
2. Let $X_1, \ldots, X_n \sim \text{Unif}(0, \theta)$. We want to test:

   $H_0 : \theta = 1$ versus $\theta > 1$.

   Consider the following test: reject $H_0$ if $\hat{\theta} > c$ where $\hat{\theta}$ is the mle.

   (a) Find $c$ so that the test has level $\alpha$.

   (b) Find the power function.


4. Let $p_0$ and $p_1$ be two density functions. Let $K = \int p_1(x) \log(p_1(x)/p_0(x)) dx$ be the Kullback-Leibler divergence. Assume that $K > 0$. Consider testing:

   $H_0 : X_1, \ldots, X_n \sim p_0$ versus $H_1 : X_1, \ldots, X_n \sim p_1$.

   Recall that the Neyman-Pearson test rejects $H_0$ when $T > c$ where $T = L(p_1)/L(p_0)$ and $c$ is chosen so that $P_0(T > c) = \alpha$. (You may assume that such a $c$ exists.) Show that if $H_1$ is true, then the power of the Neyman-Pearson test goes to 1 as $n \to \infty$.

5. Let $X_1, \ldots, X_n \sim N(\theta, 1)$. Consider testing:

   $H_0 : |\theta| \leq \epsilon$ versus $H_1 : |\theta| > \epsilon$

   where $\epsilon > 0$ is a fixed constant. Suppose we reject $H_0$ when $|\overline{X}_n| > c$. Find $c$ so that the test has size $\alpha$. Find the power function.