Quadratic probs. $\min_x \ \frac{1}{2}x^TQx + c^Tx$

Equality const. $Ax = b$

closed form

Swap out criterion with $f(x)$

↳ Newton's method reduces prob to a sequence of eq. const. quad. probs.

Added ineq. const. $h_i(x) \le 0, \ i = 1 \ldots m$

↳ Barrier method reduces to seq. of probs w/o ineq. constraints

↳ P.D. interior-point method

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perturbed KKT conditions $\quad r(x, u, v) = 0.$

$$
\begin{aligned}
0 &= r(x + \Delta x, u + \Delta u, v + \Delta v) \\
&\approx r(x, u, v) + Dr(x, u, v)\begin{pmatrix} \Delta x \\ \Delta u \\ \Delta v \end{pmatrix}
\end{aligned}
$$

Substitute in $u_i = -\frac{1}{t h_i(x)} \quad i = 1 \ldots m$ ← primal-dual interior-point methods do not do this

solve for $(\Delta x, \Delta v)$

repeat

→ Newton's steps in Barrier method

$$
r(x, u, v) = \begin{pmatrix} r_{dual} \\ r_{cent} \\ r_{prim} \end{pmatrix}
$$

$$r(x,u,v) = \begin{pmatrix} \nabla f(x) + Dh(x)^T u + A^T v \\ -\mathrm{diag}(u)\, h(x) - \frac{1}{t} \\ Ax - b \end{pmatrix}$$

$$\nabla f(x) + Dh(x)^T u + A^T v = 0$$

$$\iff$$

x minimizes $L(x,u,v)$ over x

so $g(u,v) = L(x,u,v)$

i.e. (u,v) is in domain of g (because $L(x,u,v) > -\infty$)

$\eta = \frac{m}{t}$, $t = \frac{m}{\eta}$