Homework 2
36-705
Due: Thursday Sept 10th by 3:00pm

Remember: You are not allowed to search for homework solutions on the internet.

1. Find the moment generating function for (a) the Normal distribution, (b) the Gamma distribution and (c) the Poisson distribution.

2. Suppose that we have a positive random variable $X \geq 0$, and suppose further that its mgf $M_X(t)$ exists for all $t \geq 0$. Show that,

$$\inf_{k=0,1,2,...} \frac{\mathbb{E}[X^k]}{u^k} \leq \inf_{t \geq 0} \frac{\mathbb{E}[\exp(tX)]}{\exp(tu)}.$$ 

This implies that the best moment tail bound is never worse than the best Chernoff bound. We usually use the Chernoff method because it is typically much easier to find the best $t$ than to find the best $k$.

3. Construct a random variable $X$ for which Chebyshev’s inequality is tight, that is, $P(|X - \mu| > t) = \sigma^2/t^2$.

4. Prove that

$$\text{Var}(Y) = \mathbb{E}[	ext{Var}(Y|X)] + \text{Var}[\mathbb{E}(Y|X)].$$

5. Suppose that $X \geq 0$ is a continuous random variable and that $X$ has finite mean $\mu$. Show that

$$\mu = \int_0^\infty P(X > x)dx.$$