Metropolis algorithm and distorted Brownian motion

by
Zhen-Qing Chen
University of Washington
Department of Mathematics
Seattle, WA 98195
zqchen@u.washington.edu

Abstract

Given a bounded domain $D$ in $\mathbb{R}^n$ and a continuous function $\rho$ that is bounded between two positive constants. Consider the following Metropolis Markov chain $\{X_j, j \neq 1\}$ on $D$. At time $j + 1$, pick a point $y$ at random from the ball with radius $h$ centered at $X_j$. If $y$ is in $D$ and $\rho(y) \geq \rho(x)$, let $X_{j+1} = y$; if $y$ is in $D$ but $\rho(y) < \rho(x)$, flip a coin with head probability $\rho(y)/\rho(x)$ and let $X_{j+1} = y$ if head appears; in all other cases, let $X_{j+1} = X_j$.

We show that after appropriate scaling in time and step $h$, the Metropolis chain converges weakly to reflecting distorted Brownian motion in $D$. We then apply it to the random placement of $N$ hard discs in the unit sphere, the original application of Metropolis algorithm.

This is joint work with K. Burdzy and P. Diaconis.