1. Suppose that $X_1, X_2, \ldots, X_m$ are discrete random variables with probability function $p(x_1, \ldots, x_m)$. Let $a_1, \ldots, a_m$ be constants. Show that

$$E \left[ \sum_{j=1}^{m} a_j X_j \right] = \sum_{j=1}^{m} a_j E[X_j].$$

2. Let $X$ be a discrete random variable such that $X \in \{1, 2, \ldots\}$. Show that $E[X] = \sum_j P(X \geq j)$.

3. A random variable $X$ is degenerate if there exists a number $a$ such that $P(X = a) = 1$.
   (a) Show that, if $X$ is degenerate, then $\text{Var}(X) = 0$.
   (b) Let $X$ be discrete. Suppose that $\text{Var}(X) = 0$. Show that $X$ is degenerate.

4. Let $X$ and $Y$ be random variables. Show that

$$\text{Cov}(a + bX, c + dY) = bc \text{Cov}(X, Y).$$

5. Let

$$Y = 5X + \epsilon$$

where $\epsilon \sim N(0, 1)$ and $X \sim \text{Unif}(-1, 1)$. Assume that $X$ and $\epsilon$ are independent.
   (a) Find the mean and variance of $Y$.
   (b) Find $E[Y^2]$.
   (c) Find $E[Y|X = x]$.
   (d) Find $E[Y^3]$.
   (e) Find $\text{Cov}(\epsilon, \epsilon^2)$. Are $\epsilon$ and $\epsilon^2$ independent?