Discussion Guide on Power

1. Definitions
   a. Type-1 error: rejecting a null hypothesis when it is true
   b. Alpha (significance level): Rate at which type-1 errors are controlled by a decision process (among repeat experiments when $H_0$ is true)
   c. Type-2 error: retaining a null hypothesis when it is false
   d. Beta: Rate at which type-2 errors are controlled by a decision process for a particular alternative hypothesis (among repeat experiments when that particular $H_A$ is true)
   e. Power (1-β): Rate at which correct rejection of a false null hypothesis is made for a decision process (among repeat experiments for which the specific $H_A$ is true)
   f. Precision: 1/variance. In this context, (half)-width of confidence intervals (which is proportional to the standard deviation) is a measure of imprecision.
   g. Accuracy: lack of bias, i.e., how close an estimate is on average to the true value
   h. False discovery rate: An alternative to type-1 error for multiple testing; the expected rate of type-1 errors among all $H_0$ rejections
   i. Predictive value of a positive: Rate of true $H_0$ rejections among all $H_0$ rejections
   j. Predictive value of a negative: Rate of true $H_0$ rejections among all $H_0$ rejections

2. Goals

3. Weaknesses of Cohen’s effect sizes

4. Non-equivalence testing: accepting $H_0$

5. Simulation for calculation of power

6. Illogic of post-hoc power analysis

7. How to increase power

8. Software: PinT (free download)

9. PinT example: This study involves randomized collection of data from students within schools. DV is Achievement (standardized math test). School level IV is Policy (without or with manipulables). The policy was not randomized, so this is not an experiment and causal conclusions are unwarranted. The policy was known to be especially likely to be applied in schools with more low SES students. Student level IVs are standardized IQ and standardized SES. Interest is in the effect of Policy, including how it may differentially affect students by SES. Policy is recorded as 0 or 1 with 30% of schools using manipulables.
   a. Write a reasonable model as an equation using $U_{0j}$ for a random intercept effect and $U_{ij}$ for a random slope effect and $R_{ij}$ for residual error. Write the SAS code.
Write the matrix form of the model. (PinT assumes that IVs with random slopes are recorded as within-group deviations that have been converted to z-scores. PinT also assumes that level 1 variables without a random effect are included as group means in level 2.)

b. PinT input

i. Initially “Cancel” the “User parameter file”. The program will create a parameter file from your input, which can be used (and modified) in the future.

ii. Number of variables: total level 1 (student); number of level 1 without a random slope; level 2 (school)

iii. Means of the level 1 IVs that do not have random slope (0 if z-scores)

iv. Means of the level 2 IVs (0 if z-scores)

v. Within-group (school) covariance matrix for level 1 variables: On the diagonal, put those without a random slope first. Assuming z-scores, the total variance is 1, so we need to guessimate the fraction of this variance that is within groups. (Running a random intercept model with this IV as the DV on pilot or previous data is good if available.) For treatments applied at group level, the variance is 0. For variables with a random slope, the within-group variance is 1 due to the use of z-scores. Off diagonal covariances are usually calculated from an educated guess of the correlation (covariance = correlation x sd1 x sd2).

vi. Between-group covariance matrix for level 2 IVs (first), then for the group means of the level 1 IVs with no random slope (included automatically by PinT). The between-group variance of a level 2 IV can be computed by using PROC SORT NODUP KEY followed by PROC MEANS. The between-group variance of a level 1 IV with no random slope is its total variance minus its within-group variance. Again an educated guess at the correlation(s) is needed to calculate the covariances.

vii. Residual variance at level 1 (σ²): Unexplained variance of the DV after fixed effects and the random intercept are taken into account. Applies at any values of the IVs, so it’s often easiest to think about when all IVs equal zero (so that random slope effects can be ignored). Ranges from (near) zero to the variance of the DV. Can use σ² = Var(DV) (1-R²), where R² is thought of as including the random intercept.

viii. Covariance matrix of random coefficients: For random intercept variance, remember that all level 1 variables that have a random slope are group centered so you only need to consider how much of the variance of the intercepts (from school-to-school) is accounted for by the level 2
variable(s) and the level 1 variable(s) with fixed effects; the remainder is the variance of the random intercept. One approach is to guestimate the fraction of variance in the DV that is due to group-to-group differences, then multiply by the total DV variance to get the absolute group-to-group variance, then guestimate the fraction of that variability due to level 1 fixed effects and calculate the remaining variance, then guestimate the fraction of this remaining variance due to level 2 variables and calculate the remaining variance once again. For random slope variances, guestimate the group-to-group standard deviation in slope, square to get the variance, then subtract off the fraction explained by level 2 variables. Finally guestimate the correlation of random effects and calculate the covariance(s) from correlation(s) and standard deviations.

ix. Now focus on the various ways money can be spent to collect data at different schools and within schools. Estimate the cost of collecting data from an additional student at an existing school; this dollar amount is equal to one cost unit. Divide this marginal cost into the total dollar budget to get the number to enter as the “Total Budget” (aka Budget constraint K). K is not in dollars, but in multiples of the cost of one level 1 subject. Estimate as the “Cost Parameter” the cost of adding an additional level 2 unit (school) as a multiple of the cost of an additional level 1 unit (student), noting that this is a ratio, not a dollar amount. You will also need to choose the minimum and maximum number of level 1 units in any level 2 unit (students per school). If these numbers are reasonably close together, you can use a “step size” of 1, otherwise choose as “step size” a number that make the total number of checked sizes “from min to max by step” not too unwieldy. Under your budget constraint and parameter estimates, PinT will calculate the various standard errors for parameter estimates for various combinations of numbers of schools and numbers of students per school.
The following table contains the standard errors (s.e.):

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