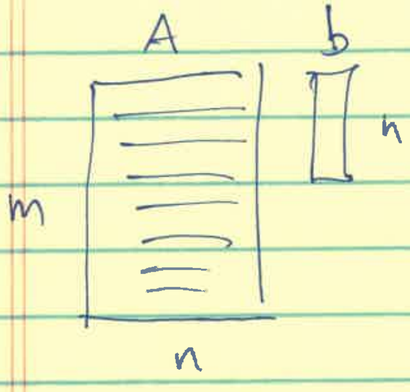
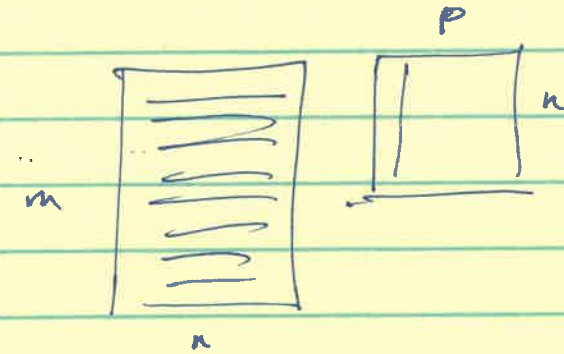
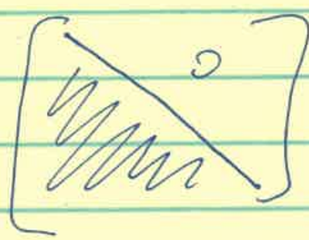
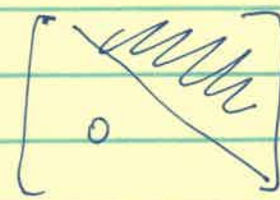




$$\nabla^2 f(x)\, v = \nabla f(x)$$
$$Av = b.$$

$$A = \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix}$$

$A =$ 



- $Ax = b, \quad Ax = c, \quad Ax = d \ldots$

$n^3 + 3n^2.$

$A \backslash B$

$Ax = b$

$QRx = b$

$Rx = Q^T b. \qquad n^2$

$x = R^{-1} Q^T b \qquad n^2.$

$A = B^T B.$

$B = QR$

$B^T B = R^T Q^T Q R$

$= R^T R$

$:= L L^T.$

$$Ax = b$$

$$\underbrace{LL^T x}_{y} = b.$$

$Ly = b.$ $\quad n^2$ forward substitution.

$y = L^{-1} b.$

$L^T x = y$

$x = (L^T)^{-1} y \quad n^2$ back substitution

$X^T X \beta = X^T y$ "normal equations".

$n \times p$ $\quad Q$ from QR

$n \times (n-p)$ $\quad \tilde{Q}$ orthogonal completes the basis

$P = [Q \;\; \tilde{Q}]$ orthogonal

$n \times n$

$$\underline{\|x\|_2^2} = \|P^T x\|_2^2 = \left\| \begin{bmatrix} Q^T \\ \tilde{Q}^T \end{bmatrix} x \right\|_2^2 = \left\| \begin{bmatrix} Q^T x \\ \tilde{Q}^T x \end{bmatrix} \right\|_2^2$$

$$= \underline{\|Q^T x\|_2^2 + \|\tilde{Q}^T x\|_2^2}.$$

$x = y - X\beta.$ $\quad$ write $X = QR$

$$\underline{\|y - X\beta\|_2^2} = \|Q^T(y - QR\beta)\|_2^2$$
$$+ \|\tilde{Q}^T(y - QR\beta)\|_2^2$$

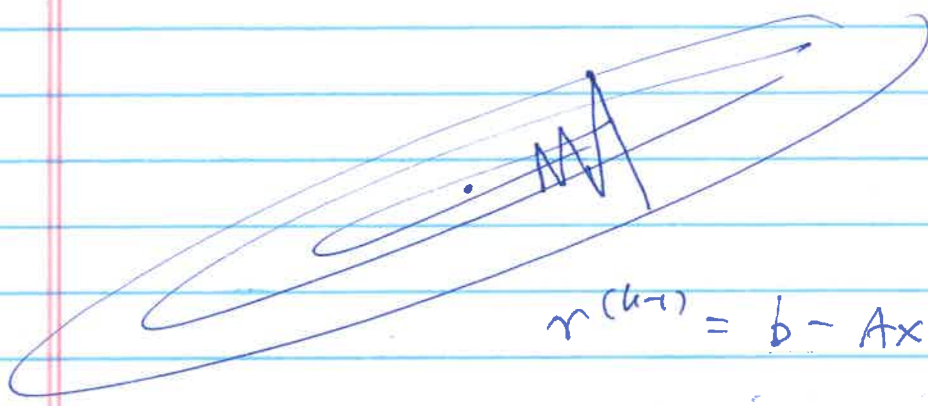
$$= \underline{\|Q^T y - R\beta\|_2^2 + \cancel{\|\tilde{Q}^T y\|_2^2}}$$

$R\beta = Q^T y$ $\quad$ solve: p^2 back substitution

$$f(x) = \frac{x^T A x}{2} - b^T x$$

$$\min f(x) \iff 0 = \nabla f(x)$$

$$Ax = b.$$



$$r^{(k-1)} = b - Ax^{(k-1)}$$

$$x^{(k)} = x^{(k-1)} + t_k p^{(k-1)}$$

$$\|y - X\beta\|_2^2$$

$$X^T X \beta = X^T y$$