# Midterm I Solutions

## Problem 1

Since f is a density function $\int_{-\infty}^{\infty} f(x)dx = 1$. That is

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = \int_{0}^{1}(a+bx^2)dx$$

$$= ax|_0^1 + b\frac{x^3}{3}\Big|_0^1 = a + \frac{b}{3} = 1 \Rightarrow a = 1 - \frac{b}{3}$$

On the other hand,

$$EX = \int_{-\infty}^{\infty} xf(x)dx = \int_{0}^{1}(ax+bx^3)dx = a\frac{x^2}{2}\Big|_0^1 + b\frac{x^4}{4}\Big|_0^1 = \frac{a}{2} + \frac{b}{4} = \frac{3}{5}$$

Replacing a by $1-\frac{b}{3}$, we get $\frac{1-\frac{b}{3}}{2} + \frac{b}{4} = \frac{3}{5} \Rightarrow b = \frac{6}{5}$ and $a = \frac{3}{5}$

## Problem 2

Let X= the number of functioning components in a 2-out-of-3 circuit
1. n= 3 components
2. two possible outcomes: the component function or not
3. the trials are independent since each component functions independently
4. the probability of success p=0.9 is constant from trial to trial

Therefore X~ Bin(n=3, p=0.9)

P(circuit functions)= P(X$\geq$2)= P(X=2)+P(X=3)= $\binom{3}{2}(0.9)^2(1-0.9)$ + $\binom{3}{3}(0.9)^3(1-0.9)^0$ = 0.243+ 0.729= 0.972

## Problem 3

**a.**
Let X= the number of defective screws in a package
1. n= 10 screws
2. two possible outcomes: the screw is defective or not
3. the trials are independent since it is know that the screws are defective independently of each other
4. the probability of success p=0.01 is constant from trial to trial

Therefore X~ Bin(n=10, p=0.01)

**b.** P(a package is replaced)= P(X$\geq$2)= 1-P(X=0)- P(X=1)= $1-\binom{10}{0}(0.01)^0(0.99)^{10}-$

$-\binom{10}{1}(0.01)(0.99)^9$ = 1-0.9043-0.0913=0.0044

Therefore the proportion of packages that must be replaced is .0044.

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**Problem 4**

Let X be the amount of dye discharged. X~N($\mu, \sigma = .4$)

**a.** Suppose $\mu = 4.5$

Let $Z = \dfrac{X-4.5}{.4} \sim N(0,1)$. Then

P(X<3)$= P\left(\dfrac{X-4.5}{.4} < \dfrac{3-4.5}{.4}\right) = P(Z <$-3.75)=$\phi(-3.75) \approx 0$

**b.** We want to find $\mu$ such that P( X> 6) = 0.01

Let $Z = \dfrac{X-\mu}{.4} \sim N(0,1)$. Then

P(X>6)= $P\left(\dfrac{X-\mu}{.4} > \dfrac{6-\mu}{.4}\right) = P\left(Z > \dfrac{6-\mu}{.4}\right) = 1-\phi\left(\dfrac{6-\mu}{.4}\right) = 0.01$

So $\phi\left(\dfrac{6-\mu}{.4}\right) =$1-0.01=.99

$\Rightarrow \dfrac{6-\mu}{.4} =$2.33 $\Rightarrow \mu$=5.068

**Problem 5**

**a.**

Let $\mu$ be the population mean diameter of the bearings

$\sigma = 0.001$ is the population standard deviation

Then , by Central Limit Theorem,

$\overline{X} = \dfrac{\sum_{i=1}^{45} X_i}{45}$ has approximately a normal distribution with mean $\mu$ and standard deviation $\dfrac{\sigma}{\sqrt{45}}$ =0.000149.

Let $Z = \dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{45}}} \sim N(0,1)$

$$P\left(|\,\overline{X} - \mu\,| < .0001\right) = P\left(-.0001 < \overline{X} - \mu < .0001\right) = P\left(-\frac{.0001\sqrt{45}}{0.001} < \frac{(\overline{X} - \mu)\sqrt{45}}{0.001} < \frac{.0001\sqrt{45}}{0.001}\right)$$

$\approx \phi(.67) - \phi(-.67) =$ 0.7486-0.2514= 0.4972

**b.** Since n=45 is sufficiently large to approximate the distribution of the sample mean, and since we didn't use in our approximation any value from the sample, the approximation in part **a**. will not be affected by the distribution of the sample.

**Problem 6**

The plot has an S-shape appearance. Despite it, taking into consideration the small number of observations, and the fact that all observed data is within the confidence bands, an assumption of population distribution normality is plausible.