$f(x)$ $\quad (\cancel{\lambda f})$

$f^*(y)$ $\quad \cancel{f^*(\lambda)}$

Hello

$$\min_{\beta} f(\beta) + \lambda \|D\beta\|_1$$

$$f^*(y) = \max_x y^T x - f(x)$$

$$\min_{\beta, z} f(\beta) + \lambda \|z\|_1$$

$$z = D\beta$$

$$\max_x y^T x - \lambda f(x) = \lambda \max_x (1/\lambda y)^T x - f(x)$$

$$\min_{\beta, z} f(\beta) + \lambda \|z\|_1 + u^T (D\beta - z)$$

$$= \min_{\beta} \left[f(\beta) + u^T D\beta \right] + \min_{z} \left[\lambda \|z\|_1 - u^T z \right] = \lambda f^*(\tfrac{1}{\lambda} y)$$

$$= -f^*(-D^T u) - I(\|u\|_\infty \leq \lambda)$$

$$\min f^*(-D^T u)$$

$$\text{s.t } \|u\|_\infty \leq \lambda$$