$$A \times B = \{(a,b) : a \in A, b \in B\}$$

$$(D_1 x + d_1,\ e_1^T x + f_1) \in Q_1$$
$$(D_2 x + d_2,\ e_2^T x + f_2) \in Q_2$$

$$\iff \cancel{(Dx + d,\ Ex + f)} \in Q_1 \times Q_2$$

$$\begin{pmatrix} D_1 x + d_1 \\ e_1^T x + f_1 \\ D_2 x + d_2 \\ e_2^T x + f_2 \end{pmatrix} \in Q_1 \times Q_2$$

$$\begin{bmatrix} tI & x \\ x^t & t \end{bmatrix} \succeq 0 \iff tI - \frac{xx^T}{t} \succeq 0$$

$$\iff t^2 I - xx^T \succeq 0.$$

$$\iff \|x\|_2^2 \le t^2$$

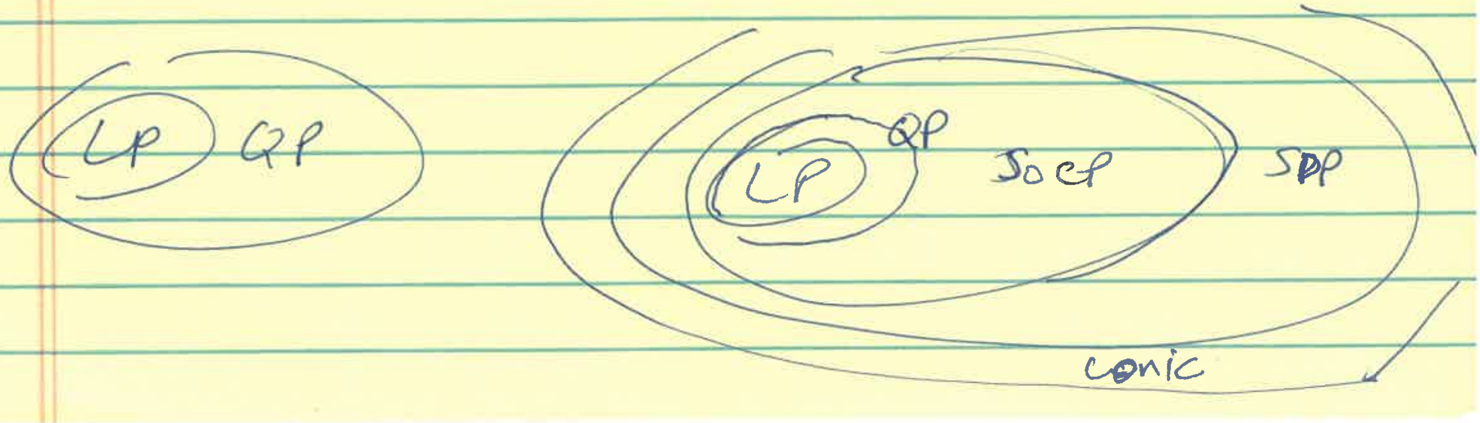
or $\|x\|_2 \le t$

Schur complement.

$$a^T(t^2 I - xx^t)a \ge 0$$

all a

$$t^2\|x\|_2^2 - \|x\|_2^4 \ge 0.$$

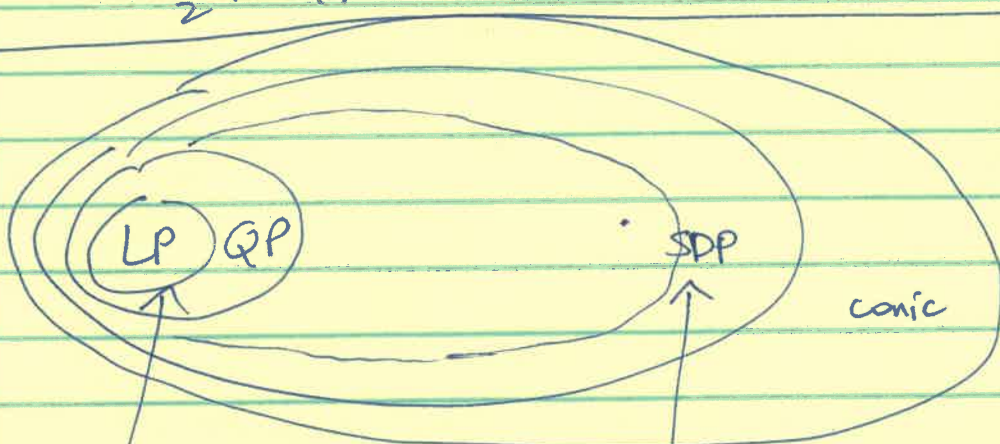


symmetric square root $Q^{1/2}Q^{1/2} = Q$.

$$\left\| \left(\frac{1}{\sqrt{2}} Q^{1/2} x, \ \frac{1}{2}(1-t) \right) \right\|_2^2 \le \frac{1}{2}(1+t)^2$$

$$\frac{1}{2} x^T Q x + \frac{1}{4}(\cancel{1} - 2t + \cancel{t^2}) \le \frac{1}{4}(\cancel{1} + 2t + \cancel{t^2})$$

$$\frac{1}{2} x^T Q x \le t. \quad \checkmark.$$

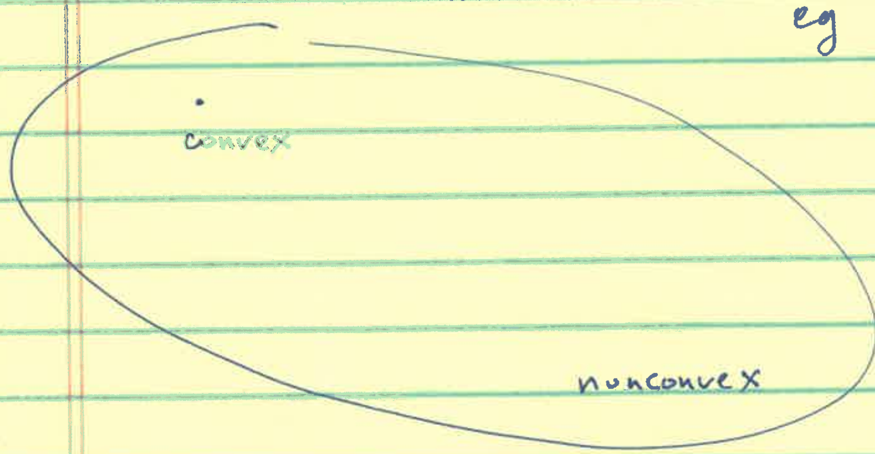


"easy" algorithmically

"hard"

only very special SDPs are easy to solve!

eg graphical lasso.
trace norm imputation



$$f(y) \approx f(x) + \nabla f(x)^T (y - x) \qquad y = x + t\Delta x$$

$$= f(x) + t \nabla f(x)^T \Delta x$$

assuming twice differentiable L Lipschitz $\iff$ $\nabla^2 f(x) \preceq LI$ all x

$$f(y) \leq f(x) + \nabla f(y)^T (y-x) + \frac{L}{2} \|y-x\|_2^2 .$$

$O(1/k)$ or to get ε-suboptimal solution

$$f(x^{(k)}) - f^* \leq \varepsilon$$

$$\frac{c}{k} = \varepsilon$$

$$k = \frac{c}{\varepsilon}$$

$$O(1/\varepsilon)$$

$O(1/k^2)$ equiv. $\frac{c}{k^2} = \varepsilon$

$$k = \sqrt{\frac{c}{\varepsilon}}$$

$$O(1/\sqrt{\varepsilon})$$

$O(c^k)$ $c^k = \varepsilon$

$$k = \underline{\log(1/\varepsilon)} \cdot \text{const.}$$

$f(\beta) = \frac{1}{2}\|y - X\beta\|_2^2$, $X^T(X\beta - y)$ $\nabla f(\beta)$ "

$\nabla^2 f(\beta) = X^T X$

$$f(y) \le f(x) + \nabla f(x)^T (y-x) + \frac{L}{2}\|y-x\|_2^2$$

all y

plug in $\underline{y = x - t\nabla f(x)} := x^+$

$$f(x^+) \le f(x) - t\|\nabla f(x)\|_2^2 + \frac{Lt^2}{2}\|\nabla f(x)\|_2^2$$

$$= f(x) - \underbrace{t\left(1 - \frac{Lt}{2}\right)}_{w}\|\nabla f(x)\|_2^2.$$

take $t \le 1/L$

$w > 0$.

$$\|\nabla f(x)\|_2^2 \le \frac{1}{w}\left(f(x) - f(x^+)\right)$$

add up this bound over $i = 0, \dots k$

$$\sum \|\nabla f(x^{(i)})\|_2^2 \le \frac{1}{w}\left(f(x^{(0)}) - f(x^{(k+1)})\right)$$

$$\le \frac{1}{w}\left(f(x^{(0)}) - f^*\right)$$

$$(k+1)\min \|\nabla f(x^{(i)})\|_2^2 \le$$

$$\min \|\nabla f(x^{(i)})\|_2^2 \le \frac{\frac{1}{w}\left(f(x^{(0)}) - f^*\right)}{k+1}.$$