$$\min \quad x + 3y$$

$$\left.\begin{array}{l} x + y \geq 2 \\ x, y \geq 0. \end{array}\right\} \Rightarrow 2y \geq 0. \Rightarrow x + 3y \geq 2.$$

lower bound $B = 2$

$$\min \quad px + qy \qquad \text{fix } a, b, c \geq 0.$$

$$\left.\begin{array}{l} x + y \geq 2 \\ x, y \geq 0. \end{array}\right\} \begin{array}{l} ax + ay \geq 2a \\ bx \geq 0. \\ cy \geq 0. \end{array}$$

$$(a+b)x + (a+c)y \geq 2a.$$

if I could find $a, b, c \geq 0$ st $a + b = p$, $a + c = q$,
then lower bound $B = 2a$.

$$\min \quad px + qy \qquad a, b \geq 0. \quad c$$

$$\left.\begin{array}{l} x \geq 0. \\ y \leq 1 \\ 3x + y = 2 \end{array}\right\} \begin{array}{l} ax \geq 0. \\ -by \geq -b. \\ 3cx + cy = 2c \end{array}$$

$$\underbrace{(a+3c)}_{=\,p}x + \underbrace{(c-b)}_{=\,q}y \geq 2c - b.$$

$u_i, v_i \geq 0$

$$\left.\begin{array}{l} a_i^T x = b_i \\ g_i^T x \leq h_i \end{array}\right\} \begin{array}{l} u_i a_i^T x = b_i u_i \\ -v_i g_i^T x \geq -v_i h_i \end{array}$$

$$\left.\begin{array}{l} Ax = b \\ Gx \leq h \end{array}\right\} \left.\begin{array}{l} u^T A x = u^T b \\ -v^T G x \geq -v^T h \end{array}\right\} \begin{array}{l} v^T(Gx - h) \\ + u^T(Ax - b) \leq 0 \end{array}$$

$$\max\{c^T y \quad \text{s.t. } y \geq 0 \quad 1^T y = 1.\} = \max_i c_i$$

$$c = P^T x. \qquad\qquad \max_i (P^T x)_i$$

$$\min_x \quad \max_i (P^T x)_i$$

$$\text{st.} \quad x \geq 0$$

$$1^T x = 1.$$

$$\Longleftrightarrow \quad \min_{x,t} \quad t \qquad\qquad \text{LP.}$$

$$x \geq 0$$

$$1^T \cancel{x} = 1$$

$$\left.\max_i (P^T x)_i \leq t.\right\} \quad P^T x \leq t.$$

$$L(x, t, u, v, y) = t - u^T x + v(1 - 1^T x) + y^T(P^T x - t1)$$

$u \geq 0.$ $y \geq 0.$

$$g(u, v, y) = \min_{x,t} L(x, t, u, v, y)$$

$$= \min_{x,t} (Py - u - v1)^T x + (1 - y^T 1)t + v$$

$$= \begin{cases} v & Py - u - v1 = 0 \text{ and } 1 - y^T 1 = 0. \\ -\infty & \text{else} \end{cases}$$

Dual of LP:

$$\max_{u,v,y} \quad v$$

s.t. $\quad Py - u - v\mathbb{1} = 0.$

$\quad 1 - y^T\mathbb{1} = 0.$

$\quad u \geq 0, \; y \geq 0.$

$\iff \max_{v,y} \quad v$

s.t. $\quad \left.\begin{array}{l} Py \geq v\mathbb{1} \end{array}\right\} \quad \min_i (Py)_i \geq v.$

$\quad 1 - y^T\mathbb{1} = 0.$

$\quad y \geq 0.$

$\iff \max_{y} \quad \min_i (Py)_i$

s.t. $\quad y \geq 0. \; \mathbb{1}^T y = 1.$

strong duality holds.

so. $f_1^* = f_2^*.$