Sparse Least Squares Estimation

by
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Abstract

Estimation of a sparse high-dimensional signal vector is well understood if the signal is directly observed with white noise. If the signal vector belongs to a small $\ell_r$ ball of radius $R$, threshold estimators at a certain level $\lambda_{mm}$ approximately attain the minimax risk. We show that in linear regression models with $n$ data points and $p > n$ unknowns, this result can be extended with penalized least squares estimators, provided that the number of significant components or $R^r/\lambda_{mm}^r$ of the signal vector is no greater than a certain $d_*$. The order of this $d_*$ could be as high as $n/\log(p/n)$. 