

Two convex sets, one open, one closed
 \exists separating hyperplane \Rightarrow disjoint

Set closed, nonempty interior, \exists supporting hyperplane at every bdy point \Rightarrow convex.

$$f(x) = h(g(x))$$

$$f'(x) = h'(g(x)) \cdot g'(x)$$

$$f''(x) = \underbrace{h''(g(x)) \cdot (g'(x))^2} + \underbrace{h'(g(x)) \cdot g''(x)}$$

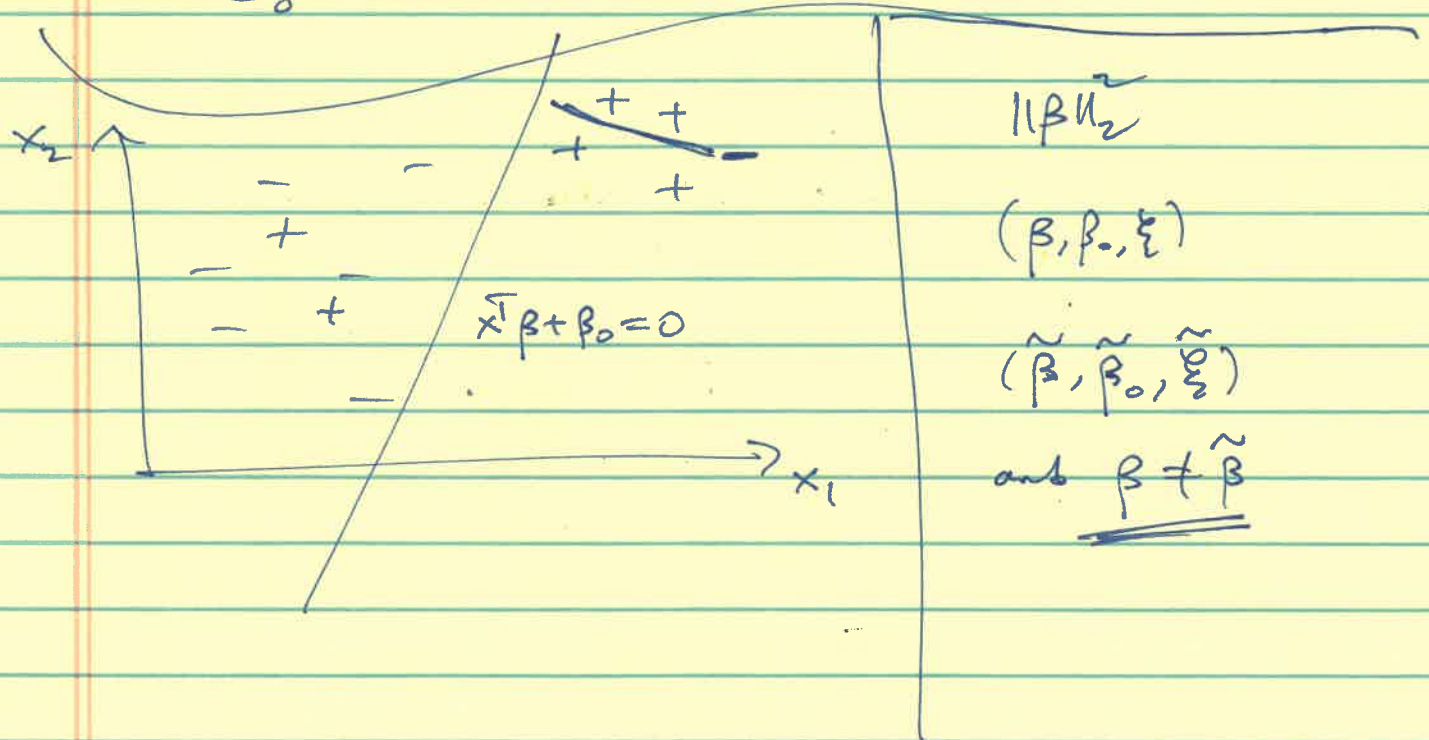
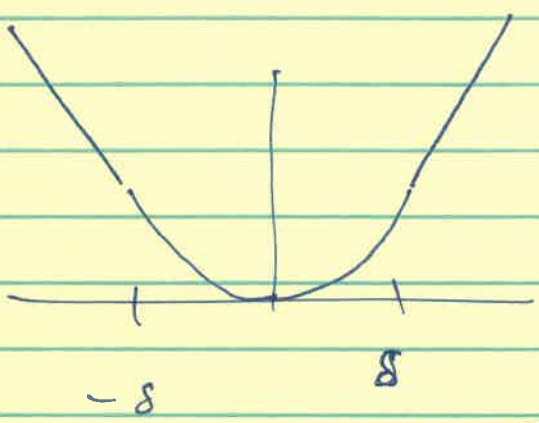
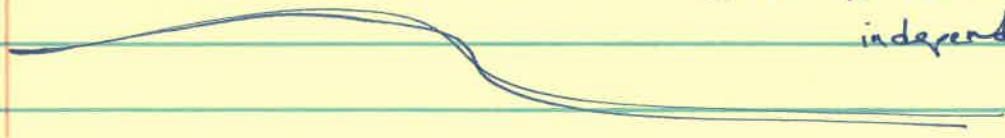
| | |
|--|--|
| $h(x) = 0$ $\Leftrightarrow \begin{cases} h(x) \leq 0 \\ -h(x) \leq 0 \end{cases}$ | $f(x) = e^{-x} \leftarrow$ $\text{min } f(x)$ <hr/> $g(\beta) = \mathbb{1} \beta \mathbb{1}_1 - s$ |
|--|--|

$$f(\beta) = \|y - X\beta\|_2^2 = \beta^T X^T X \beta - 2y^T X \beta + y^T y.$$

$$\nabla^2 f(\beta) = 2X^T X.$$

Strictly pos def $\iff X^T X$ invertible

$\iff X$ has linearly independent columns



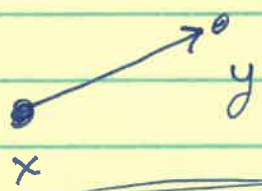
$$\nabla f(x)^T (y-x) \geq 0 \text{ all } y \in \mathbb{R}^n$$

$$\Rightarrow \nabla f(x)^T a \geq 0, \text{ all } a \in \mathbb{R}^n$$

$$\Rightarrow \nabla f(x)^T a = 0, \text{ all } a \in \mathbb{R}^n$$

$$\Rightarrow \nabla f(x) = 0.$$

$$\nabla f(x)^T (y-x) \geq 0 \text{ all } y \in C.$$



$$\max_{\theta} \text{ likelihood}(\theta) \quad \leftarrow$$

$$\Leftrightarrow \max_{\theta} \log \text{ likelihood}(\theta) \quad \leftarrow$$

pick some x_0 s.t. $Ax_0 = b$.

M have columns that span ~~all~~ $\text{null}(A)$

$$\{x : Ax = b\} = \{x_0 + My : y\}$$

