $i = 1, \ldots, N$ index the objects, and $j = 1, \ldots, J$ index the lists. Our basic model has $N \times J$ random variables, X_{ij}, such that

$$X_{ij} = \begin{cases} 1, \text{ if object } i \text{ appears on list } j; \\ 0, \text{ otherwise.} \end{cases}$$

We let $p_{ij} = P[X_{ij} = 1]$, and n be the number of objects that appear on at least one list. We wish to estimate the number of unobserved objects, $M = N - n$ or, equivalently, to estimate N.

Darroch et al. (1993), Agresti (1994) and Biggeri et al. (1999) have pointed out the need for modeling methodology that accomodates:

1. The probabilities of appearing in the various lists, i.e., capture probabilities;
2. How the lists relate to one another, i.e., list dependencies; and
3. The ways in which these capture probabilities and list dependencies vary across individuals.

A simple model that accomodates 1. and 3. is the Rasch model (1960) from educational testing,

$$\log\left\{\frac{p_{ij}}{1 - p_{ij}}\right\} = \theta_i + \beta_j, \quad i = 1, \ldots, N; \quad j = 1, \ldots, J,$$

which has been used since Sanathanan (1972) for capture-recapture work.

Our goal is to develop flexible modeling methodology based on the Rasch model, that also allows for list dependence and other features seen in epidemiological multiple-recapture problems.

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The classical capture-recapture estimates for N for pair of lists is:

$$\hat{N} = \left\lfloor \frac{n_{1+} n_{+1}}{n_{11}} \right\rfloor,$$

where n_{1+} is the number of objects in list 1, n_{+1} is the number of objects in list 2, n_{11} is the number of objects in both lists and $\lfloor x \rfloor$ is the greatest integer contained in x.

This estimate assumes homogeneous objects (e.g. Darroch et al., 1993) and no dependence between lists (e.g. Biggeri et al, 1999), and will be used to illustrate under- and over-estimation due to not modeling dependence in the $2^J - 1$ table cross-classifying lists.

Simulated Data

Using the basic Rasch model we randomly drew independent results for the presence of $N = 2000$ individuals from each of $J = 6$ lists. We simulated the values of the individual parameters θ_i, for $N = 2000$ subjects from a $N(0, 4)$ distribution, and their presence or absence from each of six lists according to list parameters $\beta = (-1, -.5, -.25, .25, .5, 1)$. The result was a 2000×6 array of ones and zeros.

We summarized this information according to the presence or absence of individuals in the 6 lists, yielding the 2^6 cross-classification of Table 1. When we analyze these data we will treat the number of individuals falling into no lists as if it were unobserved and to be estimated.

Multiple Sources For Diabetes Ascertainment

Bruno, et al. (1994) used 4 sources to identify known cases of diabetes among the residents of the area of Casale Monferrato in northern Italy on October 1, 1988 (see Table 3):

Clinics: List of all patients with a previous diagnosis of insulin-dependent diabetes mellitus (IDDM) or non-insulin dependent mellitus (NIDDM), via diabetes clinic and/or family physicians;

Hospitals: List of all patients discharged with a primary or secondary diagnosis of diabetes in all public and private hospitals in the region;

Prescriptions: Computerized database list of insulin and oral hypoglycemic prescriptions for 1988;

Reimbursements: List of all residents of region who requested a reimbursement for insulin and reagent strips.

The Number of Pages on the World Wide Web

Lawrence and Giles (1998) studied the coverage and recency of six major and widely-available World Wide Web search engines by submitting 575 queries on various scientific topics. An extract of their data is in Table 7.

The Rasch Model and Quasi-Symmetric Log-Linear Models

Let $X_1, \ldots, X_J$ be the variables cross-classified in $2^J - 1$ tables such as Tables 1 and 7. The Rasch model is a multivariate mixed-effects logistic regression model

$$P_j(\theta_i) = P[X_j = 1|\theta_i] = \frac{\exp \lambda_j(\theta_i)}{1 + \exp \lambda_j(\theta_i)}$$

where $\lambda_j(\theta_i) = \theta_i + \beta_j$ incorporates random object catchability effects θ_i and fixed list effects β_j. Assuming that the lists are independent given θ, we obtain the marginal log-linear form

$$\log \pi_{k_1 \ldots k_J} = \log P[X_1 = k_1, \ldots, X_J = k_J] = \log \int \prod_{j-1}^{J} P_j(t)^{k_j} [1 - P_j(t)]^{1-k_j} dF_\Theta(t)$$

$$= \log \int \exp \left\{ \sum_{j=1}^{J} k_j(t + \beta_j) \right\} \prod_{j=1}^{J} \frac{1}{1 + e^{t+\beta_j}} dF_\Theta(t) = \alpha + k_1\beta_1 + \cdots + k_J\beta_K + \gamma(k_+)$$

where $k_+ = \sum_{j=1}^{J} k_j$ and $\gamma(s) = \log E\left[e^{s\theta}|\mathbf{k} = \mathbf{0}\right]$. With an additional "no J-way interaction" assumption, we may estimate $\hat{N} = n + \exp \hat{\alpha}$ (e.g. Darroch et al., 1993).

Hierarchical Bayes Formulation of the Rasch Model

By adding priors $G_\beta(\cdot)$ for the β_j's and $\nu(N)$ for N, we convert the mixed effects model to a hierarchical Bayes model. This allows for flexible model expansion, and also allows us to trade "no J-way interactions" for the proper moment constraints, which seems to help.

We have extended Basu's (1998) Gibbs sampler to accomodate these models, and also implemented a version of Green's (1995) reversible-jump MCMC algorithm to select N. In principle any proper prior on N can be used but we have found that restricting N to have finite support on the integers, say $[n, N_{max}]$ for some value N_{max}, is helpful in the MCMC simulation. For the examples we present below we have typically taken N_{max} to be 10,000.

List and Latent Variable Interactions

Conditional on θ, the Rasch model is an independence model

$$\log \pi_{k_1 \cdots k_J|\theta} = \alpha(\theta) + \sum_j \lambda_j(\theta) k_j.$$

Adding terms $\lambda_{j_1 j_2}(\theta) k_{j_1} k_{j_2}$ to represent conditional interactions between lists leads to the marginal log-linear form

$$\log(\pi_{k_1 \ldots k_J}) = \alpha + k_1\beta_1 + \cdots + k_J\beta_K + \gamma(k_+) + \sum_{j_1 \neq j_2}\sum \beta_{j_1 j_2} k_{j_1} k_{j_2} + \sum_\ell \gamma'(k_\ell, k_+).$$

The latter terms $\gamma'(k_\ell, k_+)$ can also be interpreted as manifestations of multidimensional catchability effects for the objects. Analogous expansions of the Bayesian model are under development.

Table 1: 2^6 Table of 2000 Individuals Simulated From a Rasch Model.

					x6	
x1	x2	x3	x4	x5	0	1
0	0	0	0	0	304	108
0	0	0	0	1	70	55
0	0	0	1	0	37	42
0	0	0	1	1	37	50
0	0	1	0	0	30	26
0	0	1	0	1	16	24
0	0	1	1	0	16	25
0	0	1	1	1	17	46
0	1	0	0	0	21	21
0	1	0	0	1	15	37
0	1	0	1	0	9	20
0	1	0	1	1	10	52
0	1	1	0	0	10	10
0	1	1	0	1	8	28
0	1	1	1	0	8	23
0	1	1	1	1	15	97
1	0	0	0	0	11	10
1	0	0	0	1	6	12
1	0	0	1	0	8	11
1	0	0	1	1	7	28
1	0	1	0	0	5	4
1	0	1	0	1	9	18
1	0	1	1	0	1	12
1	0	1	1	1	6	58
1	1	0	0	0	1	4
1	1	0	0	1	6	12
1	1	0	1	0	1	7
1	1	0	1	1	6	58
1	1	1	0	0	2	3
1	1	1	0	1	3	30
1	1	1	1	0	4	25
1	1	1	1	1	14	331

Table 2: Traditional capture-recapture estimates for N using pairs of lists from Table 1.

Lists		$\hat{N}$
x1	x2	1253
x1	x3	1254
x1	x4	1335
x1	x5	1394
x1	x6	1472
x2	x3	1347
x2	x4	1416
x2	x5	1457
x2	x6	1512
x3	x4	1431
x3	x5	1515
x3	x6	1564
x4	x5	1534
x4	x6	1572
x5	x6	1623

Table 3: Data from prevalent cases of known diabetes melitus for residents of Casale Monferrato, Italy, on October 1, 1988, according to four sources of ascertainment.

		Clinics			
		yes		no	
		Hospitals		Hospitals	
Prescriptions	Reimbursements	yes	no	yes	no
yes	yes	58	46	14	8
yes	no	157	650	20	182
no	yes	18	12	7	10
no	no	104	709	74	?

Table 4: Traditional capture-recapture estimates for N using pairs of sources from Table 3.

Lists	$\hat{N}$
Clinics, Hospitals	2,351
Clinics, Prescriptions	2,185
Clinics, Reimbursements	2,262
Hospitals, Prescriptions	2,052
Hospitals, Reimbursements	803
Prescriptions, Reimbursements	1,555

Table 5: Six major search engines and their estimated coverages within the sample of ca. 190×10^6 pages found by 575 web search queries. *Source:* Lawrence and Giles (1998).

Search Engine	Coverage (%)	95% CI (%)
HotBot (HB)	57.5	$\pm$1.3
AltaVista (AV)	46.5	$\pm$1.3
NorthernLight (NL)	32.9	$\pm$1.1
Excite (Ex)	23.1	$\pm$0.86
InfoSeek (Is)	16.5	$\pm$1.0
Lycos (Ly)	4.41	$\pm$0.42

Table 6: Estimates of indexable web, from pairs of search engines, from the pair with the two smallest coverages relative to the total observed pages n to pair with the two largest coverages. *Source:* Lawrence and Giles (1998).

Engines	$\hat{N}$ indexable pages ($\times 10^6$)	95% CI
Lycos and InfoSeek	90	$\pm$ 6
Infoseek and Excite	220	$\pm$ 16
Excite and NorthernLight	230	$\pm$ 15
NorthernLight and AltaVista	230	$\pm$ 13
AltaVista and HotBot	320	$\pm$ 34
Actual number of unique hits	$n = 190$	—

Table 7: Multiple list data for Query 535, obtained from Lawrence and Giles (priv. comm.).

					Northern Light	
Alta Vista	Infoseek	Excite	Hot Bot	Lycos	0	1
0	0	0	0	0	–	35
0	0	0	0	1	2	2
0	0	0	1	0	79	13
0	0	0	1	1	0	3
0	0	1	0	0	21	5
0	0	1	0	1	0	1
0	0	1	1	0	3	2
0	0	1	1	1	0	1
0	1	0	0	0	8	0
0	1	0	1	0	11	5
0	1	0	1	1	1	0
0	1	1	0	0	4	2
0	1	1	0	1	1	0
0	1	1	1	0	2	2
0	1	1	1	1	1	0
1	0	0	0	0	33	7
1	0	0	0	1	1	0
1	0	0	1	0	14	7
1	0	0	1	1	1	4
1	0	1	0	0	6	1
1	0	1	1	0	0	2
1	0	1	1	1	0	1
1	1	0	0	0	0	2
1	1	0	0	1	1	2
1	1	0	1	0	5	5
1	1	0	1	1	0	1
1	1	1	1	0	2	5
1	1	1	1	1	1	0

Table 8: Traditional capture-recapture estimates for total N web pages matching Query 535, using pairs of search engines.

WWW Search Engines		$\hat{N}$
AltaVista	Infoseek	256
AltaVista	HotBot	359
AltaVista	NorthernLight	294
AltaVista	Excite	353
AltaVista	Lycos	202
Infoseek	HotBot	254
Infoseek	NorthernLight	274
Infoseek	Excite	192
Infoseek	Lycos	183
HotBot	NorthernLight	362
HotBot	Excite	489
HotBot	Lycos	293
NorthernLight	Excite	309
NorthernLight	Lycos	172
Excite	Lycos	252
Total observed in $2^6 - 1$ table:		$n = 305$

Table 9: Estimates of the population size for 2000 objects and six lists; data simulated from a Rasch model.

Model	df	Deviance	Point Estimate	95% Interval
Independence:	56	1335.44	1701	[1698,1707]
QS with no 3-way or higher	55	50.16	1974	[1914,2047]
QS with no 4-way or higher	54	42.05	1859	[1799,1950]
QS with no 5-way or higher	53	41.46	1932	[1779,2362]
QS with no 6-way	41.45	52	1904	[1701,9500]
Bayesian Rasch model			Median 2019	[1939,2128]
		Observed: $n = 1696$		

Table 10: MCMC estimated posterior mean and and quantiles for the list parameters, $\{\beta_j\}$, and prior standard deviation σ on the random catchability effects, $\{\theta_i\}$, based on 2000 objects simulated from the Rasch model. Actual parameters used in the simulation of the data are given in the rightmost column.

Name	mean	2.5%ile	median	97.5%ile	actual
List 1	-1.03	-1.27	-1.02	-0.81	-1.00
List 2	-0.40	-0.64	-0.40	-0.19	-0.50
List 3	-0.29	-0.53	-0.29	-0.08	-0.25
List 4	0.24	0.00	0.24	0.45	0.25
List 5	0.58	0.33	0.58	0.79	0.50
List 6	0.95	0.70	0.94	1.17	1.0
σ	2.10	1.90	2.10	2.32	2.00
$N\ (m = 1696)$	2022	1939	2019	2128	2000

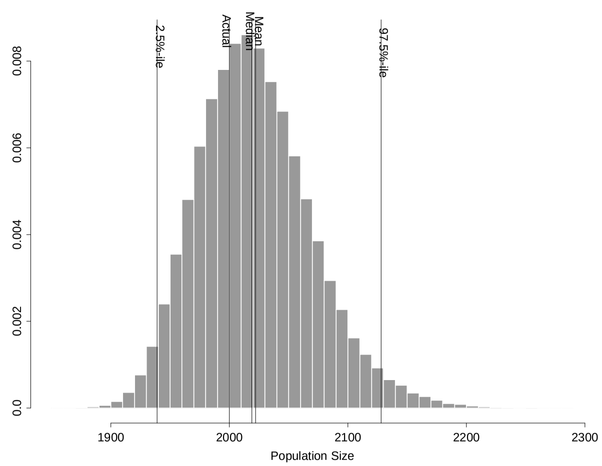

Figure 1: Posterior Distribution for Unknown Sample Size for Simulated Data (Actual=2000).

Table 11: Estimates for the number of diabetes melitus cases in Casale Monferarato, Italy, on October 1, 1988 using various methods.

Model	df	Deviance	Point Estimate	95% Interval
Independence	10	217.48	2251	[2217,2289]
All 1st-order interactions, except Reimbursements × Clinics	5	7.62	2771	[2536,3119]
QS no 3-way or higher	9	105.63	2669	[2527,2848]
QS no 4-way	8	93.95	2239	[2145,2437]
Saturated	0	0	5367	
Bayesian Rasch model			Median 2697 Mode 2664 Mean 2705	[2560,2917]
			Observed: $n = 2069$	

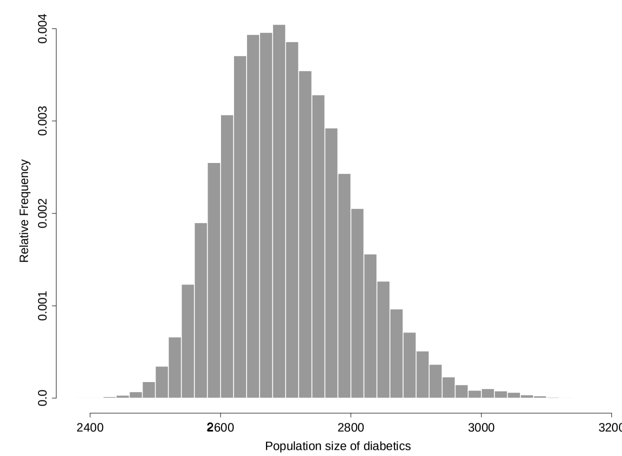

Figure 2: Posterior distribution of the number of individuals with diabetes in Casale Monferrato, Italy.

Table 12: Estimates of the number of world wide web pages on the topic “Query #535” using various estimation methods.

Model	df	Deviance	Point Estimate	95% Interval
Independence	56	148.73	373	[352,400]
BIC-based log-linear model	46	65.62	602	[484,797]
QS no 3-way or higher	55	88.61	614	[501,783]
QS no 4-way or higher	54	83.33	1266	[634,3337]
QS no 5-way or higher	53	82.43	508	[309,5778]
QS no 6-way	52	81.71	861882	[306,∞]
Bayesian Rasch model			Median 773 Mode 671 Mean 876	[528,2005]
		Observed: $n = 305$		

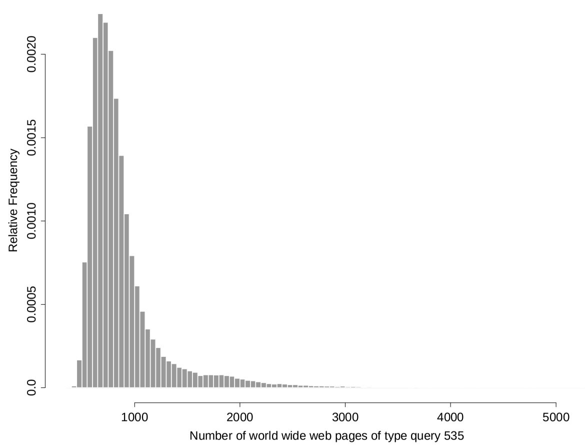

Figure 3: Histogram of the posterior distribution of the number of web pages of corresponding to query # 535.

Table 13: Estimates of the number of diabetes melitus cases in Casale Monferarato, Italy with list-by-list and list-by-total interactions.

Model	df	Deviance	Point Estimate	95% Interval
BIC[1]	5	7.62	2771	[2536, 3119]
QS2[2]	9	105.64	2669	[2527, 2848]
QS3	8	93.95	2239	[2145, 2437]
QS2 + BIC[3]	6	8.32	2752	[2552, 3031]
QS3 + BIC[4]	5	2.04	4152	[2950, 7032]
QS2+$k_+\times$Prescriptions[5]	8	103.76	2699	[2599, 2877]
QS2+$k_+\times$Clinic	8	102.70	2548	[2413, 2737]
QS2+$k_+\times$Reimbursements	8	84.68	2476	[2373, 2610]
QS2+$k_+\times$Hospitals	8	90.09	2861	[2673, 3056]
QS2+$k_+\times$Hospitals +$k_+\times$Reimburse	7	80.72	2591	[2432, 2817]
QS3+$k_+\times$Prescriptions	7	88.09	2211	[2137, 2361]
QS3+$k_+\times$Clinic	7	92.01	2216	[2196, 2373]
QS3+$k_+\times$Reimbursements	7	83.24	2327	[2192, 2604]
QS3+$k_+\times$Hospitals	7	80.61	2319	[2190, 2582]
Bayesian Rasch	—	—	2664[6]	[2560, 2917]

[1] Stepwise BIC selects: independence + reimburse:hospitals + presc:reimburse + presc:clinic + presc:hospitals + clinic:hospitals.

[2] QS2 indicates the Rasch quasi-symmetry model with no 3- or higher-way interactions. Similarly QS3 indicates Rasch quasi-symmetry with no 4- or higher-way interactions.

[3] Stepwise BIC starts with QS2 and adds presc:reimburse + presc:clinic + reimburse:hospitals.

[4] Stepwise BIC starts with QS3 and adds presc:reimburse + presc:clinic + reimburse:hospitals.

[5] $k_+\times$Prescriptions indicates one list-by-total interaction involving the prescriptions list. Similarly for the other $k_+\times$List interaction models shown.

[6] Posterior mode.

Table 14: Estimates of the number of web pages of type "Query #535" using quasi-symmetry models with list-by-list and list-by-total interactions.

Model	df	Deviance	Point Estimate	95% Interval
BIC[1]	46	65.62	602	[484, 797]
QS2[2]	55	88.61	614	[501, 783]
QS3	54	83.33	1266	[634, 3337]
QS4	53	82.43	508	[309, 5778]
QS5	52	81.71	861,882	[306, ∞]
QS2 + Dep[3]	41	59.14	598	[475, 804]
QS3 + Dep	40	52.94	1328	[635, 3701]
QS4 + Dep	39	51.91	499	[307, 8396]
QS2 + BIC[4]	44	60.69	588	[470, 782]
QS3 + BIC[5]	44	55.99	1370	[650, 3829]
QS4 + BIC[6]	43	55.09	526	[309, 6570]
QS2[1] + $k_+ \times$AltaVista[7]	54	88.53	613	[500, 782]
QS2 + $k_+ \times$Infoseek	54	82.98	583	[478, 741]
QS2 + $k_+ \times$Excite	54	82.72	636	[514, 820]
QS2 + $k_+ \times$HotBot	54	88.19	595	[482, 773]
QS2 + $k_+ \times$Lycos	54	87.87	595	[484, 765]
QS2 + $k_+ \times$NorthernLight	54	88.42	617	[502, 790]
QS2 + $k_+ \times$Infoseek + $k_+ \times$Excite	53	79.20	604	[499, 758]
QS3 + $k_+ \times$AltaVista	53	83.13	1276	[637, 3371]
QS3 + $k_+ \times$Infoseek	53	77.88	1169	[594, 3074]
QS3 + $k_+ \times$Excite	53	78.21	1251	[628, 3289]
QS3 + $k_+ \times$HotBot	53	82.60	1225	[617, 3224]
QS3 + $k_+ \times$Lycos	53	81.05	1332	[656, 3540]
QS3 + $k_+ \times$NorthernLight	53	83.30	1261	[631, 3327]
QS4 + $k_+ \times$AltaVista	52	82.21	508	[309, 5806]
QS4 + $k_+ \times$Infoseek	52	77.19	524	[309, 6254]
QS4 + $k_+ \times$Excite	52	77.28	501	[308, 5588]
QS4 + $k_+ \times$HotBot	52	81.71	502	[308, 5633]
QS4 + $k_+ \times$Lycos	52	80.04	504	[308, 5692]
QS4 + $k_+ \times$NorthernLight	52	82.39	507	[309, 5757]
Bayesian Rasch	—	—	671[8]	[528, 2005]

[1] Stepwise BIC selects: (AV:Is) + (AV:HB) + (AV:Ly) + (AV:NL) + (Is:Ex) + (Is:HB) + (Ex:NL) + (Ly:NL).

[2] QS2 indicates the Rasch quasi-symmetry model with no 3- or higher-way interactions. Similarly QS3 indicates Rasch quasi-symmetry with no 4- or higher-way interactions, etc.

[3] Dep indicates that all two-way interactions were also fitted, in addition to the quasi-symmetry model.

[4] Stepwise BIC starts with QS2 and adds AltaVista:Infoseek + AltaVista:Excite + AltaVista:HotBot + AltaVista:NorthernLight + Infoseek:Lycos + Infoseek:NorthernLight + Excite:HotBot + Excite:Lycos+ Excite:NorthernLight + HotBot:Lycos + HotBot:NorthernLight.

[5] Stepwise BIC starts with QS3 and adds AltaVista:Infoseek + AltaVista:Excite + AltaVista:HotBot + AltaVista:NorthernLight + Infoseek:NorthernLight + Excite:HotBot + Excite:Lycos + Excite: NorthernLight + HotBot:Lycos + HotBot:NorthernLight.

[6] Stepwsie BIC starts with QS4 and adds AltaVista:Infoseek + AltaVista:Excite + AltaVista:HotBot + AltaVista:NorthernLight + Infoseek:NorthernLight + Excite:HotBot + Excite:Lycos + Excite:NorthernLight + HotBot:Lycos + HotBot:NorthernLight.

[7] $k_+ \times$ AltaVista indicates one list-by-total interaction involving the AltaVista list. Similarly for the other $k_+ \times$ List interaction models shown.

[8] Posterior mode.

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