Kinds of Bayesians
(Comment on articles by Berger and by Goldstein)
Robert E. Kass — September 2005

Philosophy is consequential because it guides action. In statistics, our beliefs about the nature of statistical inference guide us both in selecting statistical methods to apply and in choosing problems to research. Jim Berger and Michael Goldstein have articulated clearly two opposing viewpoints. Yet, both statisticians are highly-experienced, widely-respected Bayesians and, faced with a particular set of data, it is very likely that the two of them would reach similar conclusions. So where, specifically, do they disagree? And which points of disagreement are most consequential? To help in identifying key issues I have constructed a list of questions. Answers to these may be used to classify various kinds of Bayesians.

1. Is it important for Bayesian inferences to have good frequentist operating characteristics? The answer to this question is especially consequential: we either check whether procedures have good long-run properties, or we do not.

2. Does the Bayesian paradigm do anything more than produce candidate procedures, to be judged according to frequentist criteria? A positive answer to this question identifies Bayesians, separating them from the many other statisticians who believe Bayesian methods may sometimes be useful, but who do not subscribe to Bayesian philosophy.

3. Is there a useful role for default (a.k.a. "objective") Bayesian inferences as representing approximately subjective inferences? There may be subjectivists who would deny any useful role for default priors, but I doubt it. A negative answer, instead, would likely identify a Bayesian who prefers non-subjectivist philosophies of the kind articulated by Jeffreys. See Kass and Wasserman (1995) for references and discussion.

4. Is it possible to interpret default Bayesian inference as anything other than approximately subjective? This question asks for the philosophical stance of any default Bayesian inference. If a negative answer is given, then subjectivism is taken as the foundation for Bayesian inference. A positive answer would require saying how to interpret an "objective Bayesian" inference. To shy away from answering at all seems like a cop-out.

5. Assuming that the data analyst has done a thorough and careful job, is it appropriate to interpret default Bayesian inferences as representing, approximately, what any reasonable person ought to think given the data and appropriate background information? This interpretation derives, very nearly, from Jeffreys. I say "very nearly" because an extremely important point is that Jeffreys did not

want to admit a subjective foundation for probability and inference, yet the interpretation given in this question should be quite acceptable to subjectivists subject to a crucial caveat: "thorough and careful" must entail a reluctance to report default inferences *unless* the inferences are sufficiently data-dominated that a reasonable range of prior opinions would be overwhelmed.

6. Is there any useful meaning to the word "objective," beyond signifying such overwhelming evidence that reasonable people will be forced to agree? I believe no thoughtful statistician would maintain that statistical inferences are objective in anything other than a weak sense. The fundamental Bayesian criticism of frequentist inference is that (i) the weak long-run sense of objectivity used by frequentists is too indirect (as Keynes said, "In the long run we will all be dead"), while (ii) most users of statistics fail to appreciate its meaning (and instead interpret confidence as posterior probability). In posing question 6 my intention is to ask whether there is a meaningful Bayesian, non-subjective notion of objectivity in statistical inference. Those who answer this question negatively while adopting the phrase "objective Bayesian inference" are faced with a troubling inconsistency.

7. Is the word "objective" so easy to misunderstand that its utility in the context of Bayesian inference is, on average, negative? This, I think, is the crux of the matter regarding the name "objective Bayesian." To a subjectivist, the name is objectionable philosophically because it seems to deny the subjectivist foundation of Bayesian inference. As a consequence, if "objective Bayesian inference" is supposed to be objective, the two-part criticism aimed at frequentist objectivity will apply once again. The only escape seems to be to use the name "objective Bayesian" while at the same time denying any intention to be claiming objectivity.

8. Is it important to distinguish scientific inference from decision-making? Those who answer negatively apparently view science as a series of decisions, and Bayesian inference as fundamentally concerned with optimal decision-making. Others may acknowledge the importance of improving decision-making in matters of personal or public policy, but see science as essentially different, being centrally concerned with the information content of data. This second camp then faces the following question.

9. Are there scientific settings in which formal elicitation procedures are useful? Elicitation assumes that the opinion of individual scientists is (i) worth knowing and (ii) possible to obtain with reasonable accuracy. Skeptics are dubious of both of these propositions, while enthusiasts see eliciation procedures as a vehicle for gaining important knowledge that is otherwise inaccessible.

In posing these questions, and elaborating on them, I am aware that I have oversimplified (under-explained, under-qualified, and omitted alternatives) in several places.

Nonetheless, I hope the questions are useful.[1]

What about the answers to the questions? For the record, I myself see science as essentially different than decision-making, and Bayesian inference as useful scientifically because it furnishes a vehicle for understanding the likelihood function. I believe subjectivism is the proper foundation for Bayesian inference, but I am quite content to use default priors because, in a scientific context, I would be reluctant to report a posterior probability unless I felt the likelihood dominated all relevant prior distributions. Furthermore, I think coverage probability is a good way to discriminate among alternative rules for default priors and, more importantly, I use frequentist criteria to judge likely accuracy of models. (To me, the Bayesian and frequentist paradigms provide two alternative languages for grappling with uncertainty and, like many bilingual speakers, while I am most comfortable in the Bayesian tongue, I often find it helpful to switch to frequentist logic.) Finally, I wish the name "objective Bayesian" would be avoided. Kass and Wasserman (1995), following a remark of Jeffreys, suggested that "reference Bayesian" would be more appropriate, even though it conflicts with Bernardo's unfortunate use in "reference prior" to refer to a particular rule rather than a generic approach. Another preferable alternative would be "pragmatic Bayesian." This has a strong positive connotation, without the philosophical baggage of "objective."

I invite Berger and Goldstein to further clarify their views in this fashion.

References

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[1] I might add that none of the questions involve probability. My title, "Kinds of Bayesians," is reminiscent of Good (1971), who had previously written an article titled "Kinds of Probability" (Good, 1959). Good (1971) is useful in understanding Good's own philosophy, but not very helpful in distinguishing dominant philosophical differences. On the other hand, Good (1959) remains lucid and important, making the simple but profound observation that it is possible—indeed, reasonable—to adopt a subjective interpretation of probability while carrying out frequentist calculations. Furthermore, I can't see a fundamental inconsistency in saying that probability should be defined by long-run frequencies and yet believing there is an important role for Bayesian inference, understood subjectively (though this seems to require distinguishing two notions of probability, often called "epistemic" and "alleatory," and defining the former in terms of the latter). For this reason, I do not think we must adopt a stance on probability before determining how we feel about Bayesian inference.