$$\boxed{\begin{aligned} \min_{x} \quad & \tfrac{1}{2} x^T Q x + c^T x \\ \text{s.t.} \quad & Ax = b \leftarrow v \\ & x \geq 0. \quad u \geq 0 \end{aligned}}$$

$$-x \leq -0$$

$Q \succ 0.$

$$L(x,u,v) = \tfrac{1}{2} x^T Q x + c^T x + \sum u_i (-x_i) + \sum v_j (a_j^T x - b_j)$$

$$= \tfrac{1}{2} x^T Q x + c^T x - u^T x + v^T (Ax - b) \leftarrow$$

$$= \tfrac{1}{2} x^T Q x + (c - u + A^T v)^T x - v^T b.$$

$$g(u,v) = \min_{x} L(x,u,v).$$

$Qx = -(c - u + A^T v)$

$x = -Q^{-1}(c - u + A^T v)$

$$= +\tfrac{1}{2}(c - u + A^T v) Q^{-1} Q Q (c - u + A^T v)$$

$$-(c - u + A^T v) Q^{-1} (c - u + A^T v)$$

$$- v^T b.$$

$$\boxed{= -\tfrac{1}{2}(c - u + A^T v) Q^{-1} (c - u + A^T v) - v^T b}$$

$Q \succeq 0.$ Not necessarily invertible.

$$g(u,v) = \min_{x} L(x,u,v).$$

minimize L: $Qx = -(c - u + A^T v).$

if $-(c - u + A^T v) \notin \mathrm{col}(Q)$ then $\min = -\infty.$

if $-(c - u + A^T v) \in \mathrm{col}(Q)$ then

$x = -Q^{+}(c - u + A^T v).$

$\uparrow$ generalized inverse.

$$g(u,v) = \min_x L(x,u,v)$$

$$= \min_x f(x) + \sum u_i h_i(x) + \sum v_j l_j(x)$$

$$= -\max_x \underbrace{\left(- f(x) - \sum u_i h_i(x_i) - \sum v_j l_j(x) \right)}_{g_x(u,v)}$$

affine in (u,v)

$\Rightarrow \max_x g_x(u,v)$ is convex in (u,v)

$$L(x,u) = x^4 - 50x^2 + 100x - ux - 4.5u$$

$-x \le 4.5$

$-x - 4.5 \le 0.$

$$\frac{d}{dx} L(x,u) = 4x^3 - 100x + 100 - u$$

---

f^* $\min \; c^T x$

$Ax = b$

$Gx \le h$

P.

g^* $\max \; -b^T u - h^T v$

$-A^T u - G^T v = c$

$v \geq 0.$

D.

Fact: Dual of D is P.
Slater's condition applied to P: if P feasible then $f^* = g^*$.
Slater's condition applied to D: if D feasible then $g^* = f^*$
Put together. strong duality only when BOTH P, D infeasible

$$L(\beta_0, \beta, \xi, v, w) = \frac{1}{2}\|\beta\|_2^2 + C\sum \xi_i - \sum v_i \xi_i + \sum w_i\left(1 - \xi_i - y_i(x_i^T\beta + \beta_0)\right)$$

$\xi_i \geq 0$ $\qquad v_i$

$y_i(x_i^T\beta + \beta_0) \geq 1 - \xi_i$ $\qquad w_i$

$$= \frac{1}{2}\beta^T\beta + (C\mathbf{1} - v - w)^T\xi + (\tilde{X}^T w)^T\beta + y^T w\,\beta_0.$$

$\tilde{X}$ matrix with rows $y_i x_i$

$$\min_{\beta, \beta_0, \xi} L(\beta_0, \beta, \xi, v, w) = \begin{cases} -\frac{1}{2} w \tilde{X}\tilde{X}^T w + \mathbf{1}^T w. & \text{if } y^T w = 0. \; w = C\mathbf{1} - v. \\ -\infty & \end{cases}$$