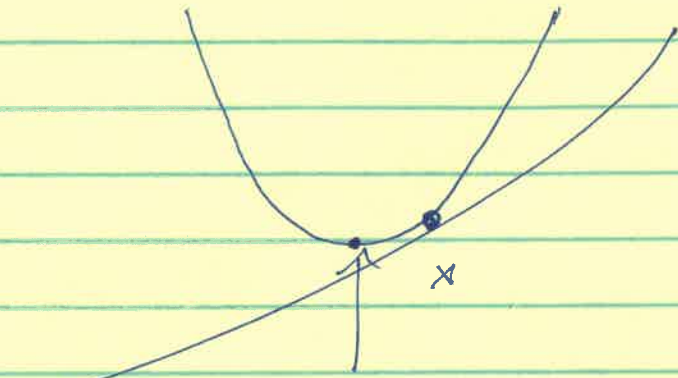


$$x^+ = x - t\nabla f(x)$$

$$\frac{1}{2t}\| z - (x - t\nabla g(x))\|_2^2 = \frac{1}{2t}\|z-x\|_2^2 + (z-x)^T \nabla g(x) + \text{const.}$$

$$\text{prox}_t(\beta) = \underset{z}{\text{argmin}} \ \frac{1}{2t}\|z-\beta\|_2^2 + \lambda\|z\|_1$$

$$= \underset{z}{\text{argmin}} \ \frac{1}{2}\|z-\beta\|_2^2 + \lambda t\|z\|_1$$

$$= S_{\lambda t}(\beta)$$ ← very cheap, $O(p)$ operations

$$\beta^+ = S_{\lambda t}(\beta - t\nabla g(\beta))$$

$$= S_{\lambda t}(\beta + tX^T(y - X\beta))$$ ← sparsity inducing

---

$$\|A\|_F^2 = \sum_{i,j} A_{ij}^2$$

$$\underset{Z}{\min} \ \frac{1}{2t}\|B - Z\|_F^2 + \lambda\|Z\|_{tr}$$

$$\Longleftrightarrow \underset{Z}{\min} \ \frac{1}{2}\|B - Z\|_F^2 + \lambda t\|Z\|_{tr}$$

$$Z - B + \lambda t \Gamma = 0.$$

for some $\Gamma \in \partial \|X\|_{tr}\big|_{X=Z}$.

Fact: $\partial \|X\|_{tr} = \{ UV^T + W : \|W\|_{op} \le 1,\ U^T W = 0,\ WV = 0 \}$

$X = U \Sigma V^T$

[illegible] $Z = S_{\lambda t}(B)$

$\cancel{\text{check: } B = Z + \lambda t \Gamma = S_{\lambda t}(Z)}$.

use $\cancel{Z = U \Sigma V^T}$

$$\cancel{Z - S_{\lambda t}(Z) = \lambda t \Gamma}$$

$$= \cancel{U(\Sigma - \Sigma_{\lambda t})V^T = \lambda t \Gamma}$$

ie, $\cancel{\Gamma = U\left(\frac{\Sigma - \Sigma_{\lambda t}}{\lambda t}\right)V^T}$

must ~~check this is a subgradient~~.

$$\cancel{\frac{\Sigma_{\lambda t} - \Sigma}{\lambda t}}$$

subgradient optimality: with this choice

$$\Gamma = U\left(\frac{\Sigma - \Sigma_{\lambda t}}{\lambda t}\right)V^T$$ is indeed a subgradient

$\|X\|_{tr}\big|_{X = S_{\lambda t}(B)}$

$$\frac{\Sigma - \Sigma_{\lambda t}}{\lambda t} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \\ & & & & \ddots \end{pmatrix}$$

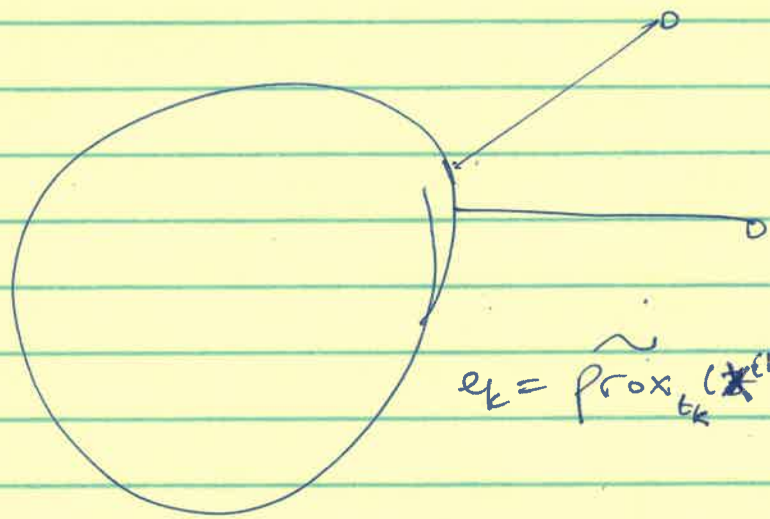
$U_{1:k} V_{1:k}^T + W$

$h = 0$

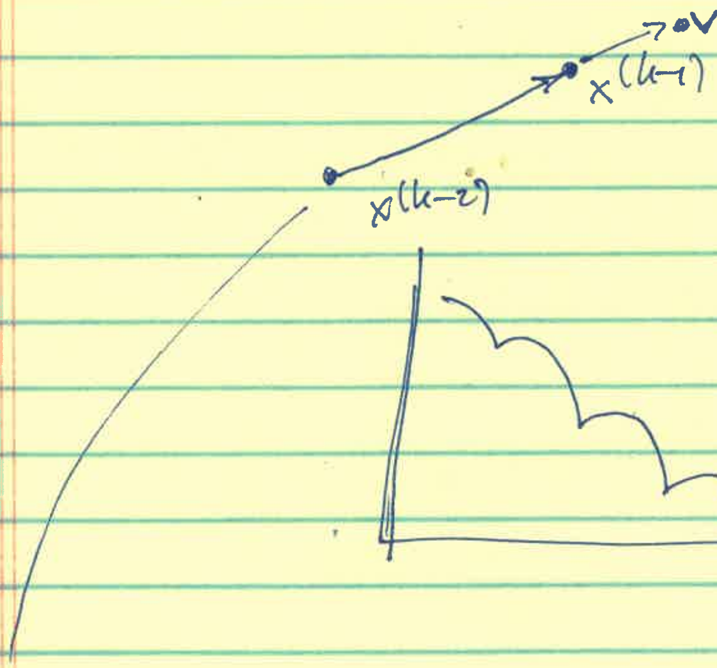
$$\mathrm{prox}_t(x) = \underset{z}{\mathrm{argmin}} \ \frac{1}{2t}\|x - z\|_2^2 + \cancel{h(z)}$$

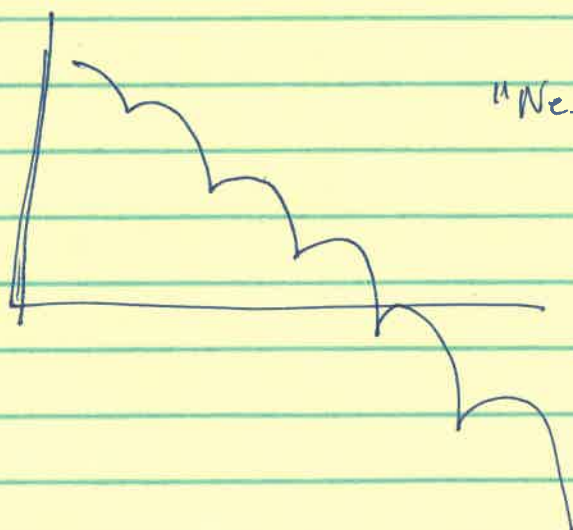
$$= x.$$

$$x^+ = \mathrm{prox}_t(x - t\nabla g(x)) = x - t\nabla g(x).$$



$$e_k = \widetilde{\mathrm{prox}}_{t_k}(x^{(k)}) - \mathrm{prox}_{t_k}(x^{(k)})$$





"Nesterov ripples".