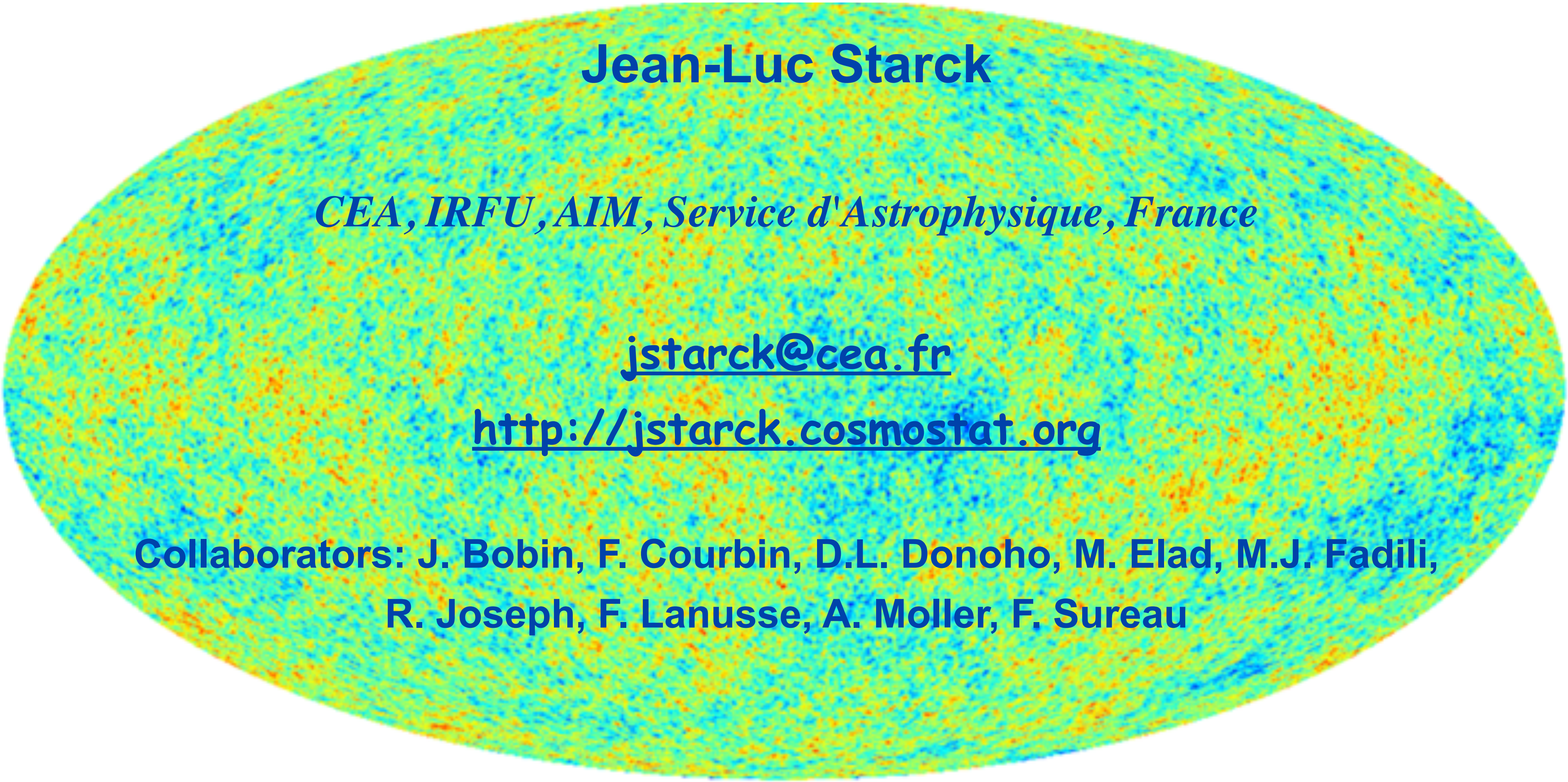
Low Complexity Models and Astrophysical Maps Reconstruction



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Mixture modeling

- Mono-channel mixture:

$$Y = X_1 + X_2 + N$$

- Hyper/Multispectral mixture:

$$Y_i = H_i * \sum_{s=1}^{S} a_{i,s} X_s + N$$

Mixture modeling

- Mono-channel mixture:

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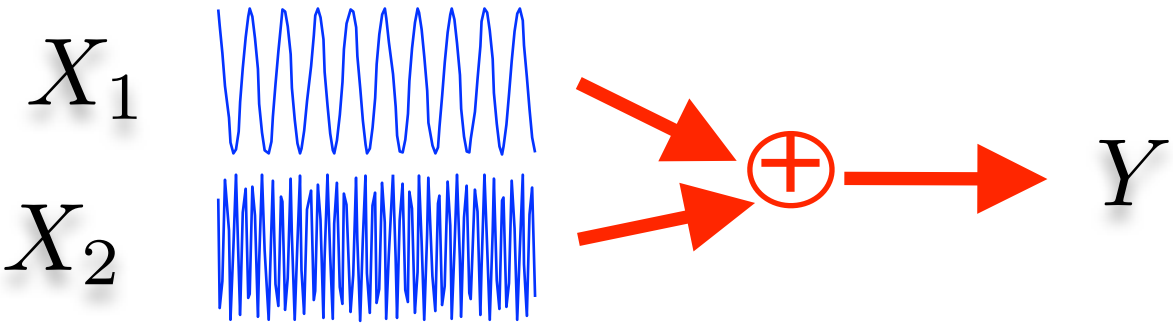


- Hyper/Multispectral mixture:

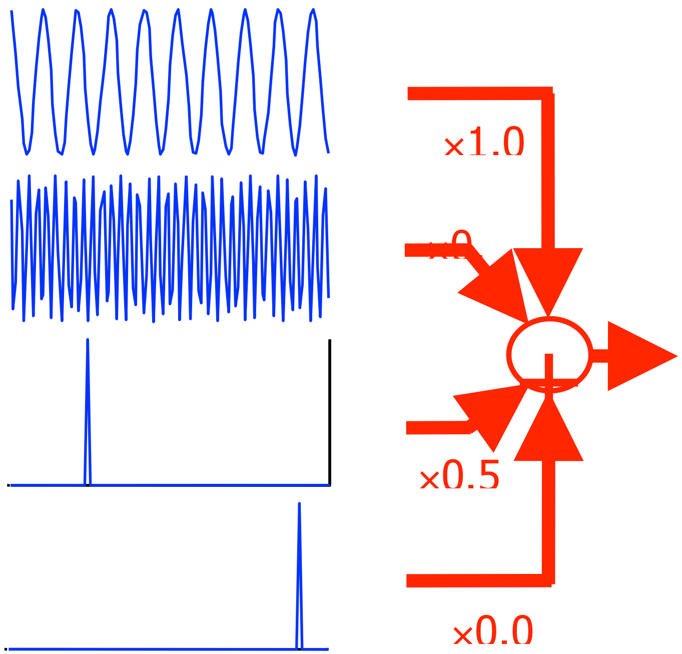
$$Y_i = H_i * \sum_{s=1}^{S} a_{i,s} X_s + N$$

Monochannel Mixture

$$min_{X_1,X_2} \parallel Y - (X_1 + X_2) \parallel^2 + C_1(X_1) + C_2(X_2)$$

C_1: C_1(X1) must be low and C_1(X2) must be high

C_2: C_2(X1) must be high and C_2(X2) must be low

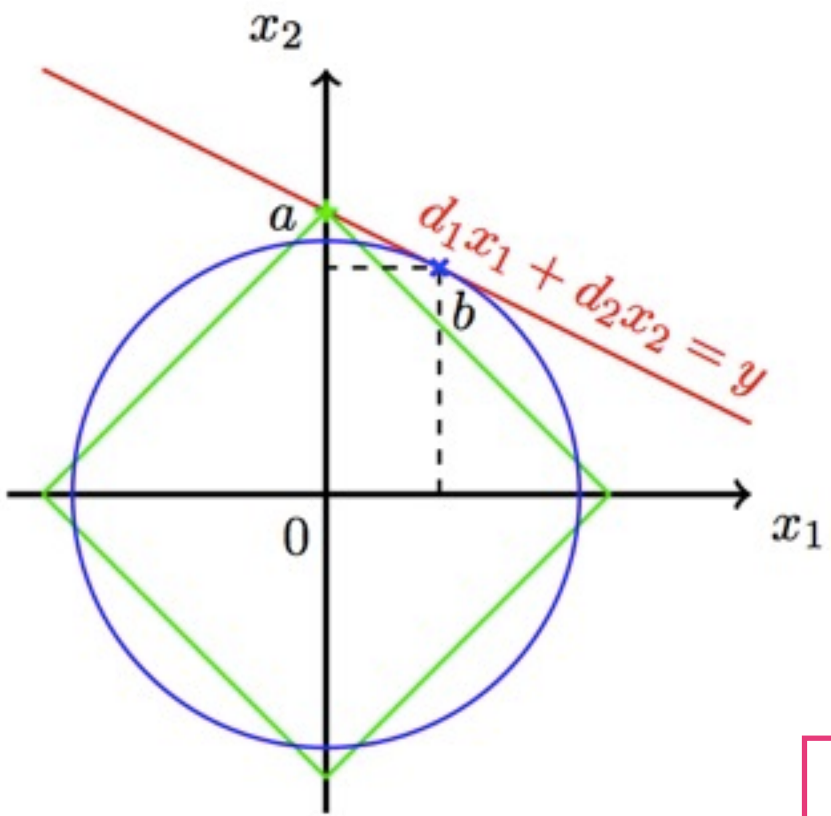


$$C_1(X_1) = \parallel \Phi_1^t X_1 \parallel_p$$

$$C_2(X_2) = \parallel \Phi_2^t X_2 \parallel_p$$

L1 Norm & Sparsity

$$\|X\|_p = (\sum_i \mid X_i \mid^p)^{\frac{1}{p}}$$



$p < 2$

Sparse Recovery & Inverse Problems

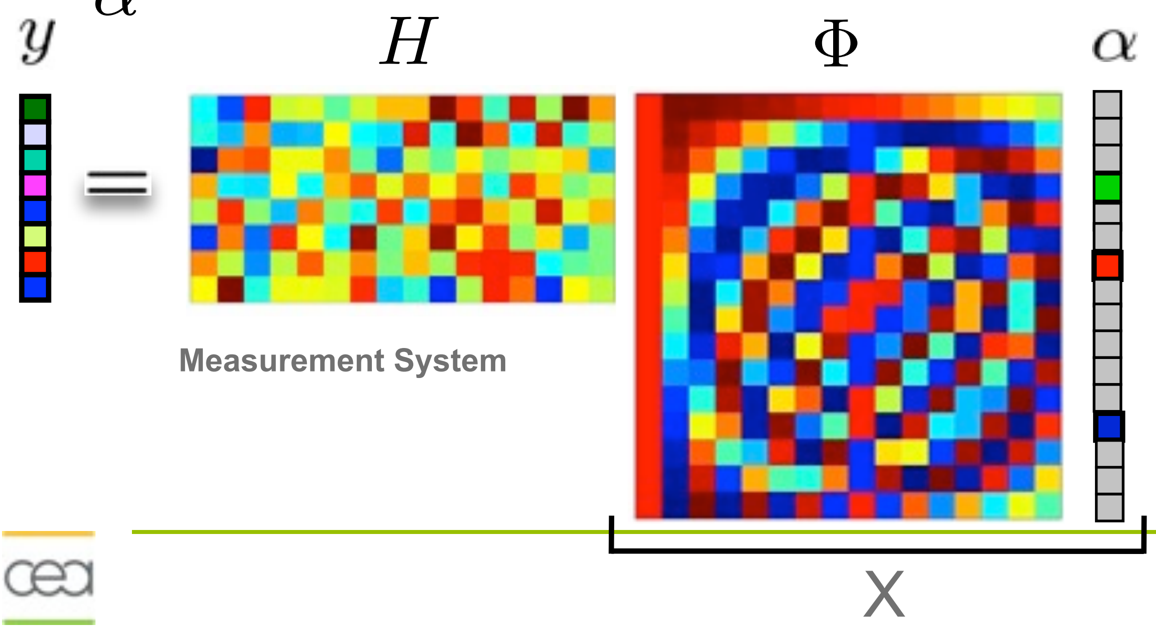
$$Y = HX + N$$

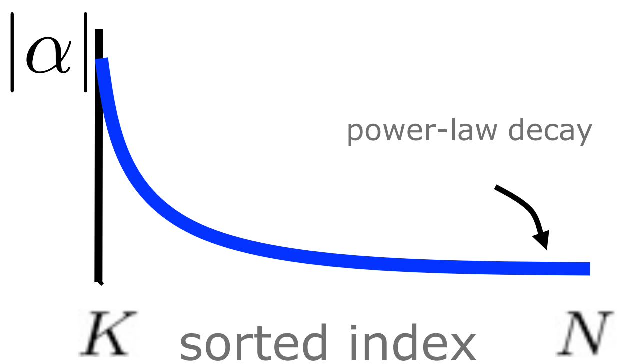
$$X = \Phi\alpha$$

and α is **sparse** or **compressible**

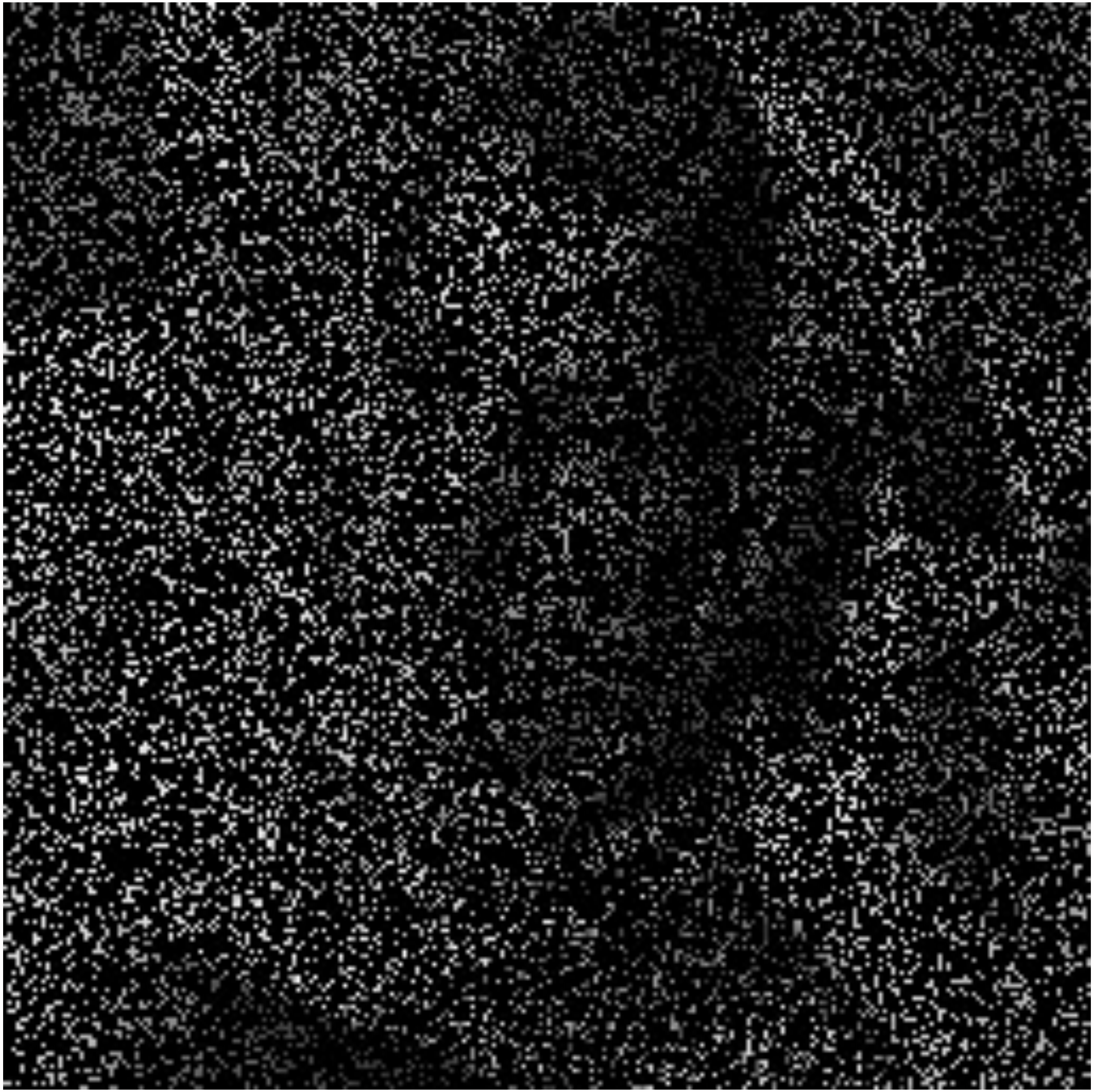
- **Denoising**
- **Deconvolution**
- **Component Separation**
- **Inpainting**
- **Blind Source Separation**
- **Minimization algorithms**
- **Compressed Sensing**

$$\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - H\Phi\alpha\|^2 \leq \epsilon$$





80%



80%

Fundamental Sparse Ingredients

$$\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - H\Phi\alpha\|^2 \leq \epsilon$$

1 - Which Norm ? P in [0,1] $\|X\|_p = (\sum_i \mid X_i \mid^p)^{\frac{1}{p}}$

2 -Constraint versus **Lagrangian formulation**

Constraint formulation: $\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - H\Phi\alpha\|^2 \leq \epsilon$

Lagrangian formulation: $\min_{\alpha} \|Y - H\Phi\alpha\|^2 + \lambda\|\alpha\|_p^p$

3 - **Analysis** versus Synthesis ?

Synthesis form: $\min_{\alpha} \|Y - H\Phi\alpha\|^2 + \lambda\|\alpha\|_p^p$

Analysis form: $\min_{X} \|Y - HX\|^2 + \lambda\|\Phi^t X\|_p^p$

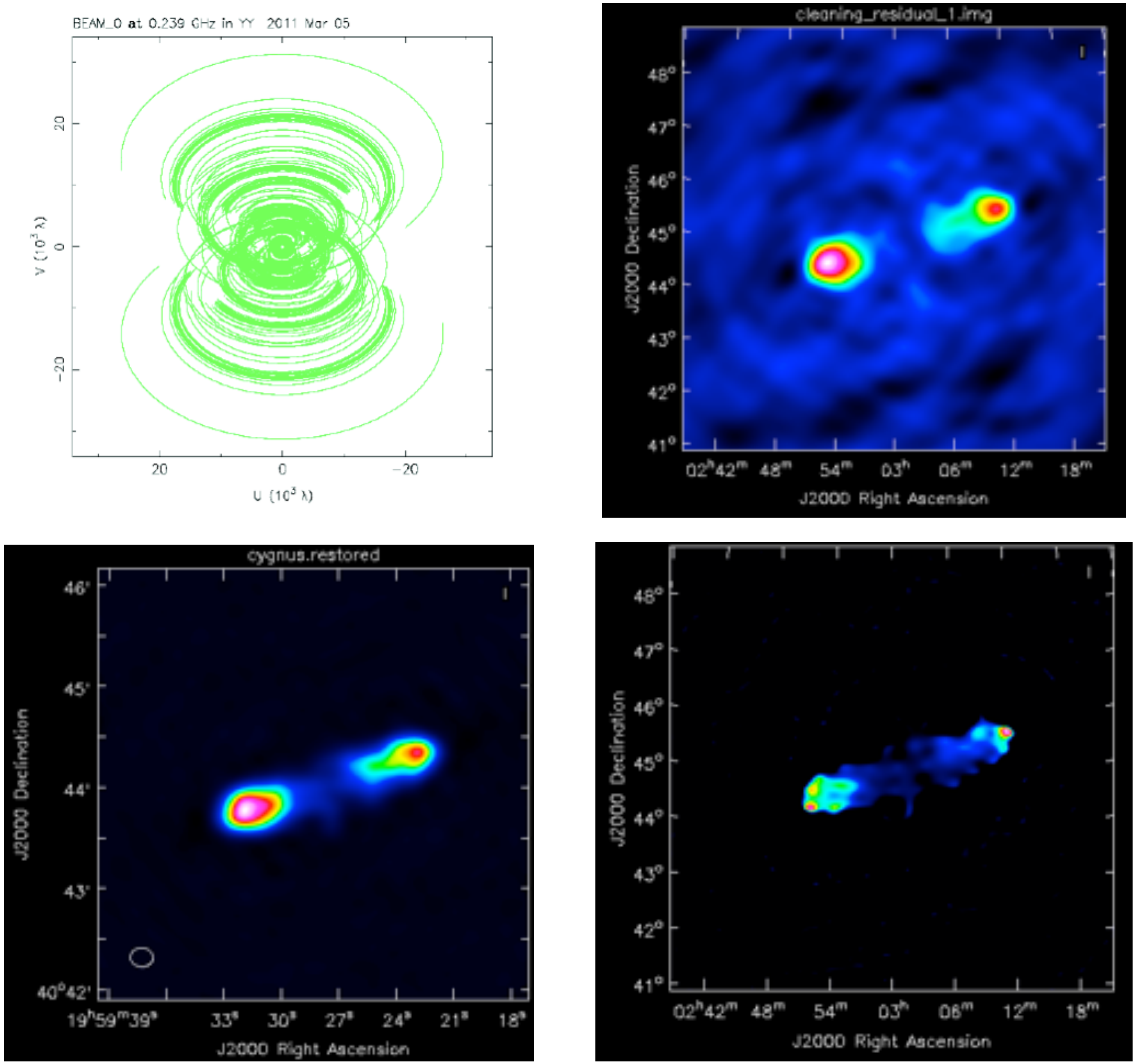
4 - Which dictionary ?

5 - Which noise model ?

6 - Which minimization method ?

7 - How to fix the regularization parameter ?

Compressed Sensing & LOFAR Cygnus A Data



Garsden et al, "**LOFAR Image Sparse Reconstruction**", A&A, 575, A90, 2015.

http://arxiv.org/abs/1406.7242

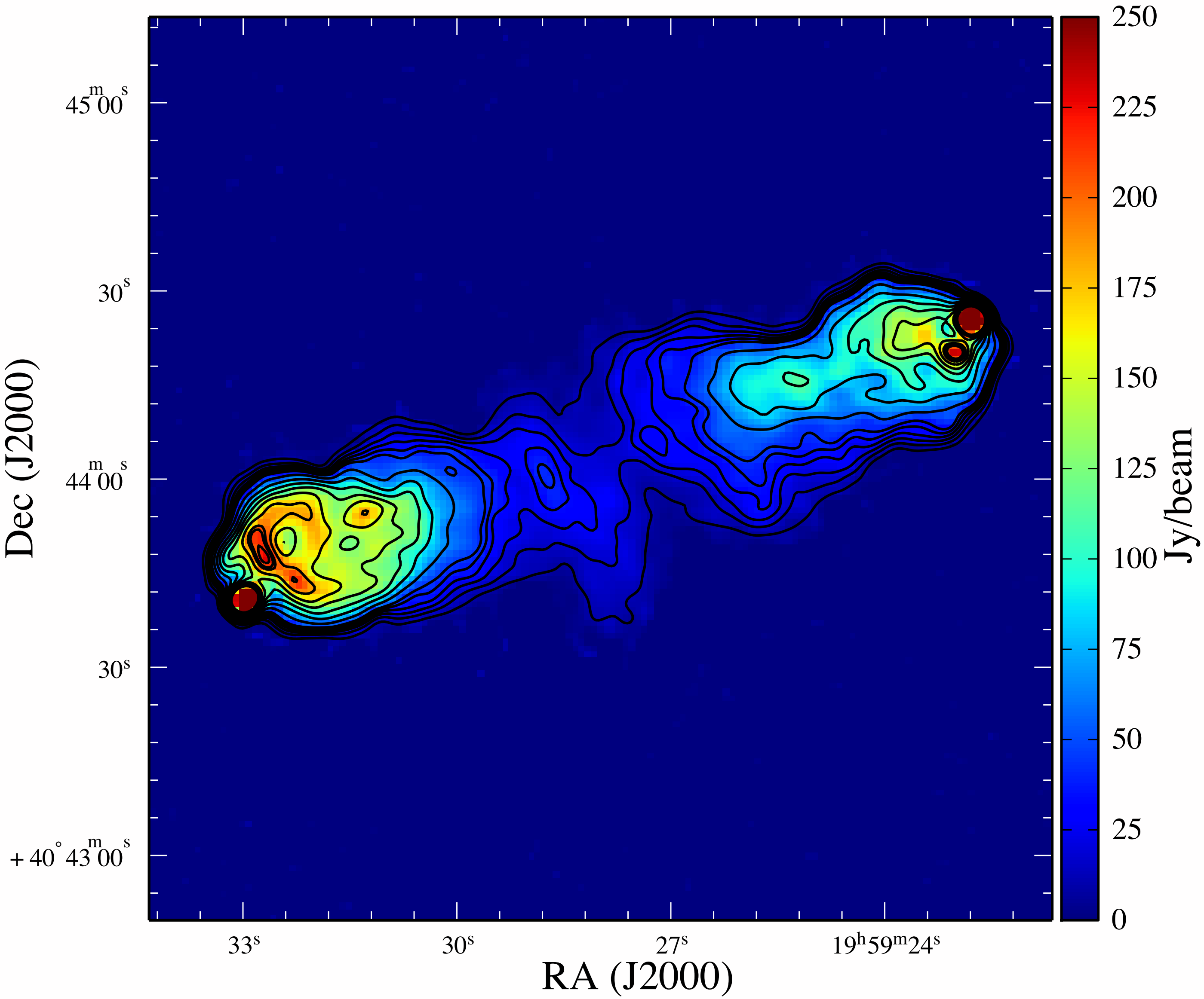
Garsden et al, “LOFAR Image Sparse Reconstruction”, A&A, 575, A90, 2015, ArXiv:1406.7242.

J. Girard

H. Garsden

S. Corbel

C. Tasse



Colorscale: reconstructed 512x512 image of Cygnus A at 151 MHz (with resolution 2.8” and a pixel size of 1”). Contours levels are [1,2,3,4,5,6,9,13,17,21,25,30,35,37,40] Jy/Beam from a 327.5 MHz Cyg A VLA image (Project AK570) at 2.5” angular resolution and a pixel size of 0.5”. **Recovered features in the CS image correspond to real structures observed at higher frequencies.**

Unmixing Using Morphological Diversity

•*J.-L. Starck, M. Elad, and D.L. Donoho, Redundant Multiscale Transforms and their Application for Morphological Component Analysis, Advances in Imaging and Electron Physics, 132, 2004.*

Sparsity Model: we consider a signal as a sum of K components s_k, each of them being sparse in a given dictionary :

$$Y = X_1 + X_2$$

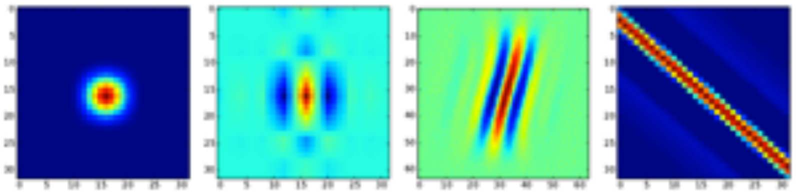
X_1 can be well approximated with few coefficients in a given domain.

X_2 can be well approximated with few coefficients in **another** domain.

$$min_{X_1, X_2} \parallel Y - (X_1 + X_2) \parallel^2 + C_1(X_1) + C_2(X_2)$$

$$C_1(X_1) = \parallel \Phi_1^t X_1 \parallel_p$$

$$C_2(X_2) = \parallel \Phi_2^t X_2 \parallel_p$$



Morphological Component Analysis (MCA)

$$X = \sum_{k=1}^{L} x_k \qquad \min_{X} \| Y - \sum_{k=1}^{L} x_k \|^2 + \lambda \sum_{k=1}^{L} \| \phi_k^* x_k \|_p$$

- Initialize all x_k to zero
- Iterate j=1,...,Niter
  - Iterate k=1,..,L

    Update the kth part of the current solution by fixing all other parts and minimizing:

$$\min_{x_k} \| Y - \sum_{i=1, i \neq k}^{L} x_i - x_k \|^2 + \lambda \| \phi_k^* x_k \|_p$$

Which is obtained by a simple **hard**/soft thresholding of :

- Decrease the threshold $\lambda^{(j)}$

MCA based artifact removal for SNe detection

SNa and its host galaxy

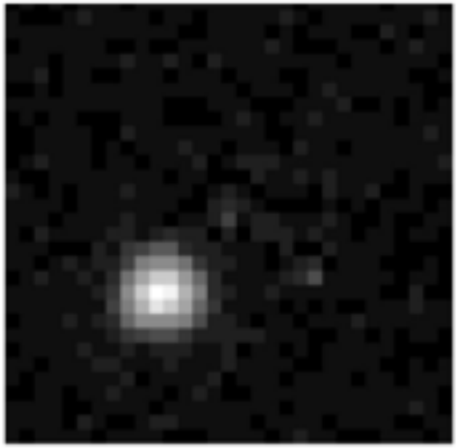
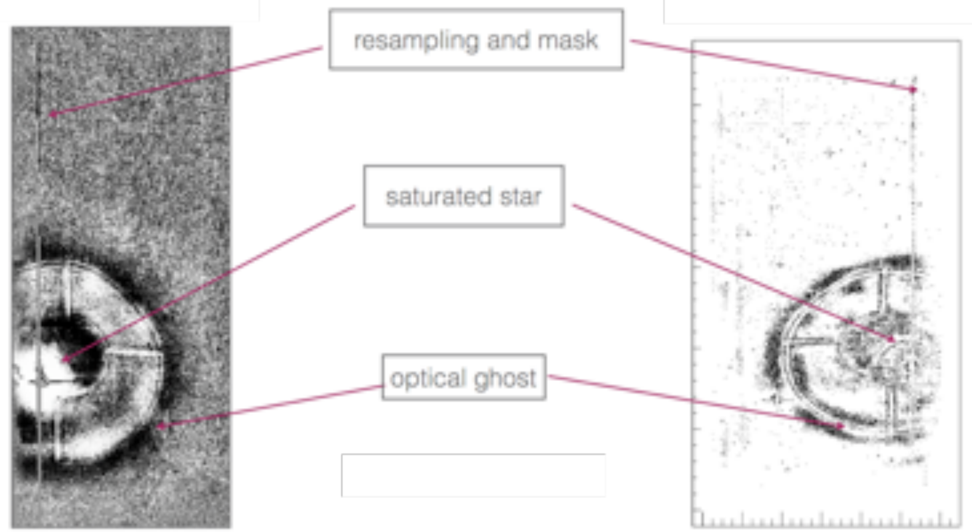


Image after subtraction of the reference sky



Subtracted image

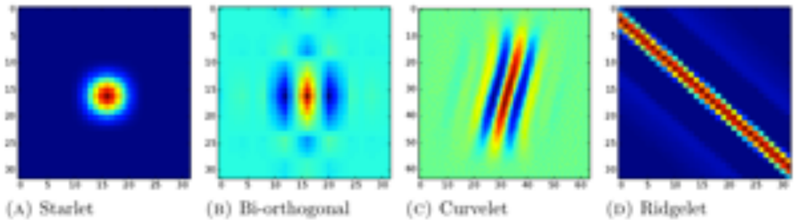
Detection catalogue

- SNe are detected by subtraction of a reference image.

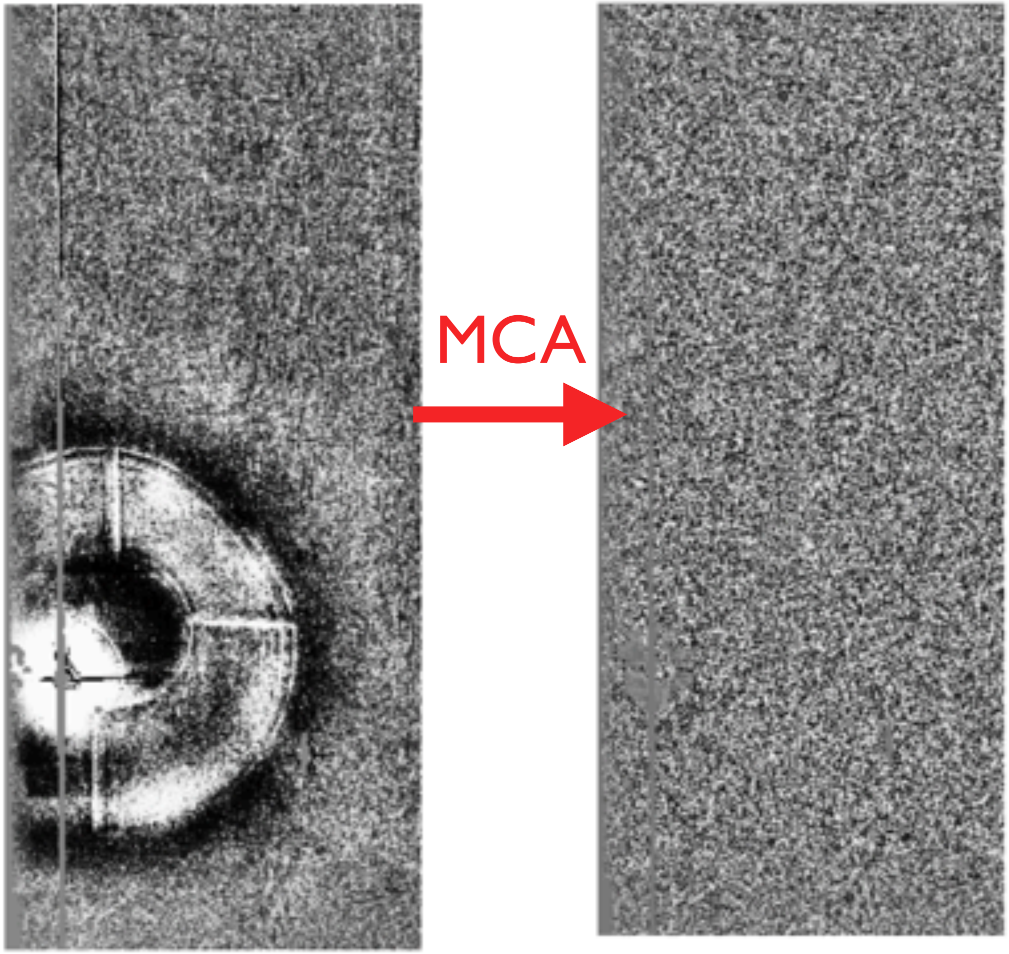
- In practice, subtracted image are contaminated by artifact which make the detection difficult

Artifact removal for SNe detection

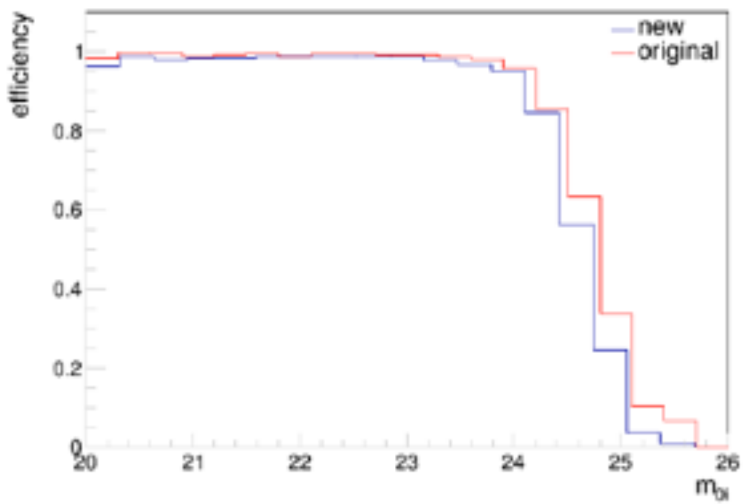
- Möller, et al, 2015, SNIa detection in the SNLS photometric analysis using Morphological Component Analysis, 04, Id 041, JCAP, arxiv:1501.02110.



Dictionaries used for the analysis



MCA cleaning of a subtracted image



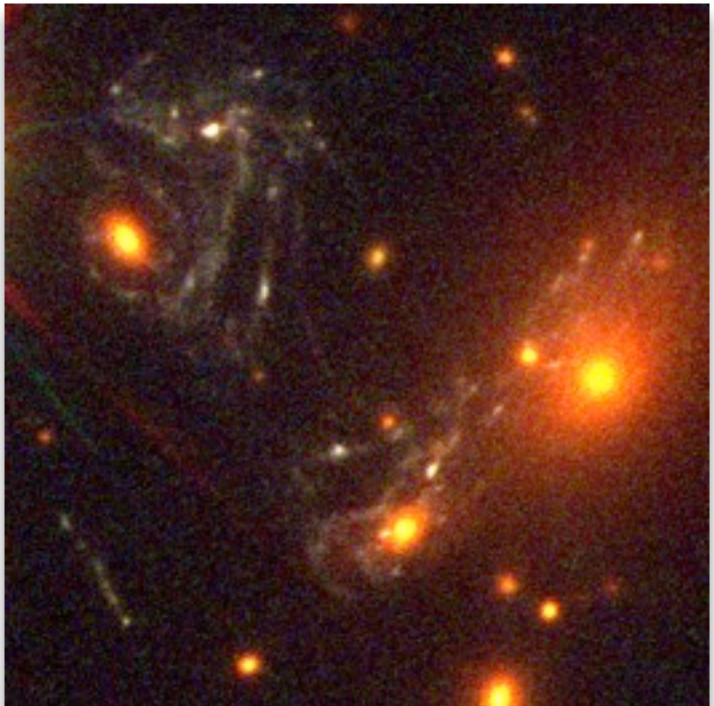
Similar detection efficiency but greatly reduced number of spurious detections

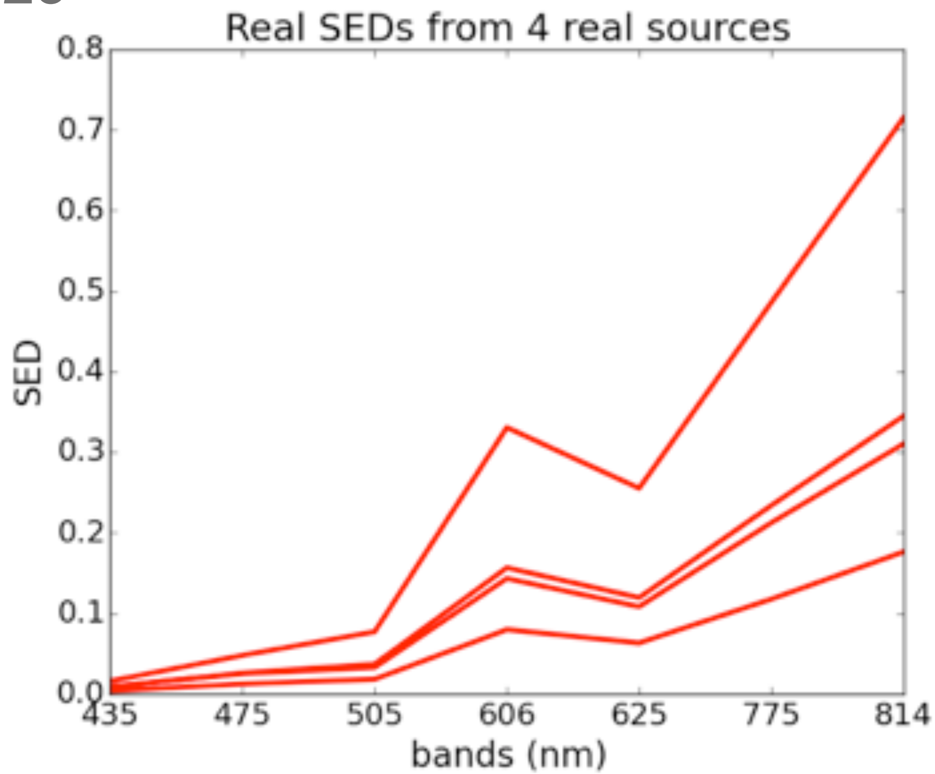
Multichannel data

GOAL: separate the foreground cluster galaxies (red) from the background lensed galaxy (blue).

$$Y_i = H_i * \sum_{s=1}^{S} a_{i,s} X_s + N$$

galaxy cluster MACS~J1149+2223





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Morpho-Spectral Diversity

$$Y_i = H_i * \sum_{s=1}^{S} a_{i,s} X_s + N$$

$\mathrm{H}_i = \mathrm{Id}$

The fixing matrix A is assumed to be known

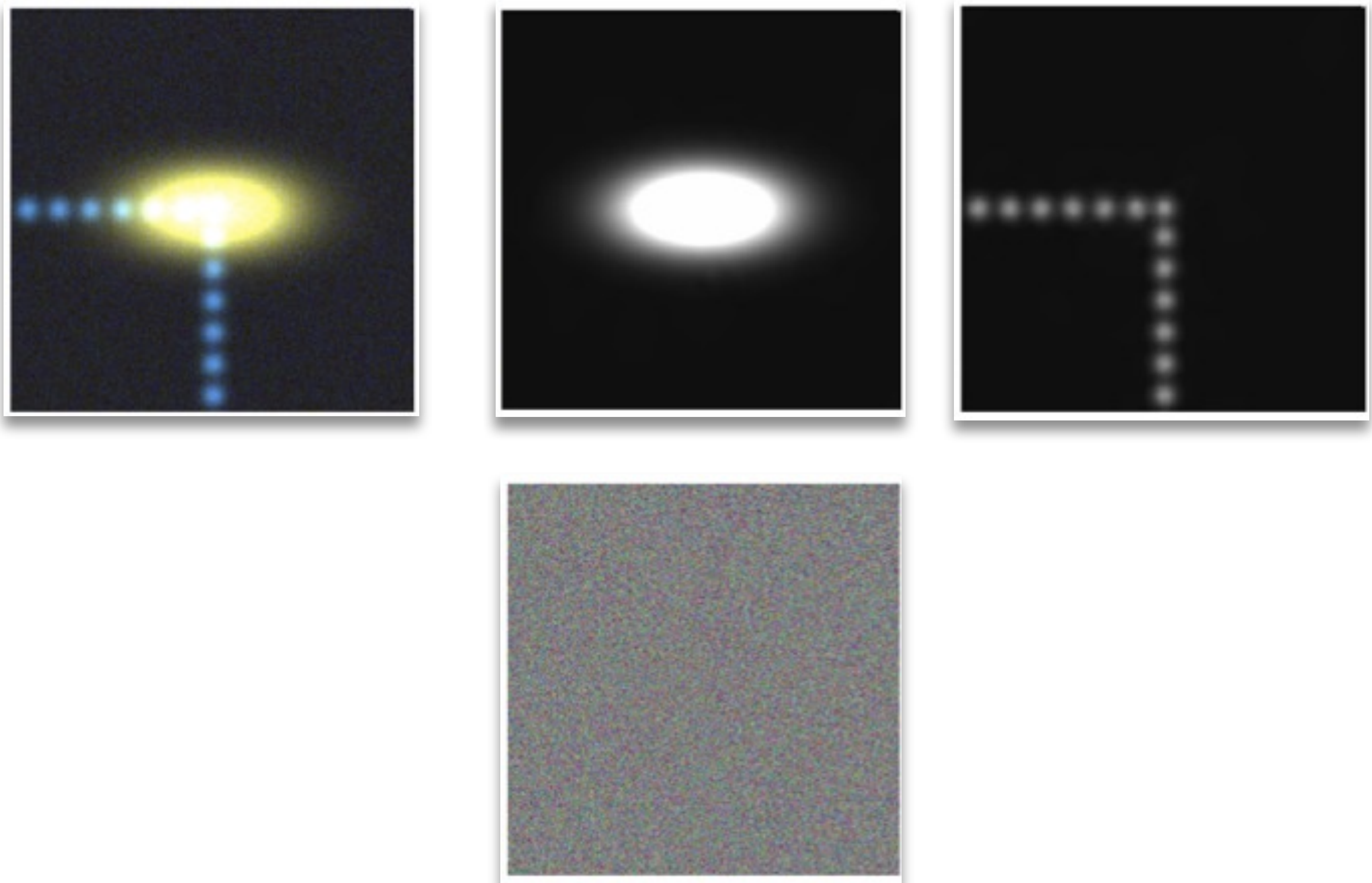
X_s is sparse in $\Phi_s = \mathcal{S}_s \Psi_s$

$\forall s, \Psi_s = \Psi$, where Ψ is the starlet transform.

$$\min_X \parallel Y - AX \parallel^2 + \sum_{j=1}^{J} \lambda_j \parallel \Psi^* x_j \parallel_0$$

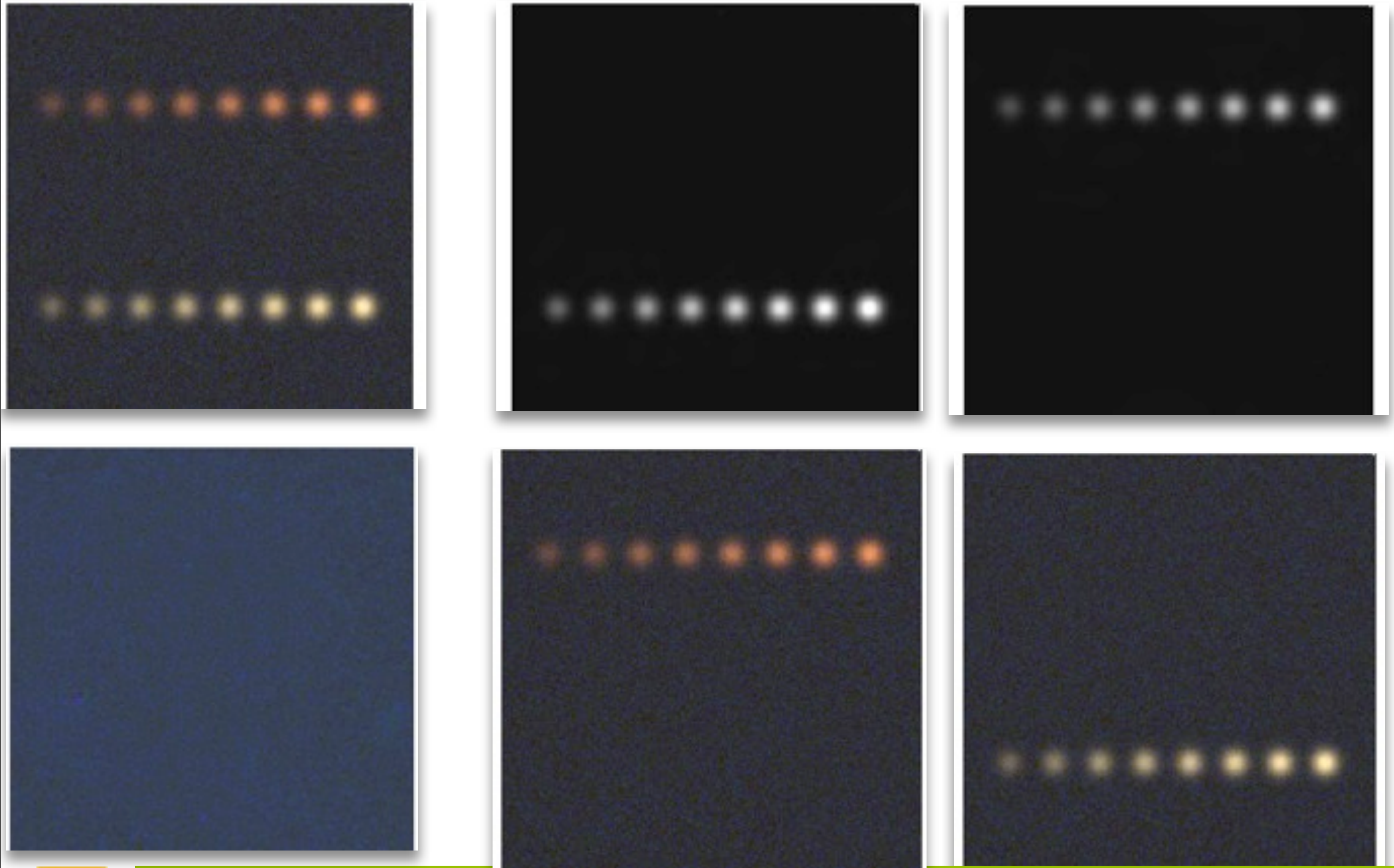
R. Jospeh, F. Courbin and J.-L. Starck, "Multi-band morpho-Spectral Component Analysis Deblending Tool (MuSCADeT): deblending colourful objects", A&A, 589, id.A2, pp 10, 2016.

Multichannel data



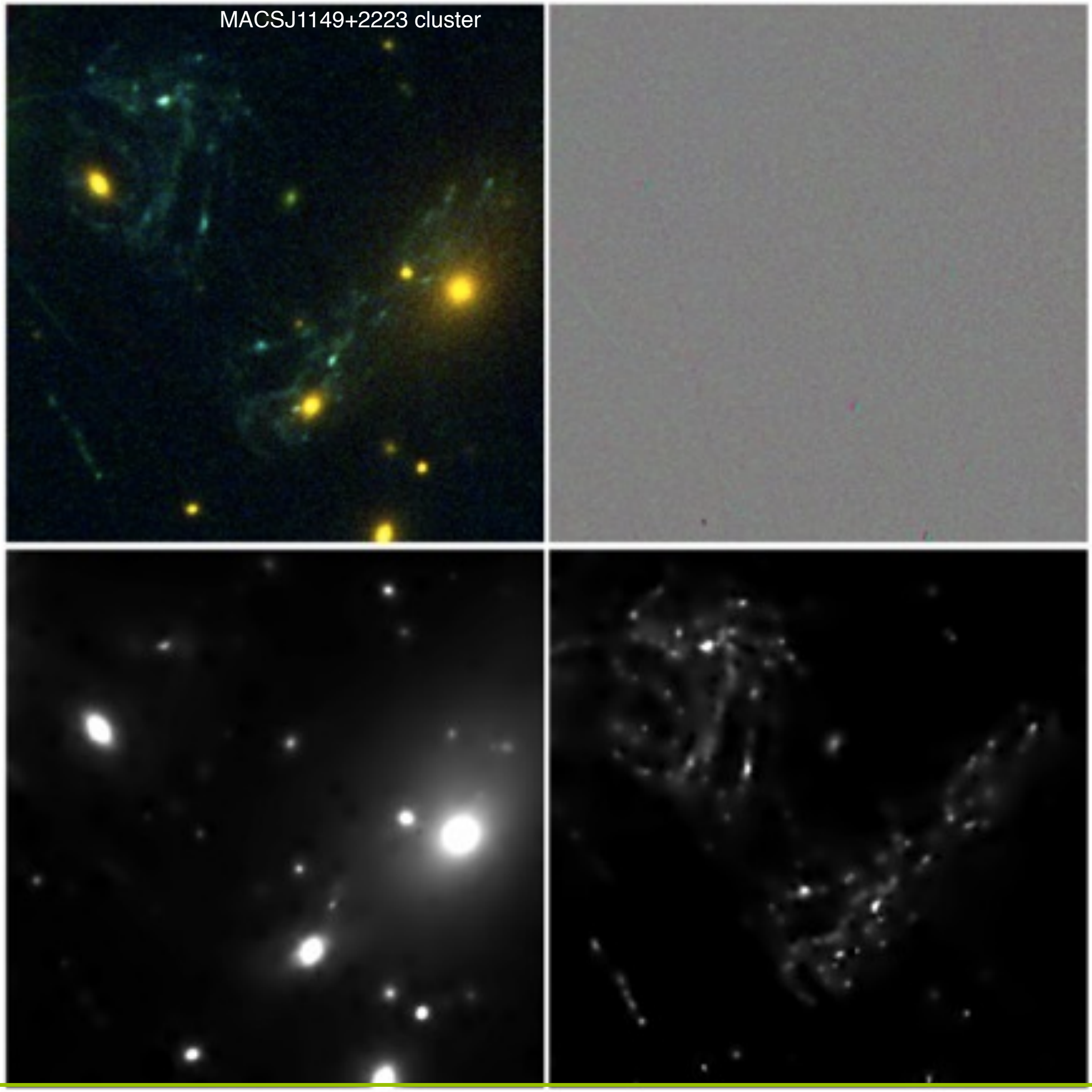
No SED variation.

Multichannel data



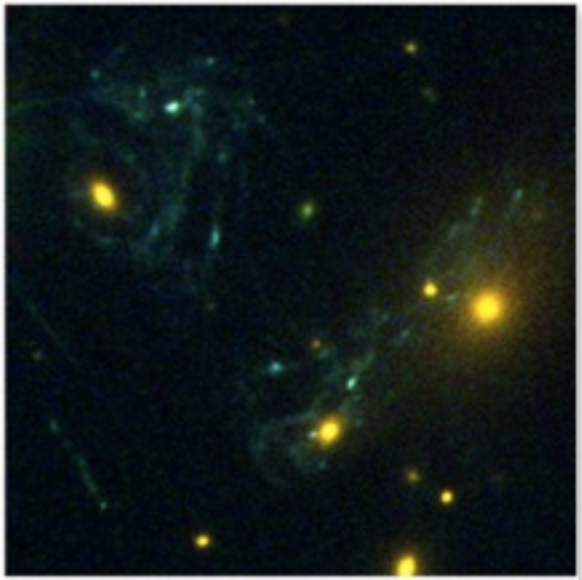
realistic SED variation.

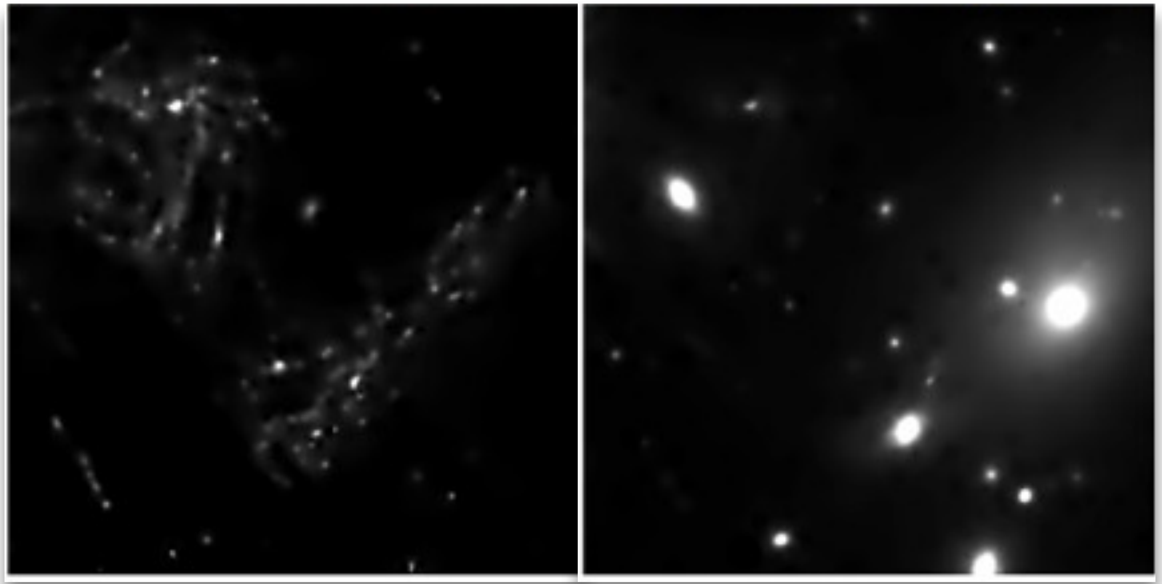
Multichannel data

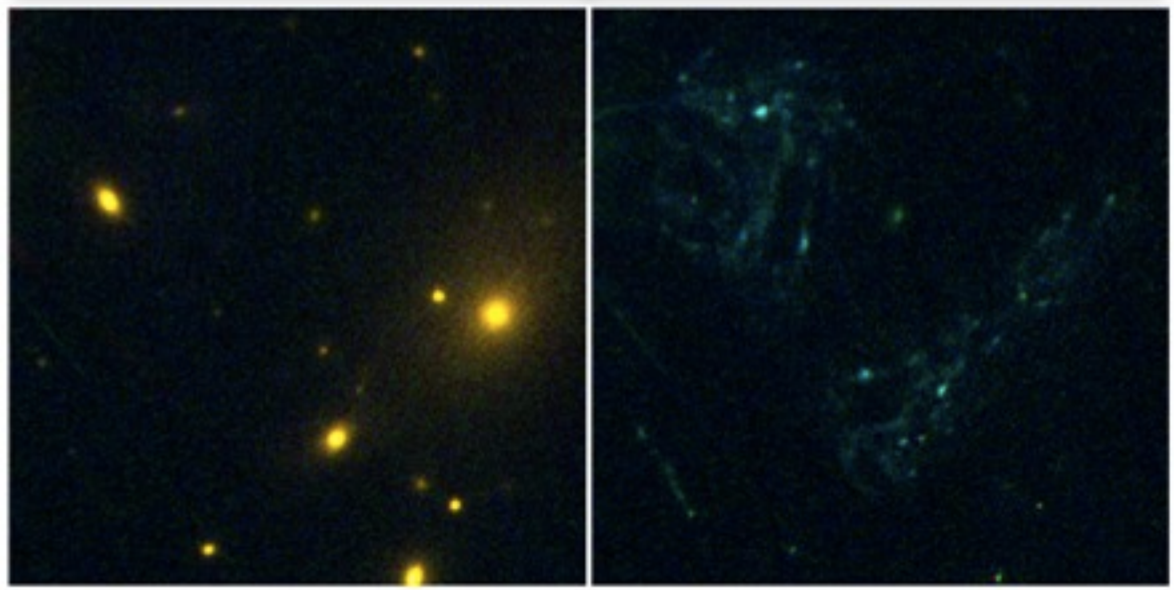


Realistic SEDs variation.

Multichannel data



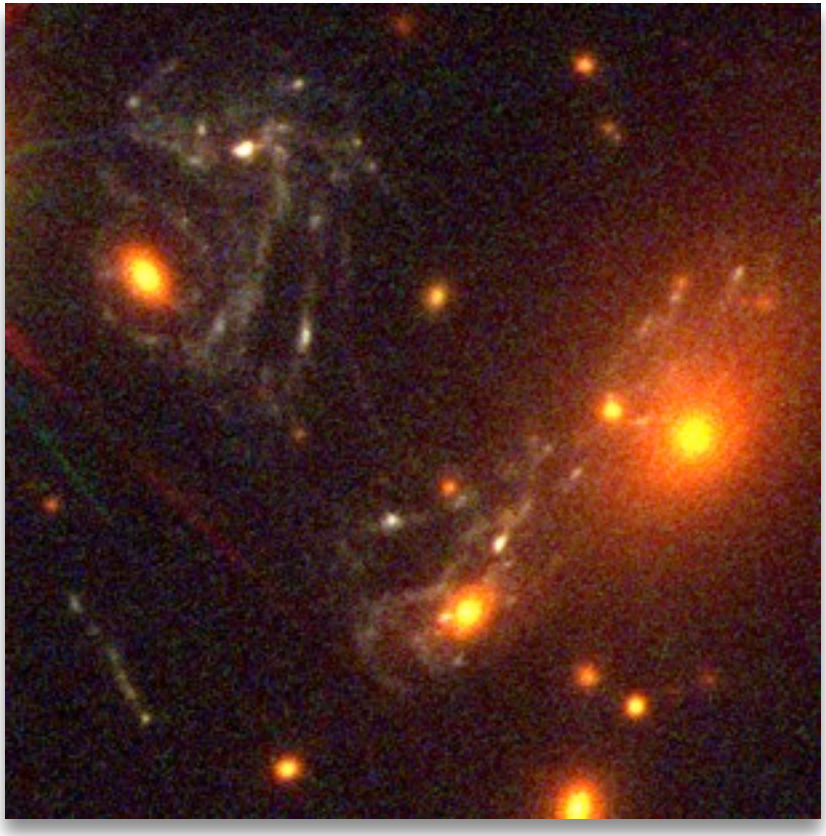


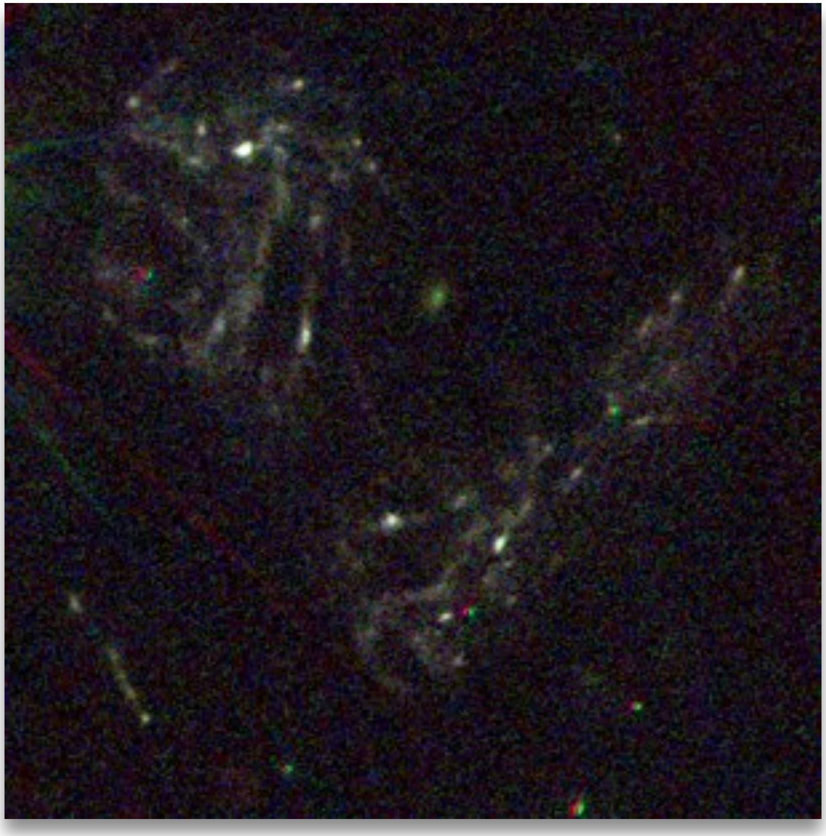


CosmoStat Lab

Multichannel data

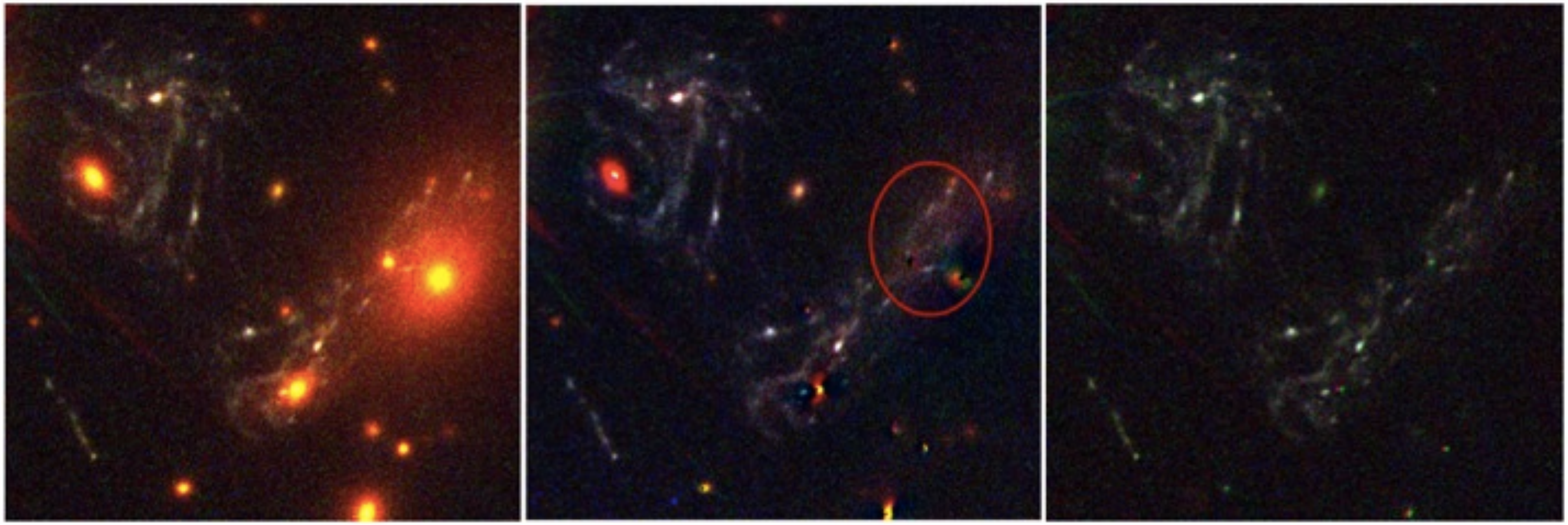
galaxy cluster MACS~J1149+2223





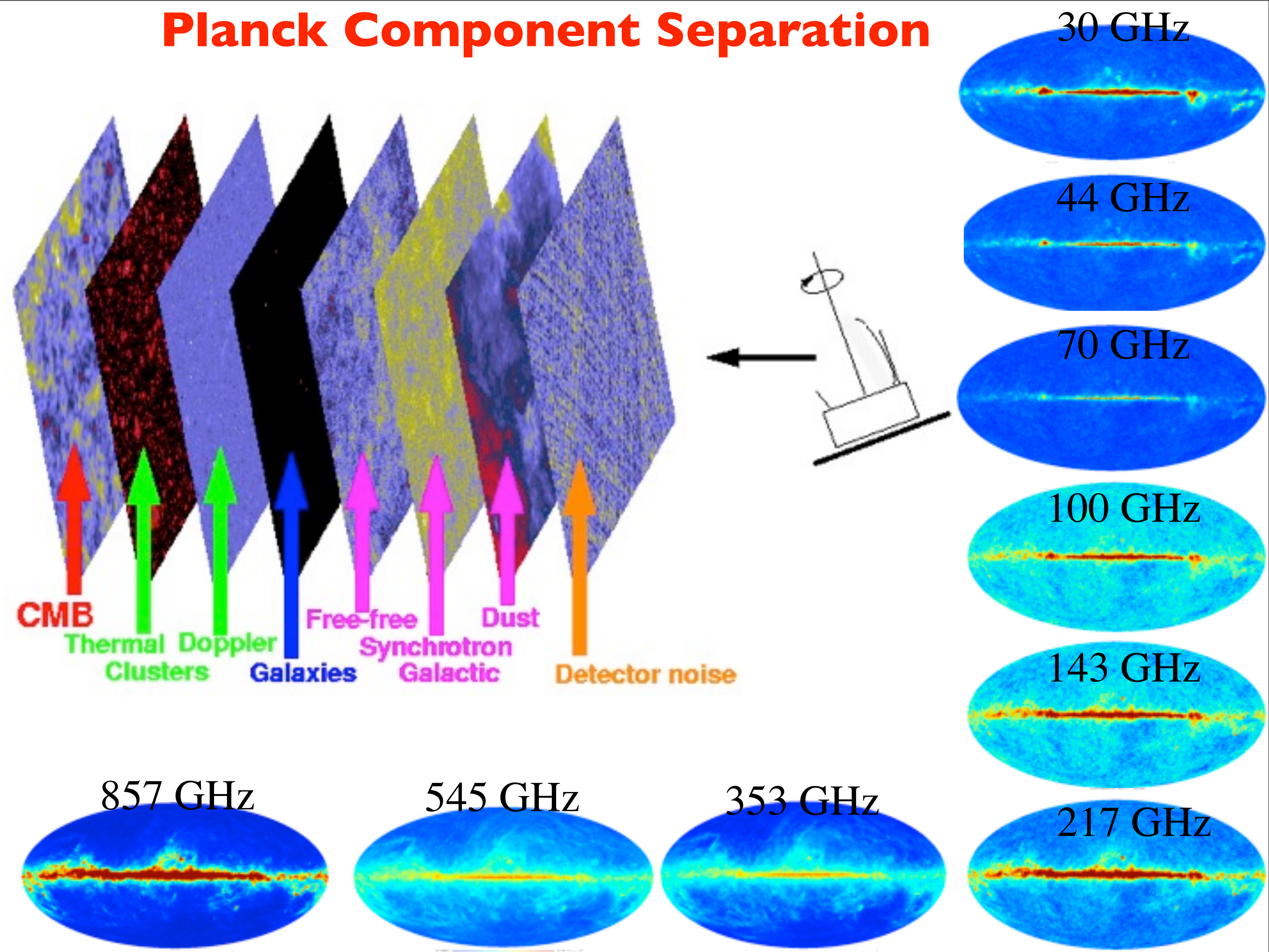
Multichannel data

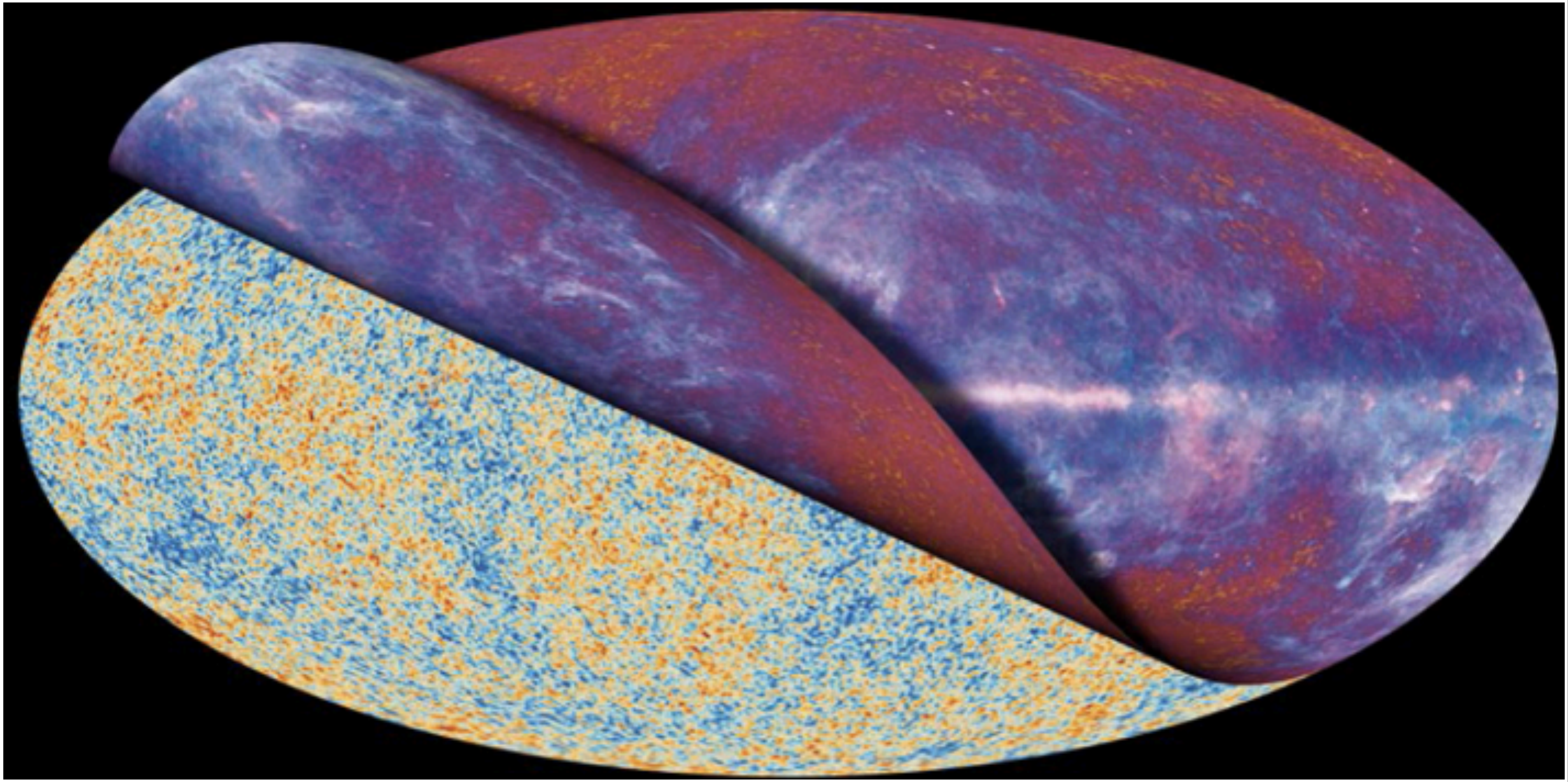
MACS~J1149+2223 cluster



galfit subtraction of the galaxy members

Planck Component Separation





Sparse Component Separation: the GMCA Method

A and S are estimated alternately and iteratively in two steps :

•J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Sparsity, Morphological Diversity and Blind Source Separation", IEEE Trans. o Image Processing, Vol 16, No 11, pp 2662 - 2674, 2007.
•.J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Blind Source Separation: The Sparsity Revolution", Advances in Imaging and Electron Physics , Vol 152, pp 221 -- 306, 2008.

$$\mathbf{X} = \mathbf{A}\mathbf{S}$$

1) Estimate S assuming A is fixed (iterative thresholding) :

$$\{S\} = \mathrm{Argmin}_S \sum_j \lambda_j \| s_j \mathbf{W} \|_1 + \| \mathbf{X} - \mathbf{A}\mathbf{S} \|^2_{F,\mathbf{\Sigma}}$$

2) Estimate A assuming S is fixed (a simple least square problem) :

$$\{A\} = \mathrm{Argmin}_A \| \mathbf{X} - \mathbf{A}\mathbf{S} \|^2_{F,\mathbf{\Sigma}}$$

Component Separation: more problems

1) The beam:

$$\forall i;\ y_i = b_i \star \left(\sum_j a_{ij} x_j \right) + n_i$$

Globally:

$$\mathbf{Y} = \mathcal{H}\left(\mathbf{AX}\right) + \mathbf{N}$$

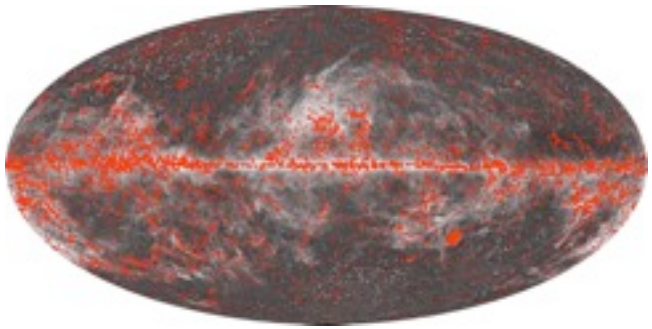
$\mathcal{H}$ **is singular !**

where $\mathcal{H}$ is the multichannel convolution operator

2) Spectral behavior **varies spatially** for some components (dust, synchroton).

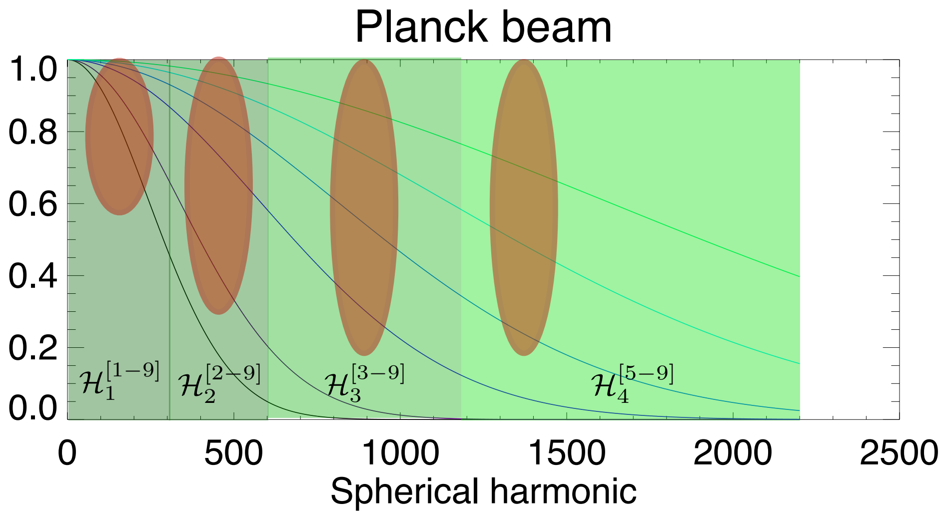
$$\mathbf{Y}[k] = \mathcal{H}\left(\mathbf{A_k X}\right)[k] + \mathbf{N}[k]$$

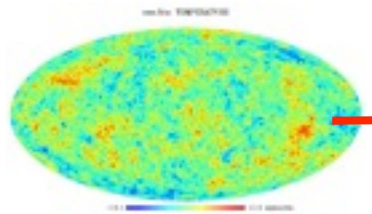
3) Point sources:

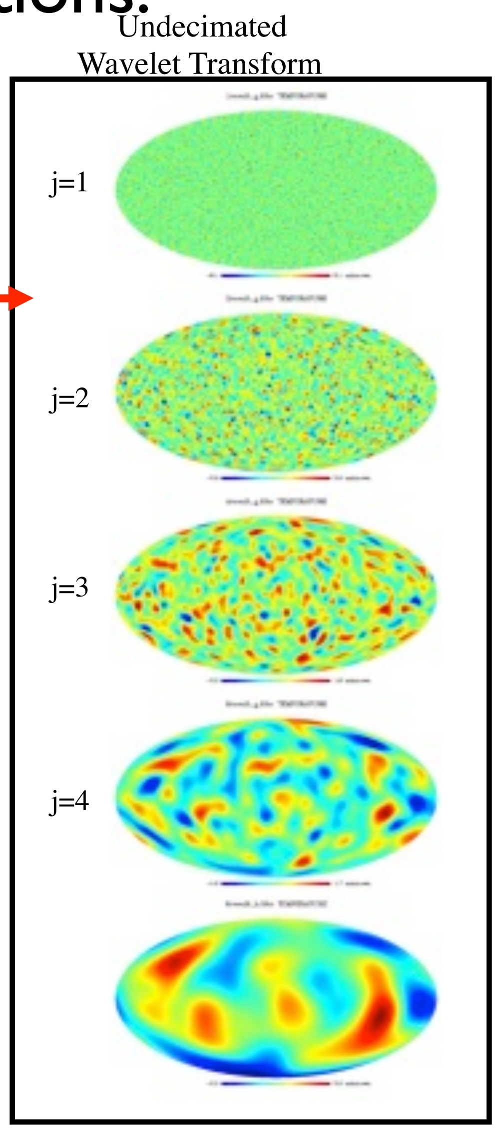


15

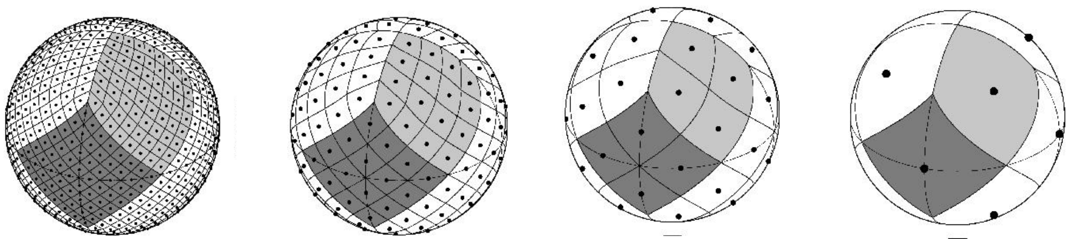
=> Use Wavelets to work at different resolutions:







=> Assume the mixing matrix varies smoothly
Partitionning of the Wavelet Scales

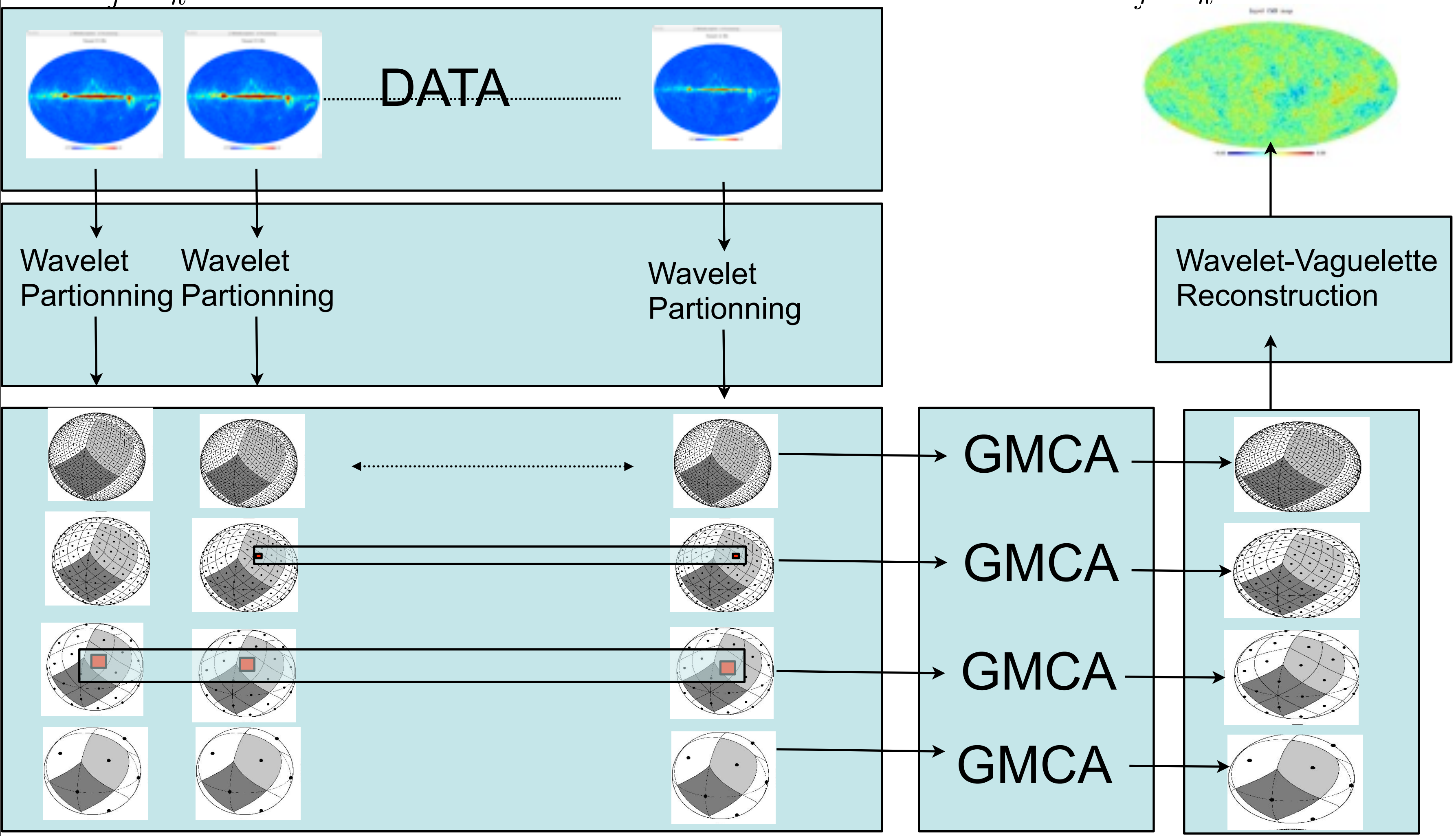


15

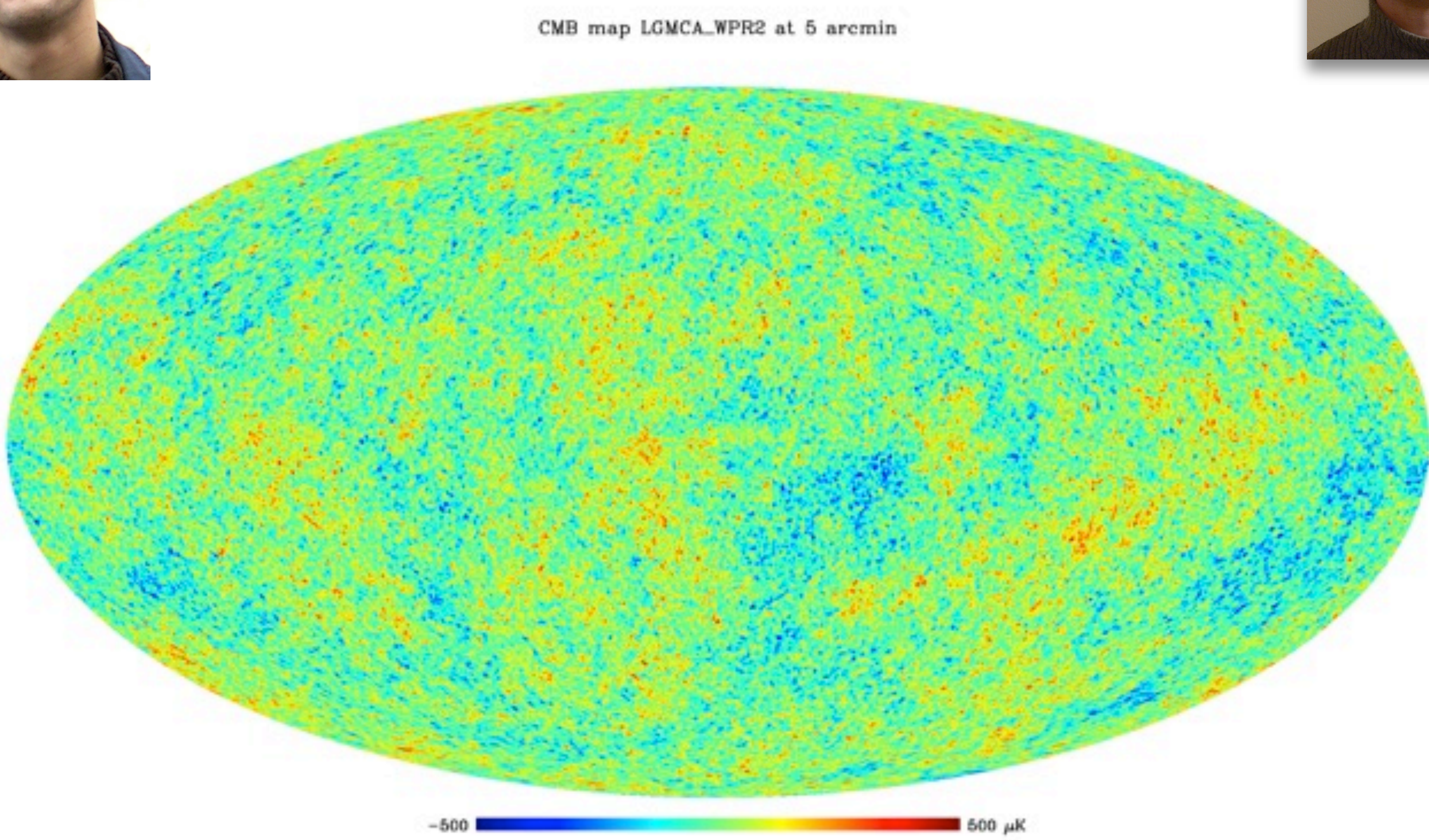
Wavelet-Vaguelette GMCA Decomposition

$$f = \sum_j \sum_k \langle Kf, \Psi_{j,k} \rangle \, \psi_j, k \quad \text{with } K^* \Psi_{j,k} = \psi_{j,k}$$

$$\tilde{f} = \sum_j \sum_k \Delta(\langle y, \Psi_{j,k} \rangle) \psi_j, k$$

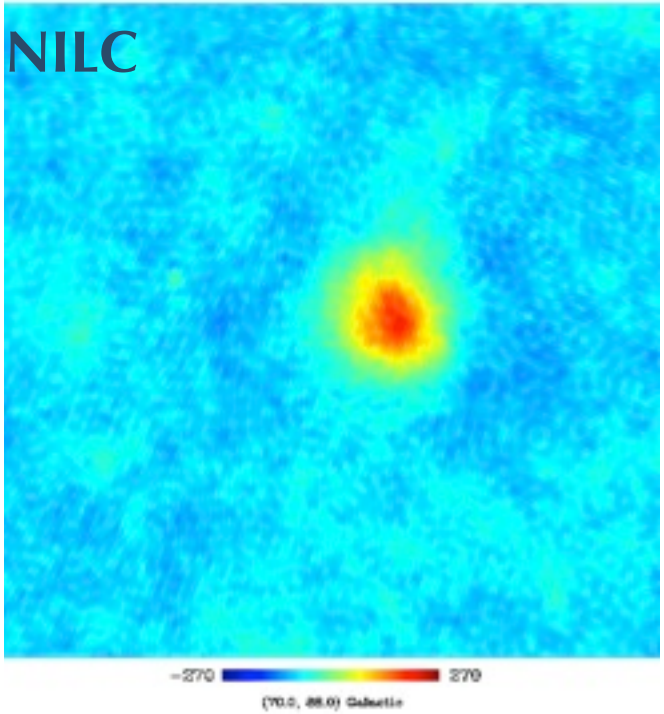


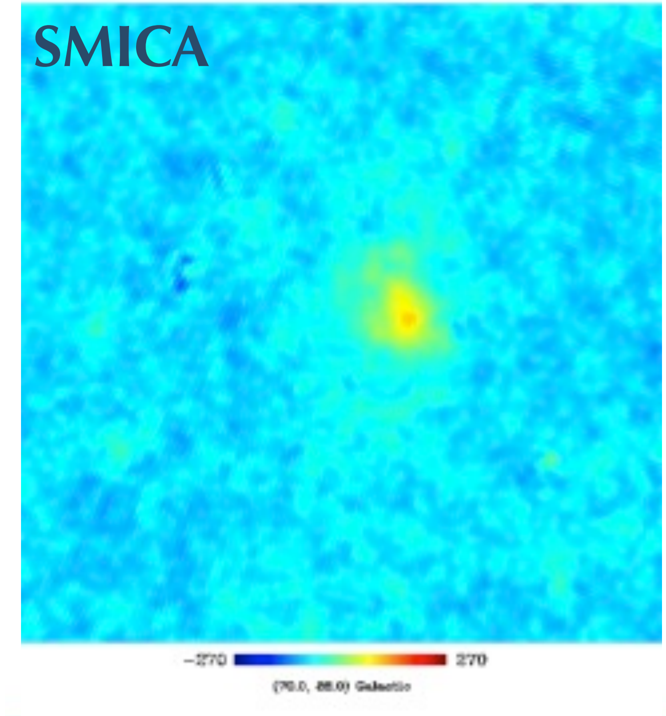
Full Sky Sparse WMAP + Planck-PR2 Map

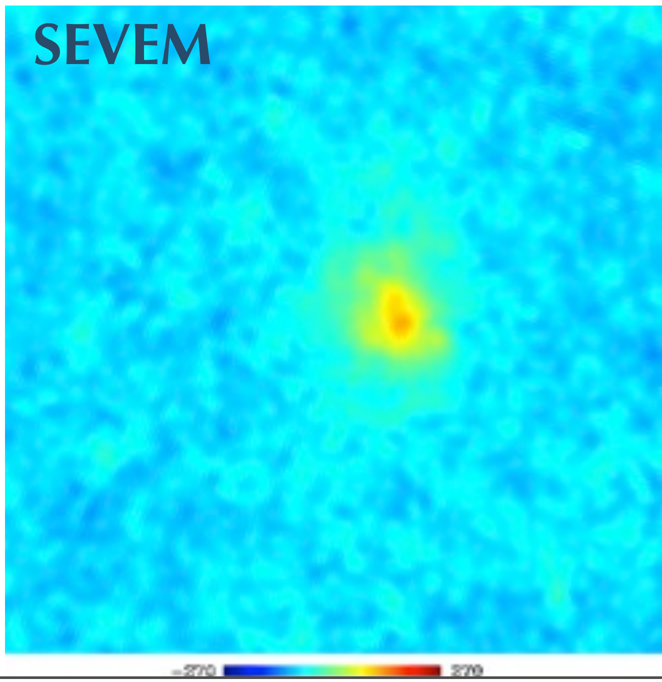


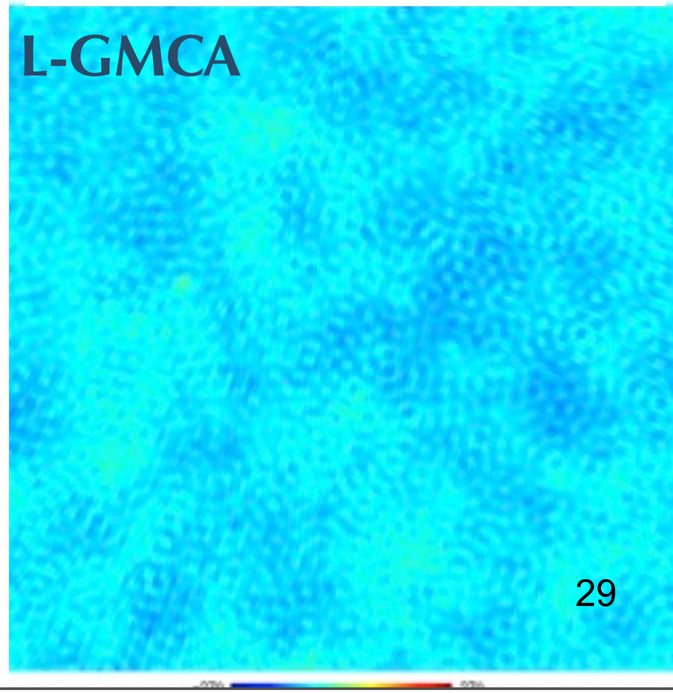
Bobin J., Sureau F., Starck J-L, Rassat A. and Paykari P., Joint Planck and WMAP CMB map reconstruction, A&A, 563, 2014

***Bobin J., Sureau F., Starck, CMB reconstruction from the WMAP and Planck PR2 data, in press, A&A, 2016.* [arXiv:1511.08690]**

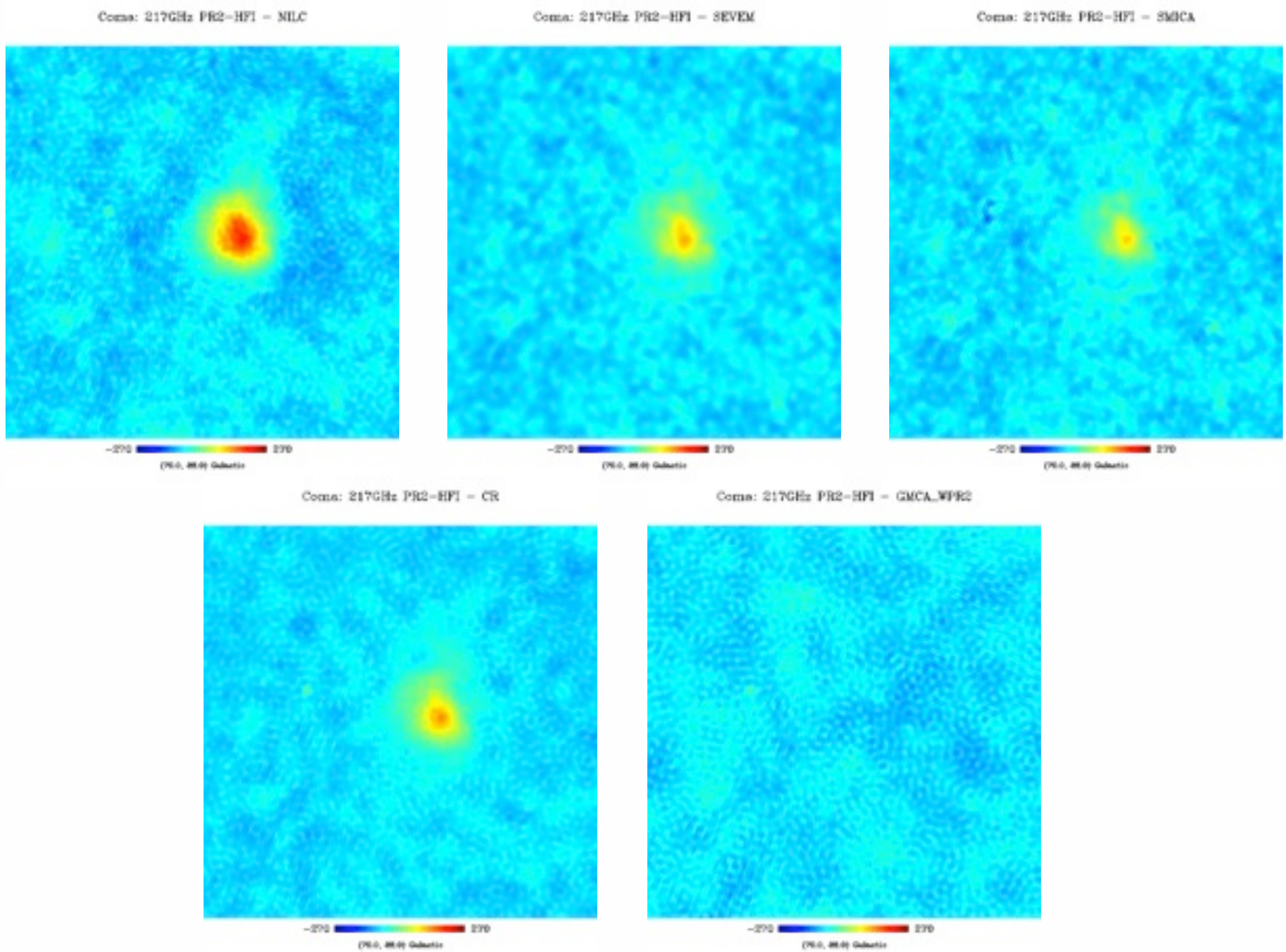

NILC
−270
270


SMICA
−270
270

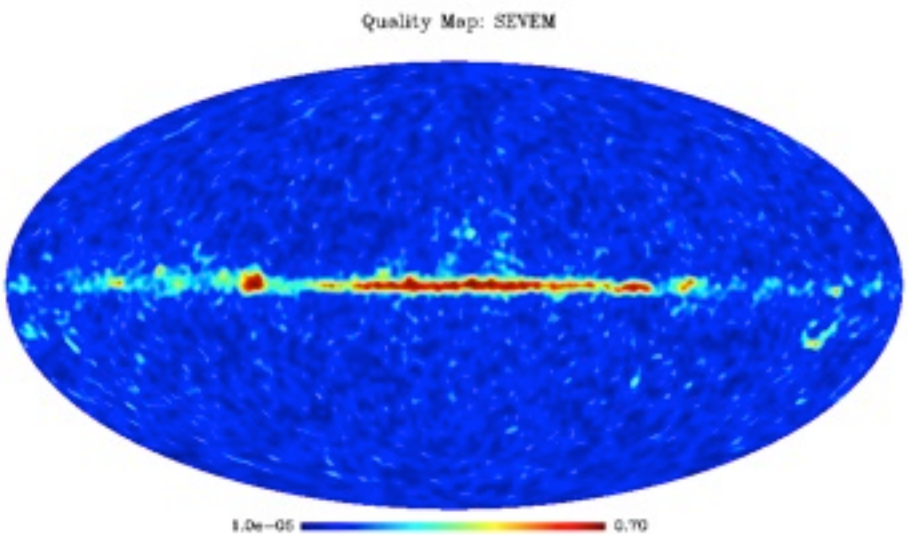

SEVEM

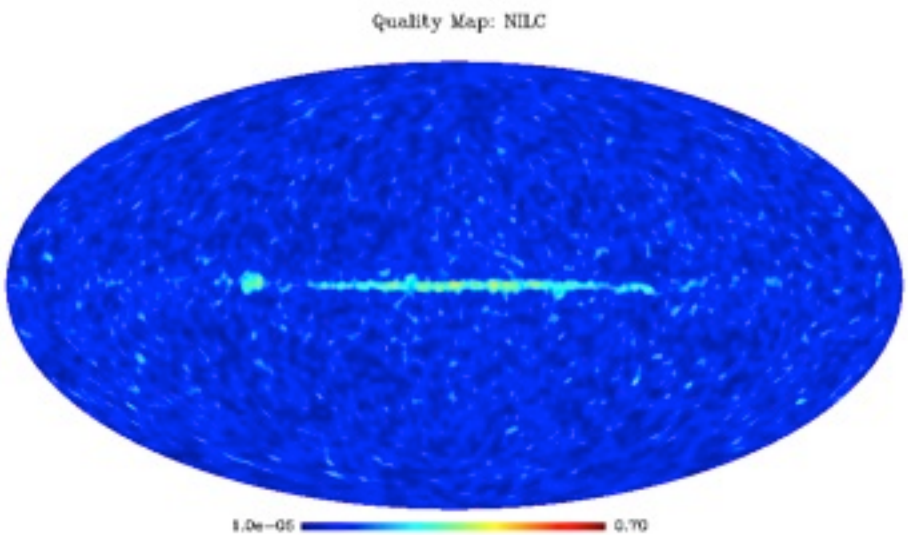

L-GMCA

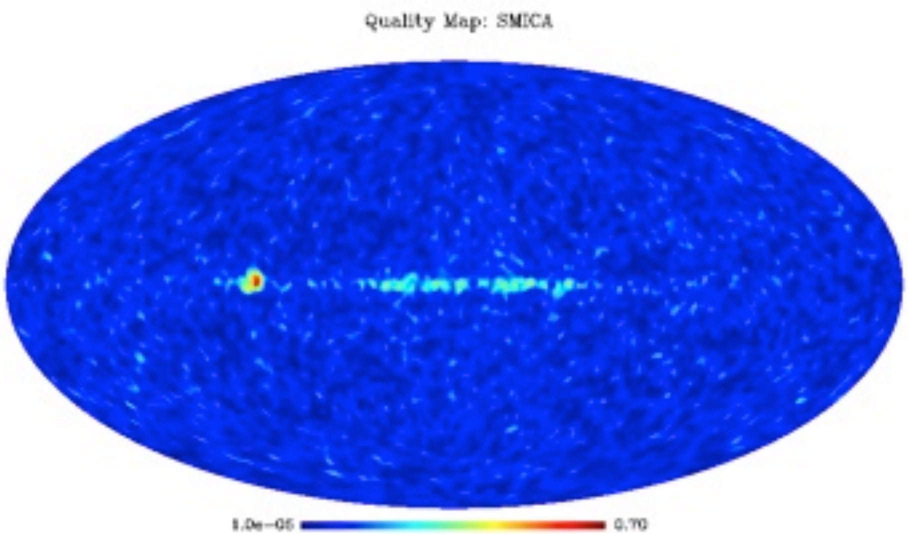
Traces of tSZ effect



Quality map


Quality Map: SEVEM
1.0e-05
0.70


Quality Map: NILC
1.0e-05
0.70


Quality Map: SMICA
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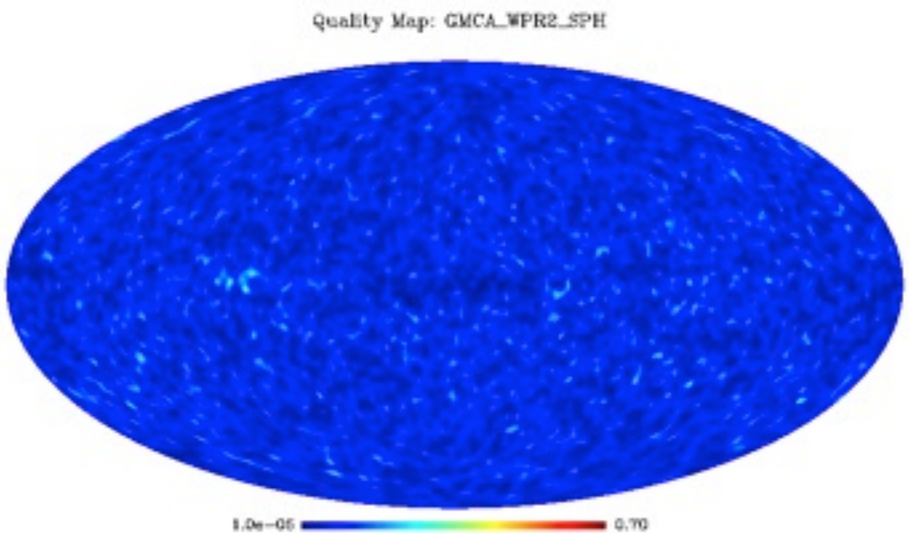
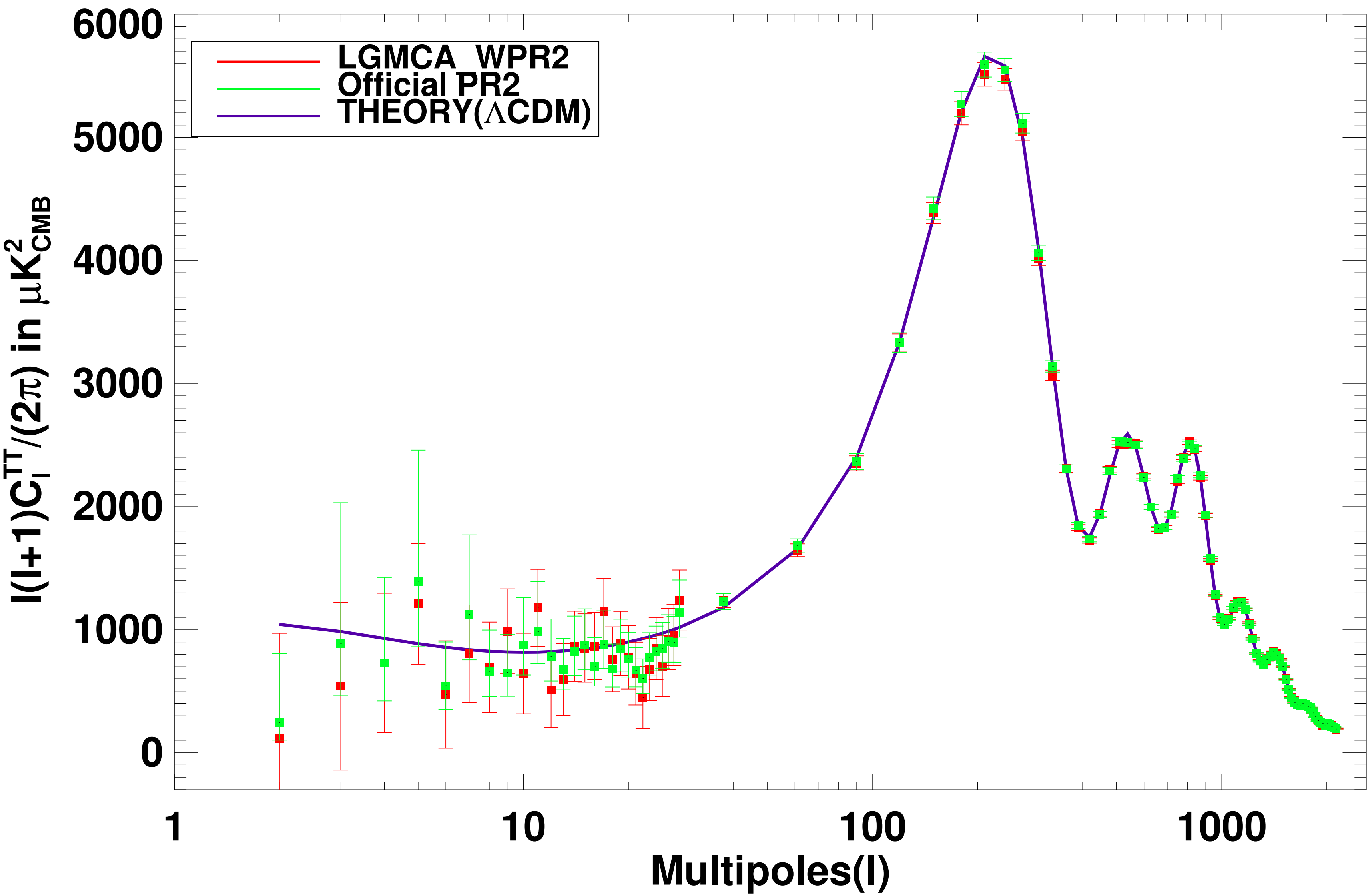
0.70

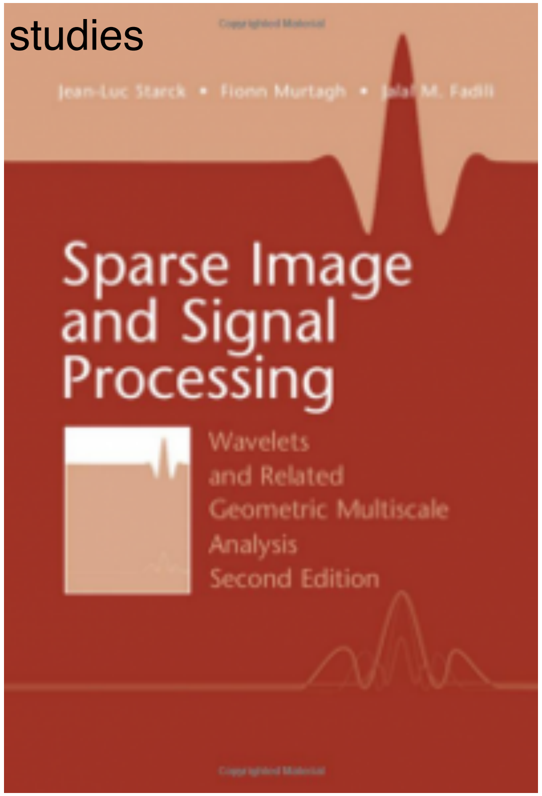

Quality Map: GMCA_WPR2_SPH
1.0e-05
0.70

Power Spectrum



Conclusions

- ✓ Sparse Regularization techniques are very efficient for
  - ➡ Component separation http://www.cosmostat.org/research/statistical-methods/gmca/
    - ★ Artefact removal
    - ★ Blue/red galaxies separation
  - ➡ Joint CMB map reconstruction from WMAP and Planck data
    - ★ High quality and full sky CMB map, from WMAP and Planck-PR2 data
    - ★ Masking is even not necessary anymore for large scale studies
    - ★ http://www.cosmostat.org/research/cmb/planck_wpr2/



- ✓ Reproducible Research

  http://www.cosmostat.org/software.html

- ✓ Perspective
  - ➡ Extend the sparse component separation to polarized data.
  - ➡ Develop sparsity techniques for SKA and LSST/Euclid (Francois Lanusse Talk this afternoon on weak lensing and sparsity.