I. Example: Brocklebank & Dickey NC Sales Data

FILENAME NCR "ncretail.dat";

TITLE 'North Carolina Retail Sales in $Millions';
TITLE2 'Monthly, Starting Jan 1983';
DATA ncretail;
   INFILE NCR;
   INPUT t sales @@;
      sales = sales / 1000000;
      date = INTNX('MONTH', '01JAN83'D, t-1);
   FORMAT date MONYY. sales COMMA8.2;
   qtr = QTR(date); /* just to demonstrate the function */
   year=YEAR(date);
RUN;

TITLE "North Carolina Sales";
PROC SG PLOT DATA=ncretail;
   SERIES X=date Y=sales;
RUN;

a. Model identification: differencing to remove trend and seasonality

PROC ARIMA DATA=ncretail PLOTS=FORECAST(ALL);
   TITLE2 "No Diff";
   IDENTIFY VAR=sales STATIONARITY=(ADF) NLAG=48;
RUN;

Name of Variable = sales
Mean of Working Series    5207.296
Standard Deviation        945.4276
Number of Observations    144
Autocorrelation Check for White Noise

To Chi-Pr >
Lag Square DF ChiSq ---------Autocorrelations---------
  6  465.92  6 <.0001 0.816 0.742 0.739 0.695 0.669 0.652
 12  784.85 12 <.0001 0.614 0.594 0.576 0.533 0.549 0.628
 18  975.15 18 <.0001 0.522 0.447 0.456 0.419 0.397 0.389
...
 48 1202.92 48 <.0001 0.064 0.066 0.062 0.034 0.077 0.127

TITLE2 "Diff 12";
IDENTIFY VAR=sales(12) SCAN ESACF MINIC NLAG=50;
Period(s) of Differencing                          12
Mean of Working Series                       289.6524
Standard Deviation                          350.7007
Number of Observations                            132
Observation(s) eliminated by differencing          12

Autocorrelation Check for White Noise

To Chi-Pr >
Lag Square DF ChiSq ---------Autocorrelations---------
  6  82.50  6 <.0001 0.301 0.291 0.330 0.300 0.374 0.291
 12 124.90 12 <.0001 0.215 0.334 0.247 0.227 0.158 0.003
 18 142.64 18 <.0001 0.138 0.222 0.175 0.124 0.045 0.044

TITLE2 "Diff 1";
IDENTIFY VAR=sales(1) STATIONARITY=(ADF) NLAG=48;

Period(s) of Differencing                           1
Mean of Working Series                       38.59098
Standard Deviation                           467.3775
Number of Observations                            143
Observation(s) eliminated by differencing           1

Autocorrelation Check for White Noise
To     Chi-     Pr  >
Lag  Square  DF  ChiSq  --------------------------Autocorrelations--------------------------
6     32.19   6  <.0001  -0.273 -0.320  0.179 -0.031 -0.042  0.086
12    86.32  12  <.0001  -0.070  0.021  0.083 -0.208 -0.126  0.521
18    104.99  18  <.0001  -0.070 -0.283  0.152 -0.026 -0.062  0.059

TITLE2 "Diff 1 12"
IDENTIFY VAR=sales(1,12) STATIONARITY=(ADF) NLAG=48;
RUN;

Period(s) of Differencing                        1,12
Mean of Working Series                       6.559084
Standard Deviation                           403.4916
Number of Observations                            131
Observation(s) eliminated by differencing          13

Autocorrelation Check for White Noise
To     Chi-     Pr  >
Lag  Square  DF  ChiSq  --------------------------Autocorrelations--------------------------
6     33.34   6  <.0001  -0.460 -0.089  0.083 -0.079  0.123  0.001
12    51.09  12  <.0001  -0.147  0.145 -0.056  0.048  0.091 -0.258
18    53.22  18  <.0001  0.070  0.073  0.012  0.024 -0.053 -0.019
b. Tentative Determination of p,d,q

i. SAS documentation quote: “Fitting ARIMA models is as much an art as it is a science. The ARIMA procedure has diagnostic options to help tentatively identify the orders of both stationary and nonstationary ARIMA processes.”

ii. SCAN (smallest canonical correlation) method
   1. Works on stationary or non-stationary ARMA processes, but poorly on seasonal data.
   2. Based on eigen-decomposition of the correlation matrix.
   3. Code is SCAN option for IDENTIFY.
   4. Interpretation: Find the upper left hand cell of the largest rectangle of all non-significant p-values.
   5. Output for sales(12):

<table>
<thead>
<tr>
<th>Lags</th>
<th>MA 0</th>
<th>MA 1</th>
<th>MA 2</th>
<th>MA 3</th>
<th>MA 4</th>
<th>MA 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 0</td>
<td>0.0002</td>
<td>0.0010</td>
<td>0.0003</td>
<td>0.0023</td>
<td>0.0003</td>
<td>0.0095</td>
</tr>
<tr>
<td>AR 1</td>
<td>0.0067</td>
<td>0.3601</td>
<td>0.3814</td>
<td>0.4463</td>
<td>0.2191</td>
<td>0.8987</td>
</tr>
<tr>
<td>AR 2</td>
<td>0.0028</td>
<td>0.3424</td>
<td>0.8462</td>
<td>0.6767</td>
<td>0.2691</td>
<td>0.1654</td>
</tr>
<tr>
<td>AR 3</td>
<td>0.0480</td>
<td>0.3533</td>
<td>0.6634</td>
<td>0.7193</td>
<td>0.1849</td>
<td>0.8552</td>
</tr>
<tr>
<td>AR 4</td>
<td>0.0017</td>
<td>0.1409</td>
<td>0.1775</td>
<td>0.0883</td>
<td>0.1530</td>
<td>0.5055</td>
</tr>
<tr>
<td>AR 5</td>
<td>0.2549</td>
<td>0.7125</td>
<td>0.1061</td>
<td>0.7919</td>
<td>0.7764</td>
<td>0.1334</td>
</tr>
</tbody>
</table>

ARMA(p+d,q) Tentative Order Selection Tests

--- SCAN ---

\[
p + d = 5, \quad q = 0
\]

This suggests ARIMA(1,0,1) or ARIMA(0,1,1) or possibly ARIMA(5,0,0) or ARIMA(4,1,0) or ... or ARIMA(0,5,0) after the 12th differencing.
6. Output for sales(1,12):

<table>
<thead>
<tr>
<th>Lags</th>
<th>MA 0</th>
<th>MA 1</th>
<th>MA 2</th>
<th>MA 3</th>
<th>MA 4</th>
<th>MA 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 0</td>
<td>&lt;.0001</td>
<td>0.3671</td>
<td>0.3985</td>
<td>0.4262</td>
<td>0.2191</td>
<td>0.9953</td>
</tr>
<tr>
<td>AR 1</td>
<td>&lt;.0001</td>
<td>0.3579</td>
<td>0.7951</td>
<td>0.7155</td>
<td>0.2694</td>
<td>0.1692</td>
</tr>
<tr>
<td>AR 2</td>
<td>0.0044</td>
<td>0.3376</td>
<td>0.6947</td>
<td>0.7014</td>
<td>0.1925</td>
<td>0.8658</td>
</tr>
<tr>
<td>AR 3</td>
<td>0.0003</td>
<td>0.1420</td>
<td>0.1781</td>
<td>0.0913</td>
<td>0.1526</td>
<td>0.5053</td>
</tr>
<tr>
<td>AR 4</td>
<td>0.1592</td>
<td>0.7583</td>
<td>0.1081</td>
<td>0.7829</td>
<td>0.7813</td>
<td>0.1452</td>
</tr>
<tr>
<td>AR 5</td>
<td>0.8501</td>
<td>0.2339</td>
<td>0.2241</td>
<td>0.8718</td>
<td>0.7188</td>
<td>0.4810</td>
</tr>
</tbody>
</table>

ARMA(p+d,q) Tentative Order Selection Tests

--- SCAN ---

<table>
<thead>
<tr>
<th>p+d</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

This suggests ARIMA(0,0,1) or possibly ARIMA(4,0,0) or ARIMA(3,1,0) or ...

iii. A test with the same goal as SCAN is ESACF (extended sample ACF). It also
works with stationary or non-stationary ARMA processes and is based on least
squares estimation of ARMA parameters.

1. Output for sales(1,12):

<table>
<thead>
<tr>
<th>Lags</th>
<th>MA 0</th>
<th>MA 1</th>
<th>MA 2</th>
<th>MA 3</th>
<th>MA 4</th>
<th>MA 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 0</td>
<td>&lt;.0001</td>
<td>0.3955</td>
<td>0.4287</td>
<td>0.4539</td>
<td>0.2460</td>
<td>0.9956</td>
</tr>
<tr>
<td>AR 1</td>
<td>&lt;.0001</td>
<td>0.7747</td>
<td>0.8112</td>
<td>0.2232</td>
<td>0.9867</td>
<td></td>
</tr>
<tr>
<td>AR 2</td>
<td>&lt;.0001</td>
<td>0.4164</td>
<td>0.8587</td>
<td>0.1551</td>
<td>0.9115</td>
<td></td>
</tr>
<tr>
<td>AR 3</td>
<td>&lt;.0001</td>
<td>0.7016</td>
<td>0.9997</td>
<td>0.1519</td>
<td>0.6463</td>
<td></td>
</tr>
<tr>
<td>AR 4</td>
<td>0.0002</td>
<td>0.4037</td>
<td>0.0001</td>
<td>0.2773</td>
<td>0.2464</td>
<td>0.4336</td>
</tr>
<tr>
<td>AR 5</td>
<td>0.1541</td>
<td>0.1335</td>
<td>&lt;.0001</td>
<td>0.1298</td>
<td>0.5656</td>
<td>0.1919</td>
</tr>
</tbody>
</table>

2. Interpretation: Find the maximal right triangle with right angle at top
right containing all non-significant p-values; the upper left vertex is the
suggested order. Here the triangle is found at AR0/MA1. As for SCAN,
the AR term is p+d, so the candidate model is ARIMA(0,0,1). If the
vertex were AR1/MA2, we’d consider ARIMA(1,0,2) and ARIMA(0,1,2).

3. Additional output is a table of suggested models in the form of p+d and q.
iv. A third test is **MINIC** (minimum information criterion), but useful only for 
*stationary* ARMA processes.

1. Output for sales(1,12):

<table>
<thead>
<tr>
<th>Lags</th>
<th>MA 0</th>
<th>MA 1</th>
<th>MA 2</th>
<th>MA 3</th>
<th>MA 4</th>
<th>MA 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 0</td>
<td>11.9889</td>
<td>11.4820</td>
<td>11.5009</td>
<td>11.5331</td>
<td>11.5652</td>
<td>11.5935</td>
</tr>
<tr>
<td>AR 1</td>
<td>11.7682</td>
<td>11.5150</td>
<td>11.5109</td>
<td>11.544</td>
<td>11.5795</td>
<td>11.6087</td>
</tr>
<tr>
<td>AR 3</td>
<td>11.6087</td>
<td>11.5720</td>
<td>11.5800</td>
<td>11.6095</td>
<td>11.6435</td>
<td>11.6622</td>
</tr>
<tr>
<td>AR 5</td>
<td>11.5556</td>
<td>11.5920</td>
<td>11.6226</td>
<td>11.6487</td>
<td>11.6854</td>
<td>11.707</td>
</tr>
</tbody>
</table>

Error series model: AR(7)  
Minimum Table Value: BIC(0,1) = 11.48201

2. Interpretation: Find the model with the minimum BIC; here it is the AR0/MA1 model.

v. **Other Stationarity (Unit Root Detection) Tests**

1. Earlier we use the **Augmented Dickey Fuller** test to determine whether models with $p+d \geq 0$ have unit roots, i.e., $d>0$.

2. SAS has several additional tests for stationarity, including **Phillips-Perron** test and the **Random Walk** test. Unfortunately there is little information available on how to interpret tests other than ADF (and even there, which lag to examine is somewhat unclear).

3. ADF is popular, but I recommend at least supplementing with plots, and analysis of differenced time series.

II. **Estimation and evaluation of candidate models**

a. Further analyze the data set or differenced data set that is specified in the most recent **IDENTIFY** statement.

b. **ESTIMATE** $P=p$ $Q=q$; estimates a mean ($\mu$) and the ARMA parameters using conditional least squares (conditional on the first few observed values needed to compute $Y_{t-k}$). Using method="ML" is better. **NOCONSTANT** drops $\mu$.

c. Standard output includes estimates of $\mu$ and the AR and MA parameters with standard errors, t-values and approximate p-values. Under “Fit Statistics” you get an estimate of the white noise variance, and values for AIC and BIC. SAS also shows you the “Correlations of the Parameter Estimates”.

d. The “Autocorrelation Check for White Noise” checks the residuals after fitting the ARMA model. If some p-values are small, the provisional ARIMA model may be inadequate.

e. A new set of residual plots is produced including ACF/PACF and plots to check for residual normality. The new fourth plot is p-values for white noise at each lag.
f. The “Model Filters” output shows the formal estimated model, e.g., non-periodic, mean=3, AR factor = (1-0.8B) with IDENTIFY VAR=v (1) corresponds to
\[(1 - 0.8B)(1 - B)Y_t = 3 + \varepsilon_t\]

or
\[(1 - B)Y_t = \frac{3 + \varepsilon_t}{(1 - 0.8B)}\]

Because AR(1) has a particularly simple MA(∞) formulation, this can also be written as:
\[Y_t = 3/5 + Y_{t-1} + \varepsilon_t + 0.8\varepsilon_{t-1} + 0.64\varepsilon_{t-2} + 0.512\varepsilon_{t-3} + \ldots\]

g. NC Sales Example

Syntax for the ESTIMATE statement: \(P= \) sets \(\Phi(B)\) for the AR term, and \(Q=\) sets \(\Theta(B)\) for the MA term. E.g., \(P=(1,2)(12)\ \ Q=(1)(12)\) determines the model:
\[(1-\phi_1 B - \phi_2 B^2)(1-0.1 B^2)Y_t = [c + (1-\theta_1 B)(1-\theta_2 B^2)]\varepsilon_t\]

A short-cut notation in SAS is \(P=p\) for \(P=(1,2,\ldots,p)\) which is \((1-\phi_1 B - \ldots - \phi_p B^p)\).

Returning to the NC retail data:
/* This ESTIMATE refers to sales(1,12) because that was the last IDENTIFY */
/* Choosing not to include NOCONSTANT. Why? */

\[
\begin{align*}
\text{ESTIMATE } & P=(1)(12); \quad \text{RUN;}
\end{align*}
\]

**Conditional Least Squares Estimation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Error</th>
<th>t Value</th>
<th>Pr &gt;</th>
<th>Approx Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>MU</td>
<td>3.30925</td>
<td>14.95821</td>
<td>0.22</td>
<td>0.8253</td>
<td>0</td>
</tr>
<tr>
<td>AR1,1</td>
<td>-0.52182</td>
<td>0.08100</td>
<td>-6.44</td>
<td>&lt;.0001</td>
<td>1</td>
</tr>
<tr>
<td>AR2,1</td>
<td>-0.33602</td>
<td>0.08862</td>
<td>-3.79</td>
<td>0.0002</td>
<td>12</td>
</tr>
</tbody>
</table>

Constant Estimate 6.728309
Variance Estimate 115775.9
Std Error Estimate 340.2585
AIC 1902.11
SBC 1910.735
Number of Residuals 131

* AIC and SBC do not include log determinant.

**Correlations of Parameter Estimates**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MU</th>
<th>AR1,1</th>
<th>AR2,1</th>
</tr>
</thead>
<tbody>
<tr>
<td>MU</td>
<td>1.000</td>
<td>0.019</td>
<td>0.010</td>
</tr>
<tr>
<td>AR1,1</td>
<td>0.019</td>
<td>1.000</td>
<td>-0.008</td>
</tr>
<tr>
<td>AR2,1</td>
<td>0.010</td>
<td>-0.008</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**Autocorrelation Check of Residuals**

<table>
<thead>
<tr>
<th>Lag</th>
<th>Chi-Square DF</th>
<th>Pr &gt; Chi-Sq</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>26.44</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>12</td>
<td>29.54</td>
<td>0.0010</td>
</tr>
<tr>
<td>18</td>
<td>36.86</td>
<td>0.0022</td>
</tr>
<tr>
<td>24</td>
<td>60.01</td>
<td>&lt;0.0001</td>
</tr>
</tbody>
</table>

---

7
Model for variable sales
Estimated Mean               3.309249
Period(s) of Differencing        1,12
Diff 1 12 P=(1)(12) with constant

Autoregressive Factors
Factor 1:  1 + 0.52182 B**(1)  
Factor 2:  1 + 0.33602 B**(12)

ESTIMATE Q=(1)(12) NOCONSTANT METHOD=ML; RUN;
WARNING: The model defined by the new estimates is unstable. The
iteration process has been terminated.

WARNING: Estimates may not have converged.

ARIMA Estimation Optimization Summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation Method</td>
<td>Maximum Likelihood</td>
</tr>
<tr>
<td>Parameters Estimated</td>
<td>2</td>
</tr>
<tr>
<td>Termination Criteria</td>
<td>Maximum Relative Change in Estimates</td>
</tr>
<tr>
<td>Iteration Stopping Value</td>
<td>0.001</td>
</tr>
<tr>
<td>Criteria Value</td>
<td>103.0445</td>
</tr>
<tr>
<td>Maximum Absolute Value of Gradient</td>
<td>1590.444</td>
</tr>
<tr>
<td>R-Square Change from Last Iteration</td>
<td>0.092972</td>
</tr>
<tr>
<td>Objective Function</td>
<td>Log Gaussian Likelihood</td>
</tr>
<tr>
<td>Objective Function Value</td>
<td>-922.602</td>
</tr>
<tr>
<td>Marquardt's Lambda Coefficient</td>
<td>0.000001</td>
</tr>
<tr>
<td>Numerical Derivative Perturbation Delta</td>
<td>0.001</td>
</tr>
<tr>
<td>Iterations</td>
<td>8</td>
</tr>
<tr>
<td>Warning Message</td>
<td>Estimates may not have converged.</td>
</tr>
</tbody>
</table>
Maximum Likelihood Estimation

| Parameter | Estimate | Error  | t Value | Pr > |t| | Lag |
|-----------|----------|--------|---------|------|---|-----|
| MA1,1     | 0.73712  | 0.06012| 12.26   | <.0001 |   | 1   |
| MA2,1     | 0.99980  | 86.83175| 0.01    | 0.9908 | 12|

```
ESTIMATE Q=(1) NOCONSTANT METHOD=ML; RUN;
```

Maximum Likelihood Estimation

| Parameter | Estimate | Error  | t Value | Pr > |t| | Lag |
|-----------|----------|--------|---------|------|---|-----|
| MA1,1     | 0.81094  | 0.05417| 14.97   | <.0001 |   | 1   |

Variance Estimate 94196.83
Std Error Estimate 306.915
AIC 1874.191
SBC 1877.067
Number of Residuals 131

Autocorrelation Check of Residuals

To Lag | Chi-Square | DF | ChiSq | Pr > |Lag |
       | 6         | 5  | 0.3672 | -0.075 | -0.081 | 0.033 | 0.155 | 0.048 |
       | 12        | 11 | 0.0453 | -0.056 | 0.158  | 0.049 | 0.058 | -0.017 | -0.258 |
       | 18        | 17 | 0.0922 | -0.003 | 0.140  | 0.098 | 0.049 | -0.045 | -0.021 |
       | 24        | 23 | 0.0821 | 0.038  | -0.109 | -0.065 | 0.141 | 0.087 | -0.064 |
       | 30        | 29 | 0.0877 | -0.064 | -0.095 | 0.098 | -0.112 | -0.073 | 0.007 |
       | 36        | 35 | 0.1431 | 0.066  | 0.033  | -0.053 | -0.078 | -0.079 | -0.052 |
       | 42        | 41 | 0.0696 | 0.089  | 0.060  | -0.150 | 0.007 | 0.150 | -0.045 |
       | 48        | 47 | 0.0577 | -0.037 | -0.030 | 0.049 | 0.174 | -0.036 | -0.057 |

Model for variable sales
Period(s) of Differencing 1,12
No mean term in this model.

Moving Average Factors
Factor 1: 1 - 0.81094 B**(1)
ESTIMATE Q=(1) P=(12) NOCONSTANT METHOD=ML; RUN;

Maximum Likelihood Estimation

| Parameter     | Estimate | Error  | t Value | Pr > |t| | Lag |
|---------------|----------|--------|---------|-------|---|-----|
| MA1,1         | 0.78355  | 0.05859| 13.37   | <.0001| 1 |
| AR1,1         | -0.28563 | 0.09026| -3.16   | 0.0016| 12|

Variance Estimate 87244.03
Std Error Estimate 295.371
AIC 1866.067
SBC 1871.817
Number of Residuals 131

Correlations of Parameter Estimates
Parameter MA1,1 AR1,1
MA1,1 1.000 0.113
AR1,1 0.113 1.000

Autocorrelation Check of Residuals

<table>
<thead>
<tr>
<th>To</th>
<th>Chi-</th>
<th>Pr &gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag</td>
<td>Square</td>
<td>DF</td>
</tr>
<tr>
<td>6</td>
<td>6.18</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>10.45</td>
<td>10</td>
</tr>
<tr>
<td>18</td>
<td>15.63</td>
<td>16</td>
</tr>
<tr>
<td>24</td>
<td>28.54</td>
<td>22</td>
</tr>
<tr>
<td>30</td>
<td>33.45</td>
<td>28</td>
</tr>
<tr>
<td>36</td>
<td>37.55</td>
<td>34</td>
</tr>
<tr>
<td>42</td>
<td>49.03</td>
<td>40</td>
</tr>
<tr>
<td>48</td>
<td>55.40</td>
<td>46</td>
</tr>
</tbody>
</table>

Model for variable sales
Period(s) of Differencing 1,12
No mean term in this model.

Autoregressive Factors Factor 1: 1 + 0.28563 B**(12)
Moving Average Factors Factor 1: 1 - 0.78355 B**(1)

Model: \((1 - \phi_{1,1}B^{12})Y_{12}Y_t = (1 - \theta_{1,1}B)\varepsilon_t\) Also: AR(1)MA(0,d=1,q=1)(P=1,D=1,Q=0)_{12}
III. **Forecasting**

a. Forecasts based on the model of the most recent `ESTIMATE` statement.

b. `FORECAST [BACK=m] LEAD=n INTERVAL=MONTH ID=date OUT=results; RUN;`

c. Checking the model with “forecasting” of observed values

```
FORECAST ID=date BACK=12 LEAD=12 INTERVAL=MONTH OUT=foreChk; RUN;
```

This particular plot requires `PLOTS=FORECAST(ALL)` in the `PROC ARIMA` statement.
d. Forecasting the future:
   FORECAST ID=date LEAD=60 INTERVAL=MONTH OUT=NCforecast60;

   ![](image.png)

IV. Outlier Detection
   a. If you didn’t QUIT, you can run OUTLIER MAXNUM=n; RUN; to check for outliers.
      Outlier Detection Summary
      Maximum number searched           5
      Number found                      5
      Significance used              0.05

      Outlier Details
      | Obs | Type   | Estimate | Chi-Square | Prob > ChiSq |
      |-----|--------|----------|------------|--------------|
      | 87  | Additive | -1091.2  | 57.04      | <.0001       |
      | 85  | Additive | 989.11354 | 40.49     | <.0001       |
      | 144 | Additive | 931.65780 | 21.30     | <.0001       |
      | 142 | Additive | 767.66972 | 15.85     | <.0001       |
      | 82  | Shift   | -384.90076 | 14.95   | 0.0001       |

V. Comparing Predictions
   ESTIMATE Q=1 NOCONSTANT METHOD=ML; /* Predict with Q=1 */
   FORECAST ID=date LEAD=60 INTERVAL=MONTH OUT=NCforecast60D;

   DATA pred2ways;
   MERGE NCforecast60 NCforecast60D(DROP=sales residual std
      RENAME=(forecast=sfore 195=s195 u95=su95));
     BY date;
   RUN;

   NOTE: There were 204 observations read from the data set WORK.NCFORECAST60.
   NOTE: There were 204 observations read from the data set WORK.NCFORECAST60D.
   NOTE: The data set WORK.PRED2WAYS has 204 observations and 10 variables.
Conclude: Using two year’s back data (AR(12) on diff(1,12)) results in better fit, a smaller residual error, and, therefore, a narrow prediction interval.

VI. **Summary**
   a. Interactively use IDENTIFY, ESTIMATE, FORECAST and OUTLIER to fit and forecast a time series.
   b. Differencing is used to create a stationary, non-periodic time series.
   c. Examination of AIC/BIC, residual ACF/PACF, residual white noise checks, and residual normality are all useful for model assessment.
   d. Forecasting restores the effects of differencing and predicts at future time points based on the final ARMA model.
   e. Outliers in the form of individual values and/or shifts in mean may reflect un-modeled phenomena.

VII.
VIII. An R example

```r
require(fpp)

# A monthly time series of the short-term visitor departures
# from Australia
vs = departures[, "visshort"]

# Examine
class(vs) # ts
tsp(vs) # 1976.000 2012.083 12.000
plot(vs)

# Fix heteroscedasticity
lvs = log(vs)
class(lvs)
tsp(lvs)
tsdisplay(lvs)
```
# deseason
lvs.ds = diff(lvs, 12)
class(lvs.ds)
tsp(lvs.ds)
tsdisplay(lvs.ds)

```

```

# difference deseasoned data
tsdisplay(diff(lvs.ds))
m1.1.0..1.1.0 = Arima(lvs, order=c(1,1,0), seasonal=c(1,1,0))
summary(m1.1.0..1.1.0)
# ARIMA(1,1,0)(1,1,0)[12]
# Coefficients:
#     ar1  sar1
#     -0.2484 -0.3726
#     s.e.  0.0472  0.0456
#     sigma^2 estimated as 0.003497:  log likelihood=592.23
# AIC=-1178.47  AICc=-1178.41  BIC=-1166.34
tsdig(m1.1.0..1.1.0)

```

Standardized Residuals

```

```

ACF of Residuals

```

p values for Ljung-Box statistic

```

```
ml.1.0..2.1.0 = Arima(lvs, order=c(1,1,0), seasonal=c(2,1,0))
summary(ml.1.0..2.1.0)
# ARIMA(1,1,0)(2,1,0)[12]
# Coefficients:
# ar1  sar1  sar2
# -0.2614 -0.4493 -0.2107
# s.e.  0.0471  0.0481  0.0485
# AIC=-1194.81 AICc=-1194.71 BIC=-1178.64
tsdia(m1.1.0..2.1.0)

---

```
Standardized Residuals

Time
1980 1990 2000 2010
-3 1 0.0 0.5 1.0 1.5 2.0
-0.2 0.8

ACF of Residuals

Lag

p values for Ljung-Box statistic

lag
p value
```

---

ml.1.1.2..2.1.0 = Arima(lvs, order=c(1,1,2), seasonal=c(2,1,0))
summary(ml.1.1.2..2.1.0)
# ARIMA(1,1,2)(2,1,0)[12]
# Coefficients:
# ar1  ma1  ma2  sar1  sar2
# 0.5453 -0.8812  0.0861 -0.4390 -0.2055
# s.e.  0.2103  0.2202  0.1157  0.0485  0.0485
# AIC=-1210.89 AICc=-1210.69 BIC=-1186.64
#
ml.1.1.1..2.1.0 = Arima(lvs, order=c(1,1,1), seasonal=c(2,1,0))
summary(ml.1.1.1..2.1.0)
# ARIMA(1,1,1)(2,1,0)[12]
# Coefficients:
# ar1  ma1  sar1  sar2
# 0.3899 -0.7150 -0.4400 -0.2040
# s.e.  0.1039  0.0799  0.0483  0.0484
# AIC=-1212.39 AICc=-1212.25 BIC=-1192.18

tsdia(m1.1.1..2.1.0)

Model: 
\[(1 - \phi_{1,1} B^1)(1 - \phi_{2,1} B^{12} - \phi_{2,2} B^{24}) v_1 v_{12} \log(Y_t) = (1 - \theta_1 B) \varepsilon_t\]

Also: ARI(S)MA(p=1, d=1, q=1)(P=2, D=1, Q=0)
Forecasts from ARIMA(1,1,1)(2,1,0)[12]
```
plahead = forecast(m1.1.1..2.1.0, h=60)
plot(plahead, xlab="Year", ylab="Log(Short term arrivals)")

Forecasts from ARIMA(1,1,1)(2,1,0)[12]

```

```
templ = exp(plahead$lower[,2])/1000
tempu = exp(plahead$upper[,2])/1000
plot(vs/1000, xlim=c(1976, 2017.1), ylim=range(vs/1000, templ, tempu),
     xlab="Year", ylab="Short term arrivals(1000s)",
     main="Airline Predictions")
tempt = time(plahead$mean)
segments(tempt, templ, tempt, tempu, col="lightblue", lwd=2)
lines(exp(plahead$mean)/1000, col=4, lwd=2)
```