Homework 5
36-705
Due: Thursday October 22 by 3:00

1. Let \( X \sim \text{Bernoulli}(\theta) \) where \( \theta \in \{1/4, \, 3/4\} \). Let the loss function be 0-1 loss, i.e. \( L(\theta, \hat{\theta}) = I(\theta \neq \hat{\theta}) \). Find a minimax estimator for \( \theta \).

2. Chapter 9, problem 7 parts a,b,c.

3. Chapter 9, problem 8.

4. Let \( X_1, \ldots, X_n \sim N(\theta, 1) \). Suppose that \( \theta \in \Theta = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \).

   (a) Find the mle \( \hat{\theta} \).
   (b) Show that the mle is unbiased.
   (c) Show that the mle is consistent.
   (d) Prove or disprove: \( \sqrt{n}(\hat{\theta} - \theta) \) is asymptotically Normal.

5. Let \( X_1, \ldots, X_n \sim \text{Poisson}(\lambda) \). Let \( \theta = \mathbb{P}(X_i = 0) \).

   (a) Find the mle \( \hat{\theta} \) for \( \theta \).
   (b) Find the limiting distribution for \( \hat{\theta} \) (appropriately normalized).
   (c) Show that \( \hat{\theta} \) is consistent.

6. Let \( X_1, \ldots, X_n \sim \text{Uniform}(0, \theta) \). Find the method of moments estimator \( \hat{\theta}_1 \) and the mle \( \hat{\theta}_2 \). Show that \( \hat{\theta}_1 - \theta = O_P(1/\sqrt{n}) \) and that \( \hat{\theta}_2 - \theta = O_P(1/n) \).