$$\min_x \; f(x) + g(x) \qquad \text{Primal}$$

$$\max_u \; -f^*(u) - g^*(-u) \qquad \text{Dual}$$

$$x \in \arg\min_z \; f(z) - y^T z$$

$$\iff \quad \partial f(x) - y \ni 0.$$

$$\iff \quad y \in \partial f(x)$$

$$\min_x \; f(x) \quad \text{st} \quad Ax = b$$

$$L(x,u) = f(x) + u^T(Ax - b)$$

Stationarity:

at optimality

$$x^* \text{ minimizes } \underbrace{f(x) + u^{*T} A x}\; \cancel{- u^{*T} b}$$

$$\underbrace{dI}_{\substack{\text{strong}\\\text{convexity}\\\text{of } f}} \preceq \nabla^2 f(x) \preceq \underbrace{LI}_{\substack{\text{Lipschitzness}\\\text{of } \nabla f}}$$

together: give $O(\log(1/\varepsilon))$ rate for grad desc.

strong convexity alone: only gives $O(1/\varepsilon)$

$$A = [A_1 \dots A_B]$$

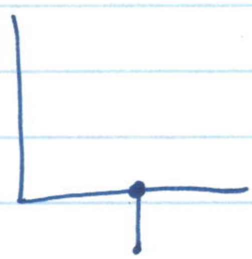
$$x = (x_1 \dots x_B)$$

$$Ax = \sum A_i x_i$$

$$\min_x \quad f(x) + u^T A x$$

$$\iff \min_x \quad \sum_i f_i(x_i) + \sum_i u^T A_i x_i$$

$$\iff \min_{x_i} \quad f_i(x_i) + u^T A_i x_i \qquad i = 1 \dots B \text{ separately}$$



$$Ax = b$$

$$A_1 x_1 + \dots A_B x_B = b$$

$a_i^T x = b_i$

a_i rows of A

$$x^+ \in \operatorname{argmin} \quad f(x) + u^T A x$$

$$\operatorname{argmin} \quad f(x) + \sum_j u_j (Ax)_j$$

~

so $x^+ \in \operatorname{argmax} \quad g(x) - \sum u_j (Ax)_j$

utility function $g = -f$ price   usage of ~~[illegible]~~ resource j

if $f(x) = \sum f_i(x_i)$

and $Ax = b \iff \sum A_i x_i = b$

then $\| \sum A_i x_i - b \|_2^2$ ~~doesn't~~ isn't decomposable!

$x^{(k)} = \text{argmin} \{ \qquad \}$

$u^{(k)} = u^{(k-1)} + t_k (Ax^{(k)} - b)$

$\uparrow$

choose to be ρ

$$L_\rho(x, z, u) = 1_C(x) + 1_D(z) + \frac{\rho}{2} \| x - z + u \|_2^2$$

min over x: projection of $z - u$ onto C

min over z: projection of $x + u$ onto D

dual update: $u^+ = u + x - z$