

$$f(\beta) = \frac{1}{2} \|y - \beta\|_2^2 + \lambda \|\beta\|_1$$

subgradients at  $\beta$  are

$$\beta - y + \lambda s$$

$$s \in \partial \|\beta\|_1$$

$$s_i = \begin{cases} \text{sign}(\beta_i) & \beta_i \neq 0 \\ \in [-1, 1] & \beta_i = 0 \end{cases} \quad i=1, \dots, n$$

$$\in [-1, 1] \quad \beta_i = 0$$

now set to 0

$$y_i - \beta_i = \lambda \text{sign}(\beta_i) \quad \beta_i \neq 0, \quad i=1, \dots, n$$

$$|y_i - \beta_i| \leq \lambda \quad \beta_i = 0.$$

plug in  $\beta = S_\lambda(y)$

o  $y_i > \lambda$ ,  $\beta_i = y_i - \lambda$ ,  $y_i - \beta_i = \lambda$  ✓

o  $y_i < -\lambda$ , similar

o  $-\lambda \leq y_i \leq \lambda$ ,  $\beta_i = 0$ ,  $|y_i| \leq \lambda$  ✓

$\varepsilon = 1/100$   
compare  $1/\varepsilon$  vs  $1/\varepsilon^2$ .



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$$\beta^+ = \beta - t \cdot \left( \sum (y_i - p_i(\beta)) x_i + \lambda s \right)$$

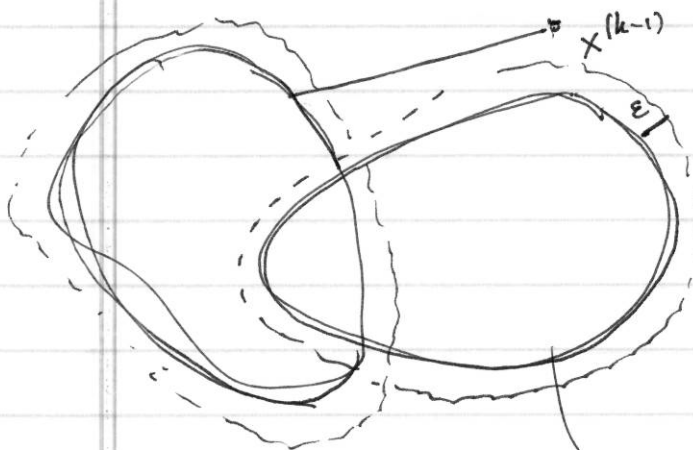
$$s_i = \begin{cases} \text{sign}(\beta_i) & \beta_i \neq 0, i=1, \dots, n \\ [-1, 1] & \text{else} \end{cases}$$

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$$\text{dist}(x, C_i) = \min_{y \in C_i} \|y - x\|_2$$

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$$f(x^{(k-1)}) = \|x^{(k-1)} - P_{C_i}(x^{(k-1)})\|_2$$



$$\{x: g_i(x) \leq b_i\}$$

$$m^2 G^2$$

(Lipschitz bd)<sup>2</sup>

$$\text{for } f = \sum_{i=1}^m f_i$$