1. Let $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$. Let $\theta = \mathbb{P}(X_i = 0)$. Note that $\theta = g(\lambda)$ for some function $g$.
   (a) Find the mle $\hat{\theta}$ for $\theta$.
   (b) Find the limiting distribution for $\hat{\theta}$ (appropriately normalized).
   (c) Show that $\hat{\theta}$ is consistent.

2. Let $X_1, \ldots, X_n \sim \text{Uniform}(0, \theta)$. Let’s compare the method of moments estimator $\hat{\theta}_1$ and the maximum likelihood estimator $\hat{\theta}_2$.
   (a) Find both estimators.
   (b) Show that they are both consistent.
   (c) Find their limiting distributions.
   (d) Show that $\hat{\theta}_1 - \theta = O_p(1/\sqrt{n})$ and that $\hat{\theta}_2 - \theta = O_p(1/n)$. Which estimator should be preferred?

3. Let $X_1, \ldots, X_n \sim \mathcal{N}(\mu, 1)$. Consider testing
   \[ H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu \neq \mu_0. \]
   Construct the Wald test and show that the p-value has a Unif(0,1) under $H_0$. Find the distribution of the p-value under $H_1$.

4. Let $X_1, \ldots, X_n \sim \mathcal{N}(\mu, \sigma^2)$ where both $\mu$ and $\sigma^2$ are unknown.
   (a) Find the Wald test for
   \[ H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu \neq \mu_0. \]
   State the rejection rule explicitly.
   (b) Find the LRT for
   \[ H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu \neq \mu_0. \]
   State the rejection rule explicitly.
   (c) Find the Wald test for
   \[ H_0 : \sigma = \sigma_0 \quad \text{versus} \quad H_1 : \sigma \neq \sigma_0. \]
   State the rejection rule explicitly.
   (d) Find the LRT for
   \[ H_0 : \sigma = \sigma_0 \quad \text{versus} \quad H_1 : \sigma \neq \sigma_0. \]
   State the rejection rule explicitly.