Homework 8
36-705
Due: Thursday October 22 by 3pm.

1. Let \( X_1, \ldots, X_n \sim N(\mu, \sigma^2) \).
   (a) Find the Fisher information matrix.
   (b) Find the limiting distribution of the mle.
   (c) Find the mle of \( \psi = \mu/\sigma \) and find its limiting distribution.

2. Let \( X \sim \text{Bin}(n, p) \). We want to test
   \[ H_0 : p = p_0 \quad \text{versus} \quad H_1 : p \neq p_0. \]
   (a) Find the Wald test statistic and the critical value. (i.e. when do we reject \( H_0 \)?)
   (b) Find the likelihood ratio test and the critical value.

3. Let \( X_1, \ldots, X_n \sim N(\mu, \sigma^2) \).
   We want to test
   \[ H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu \neq \mu_0. \]
   (a) Find the Wald test statistic and the critical value. (i.e. when do we reject \( H_0 \)?)
   (b) Find the likelihood ratio test and the critical value.

4. Another popular test is the score test. Let \( X_1, \ldots, X_n \sim p(x; \theta) \). Suppose that \( \theta \in \mathbb{R} \).
   We want to test
   \[ H_0 : \theta = \theta_0 \quad \text{versus} \quad H_1 : \theta \neq \theta_0. \]
   Let
   \[ S_n(\theta) = \sum_i \frac{\partial \log p(X_i; \theta)}{\partial \theta} \]
   be the score function. We will use \( S_n(\theta_0) \) as a test statistic. Find the limiting distribution of \( S_n(\theta_0) \) under \( H_0 \). When do we reject \( H_0 \)? Note that, to do this test, we never need to estimate \( \theta \).