

# Supplement to “Extended Comparisons of Best Subset Selection, Forward Stepwise Selection, and the Lasso”

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This supplementary document contains plots from the simulation suite described in the paper “Extended Comparisons of Best Subset Selection, Forward Stepwise Selection, and the Lasso”. The plots in Section 1 precisely follow the simulation format described in the paper. Those in Section 2 follow an analogous format, except that the tuning has been done using an “oracle”, rather than a validation set as in Section 1. Specifically, the tuning parameter for each method in each scenario is chosen to minimize the average risk over all of the repetitions.

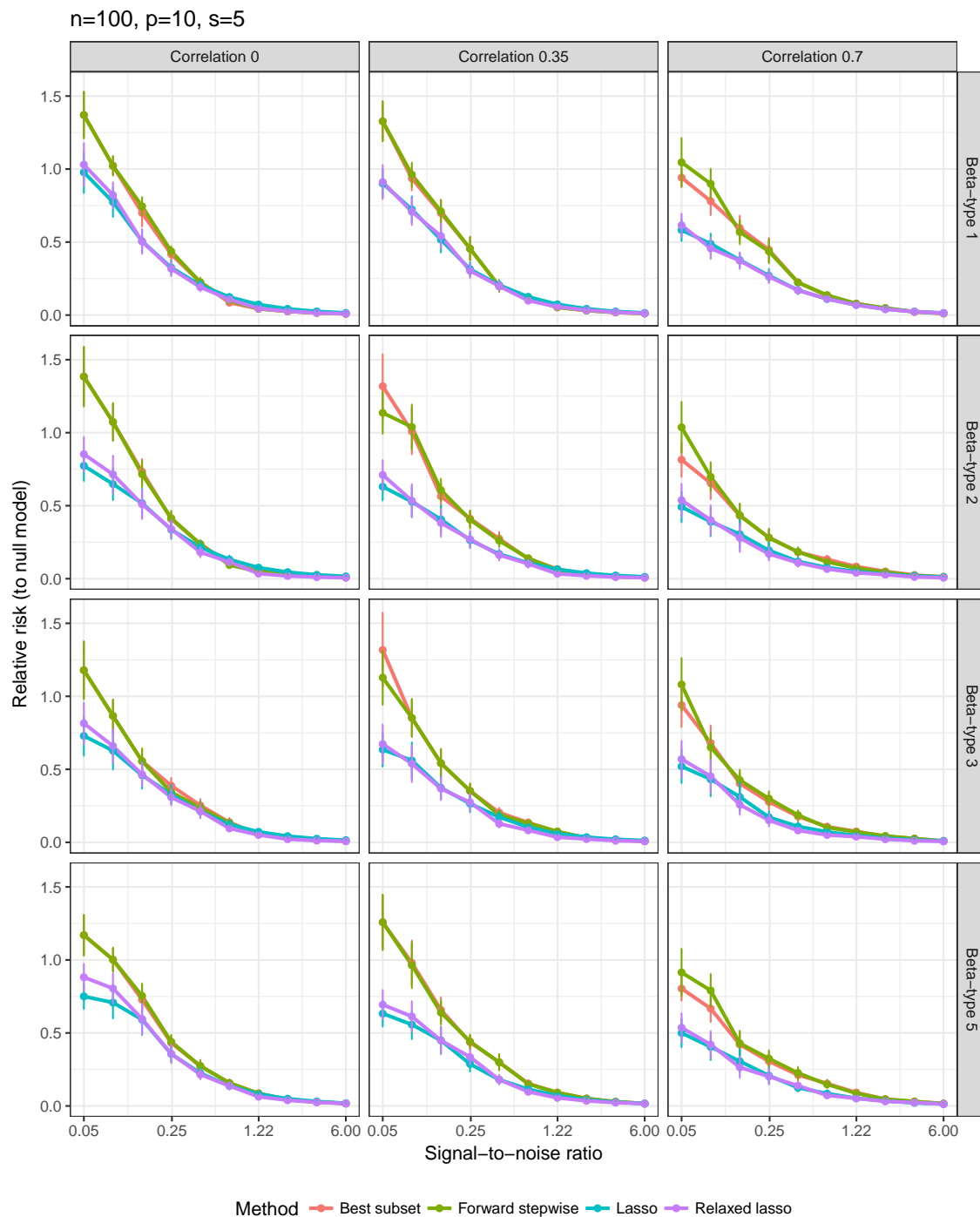
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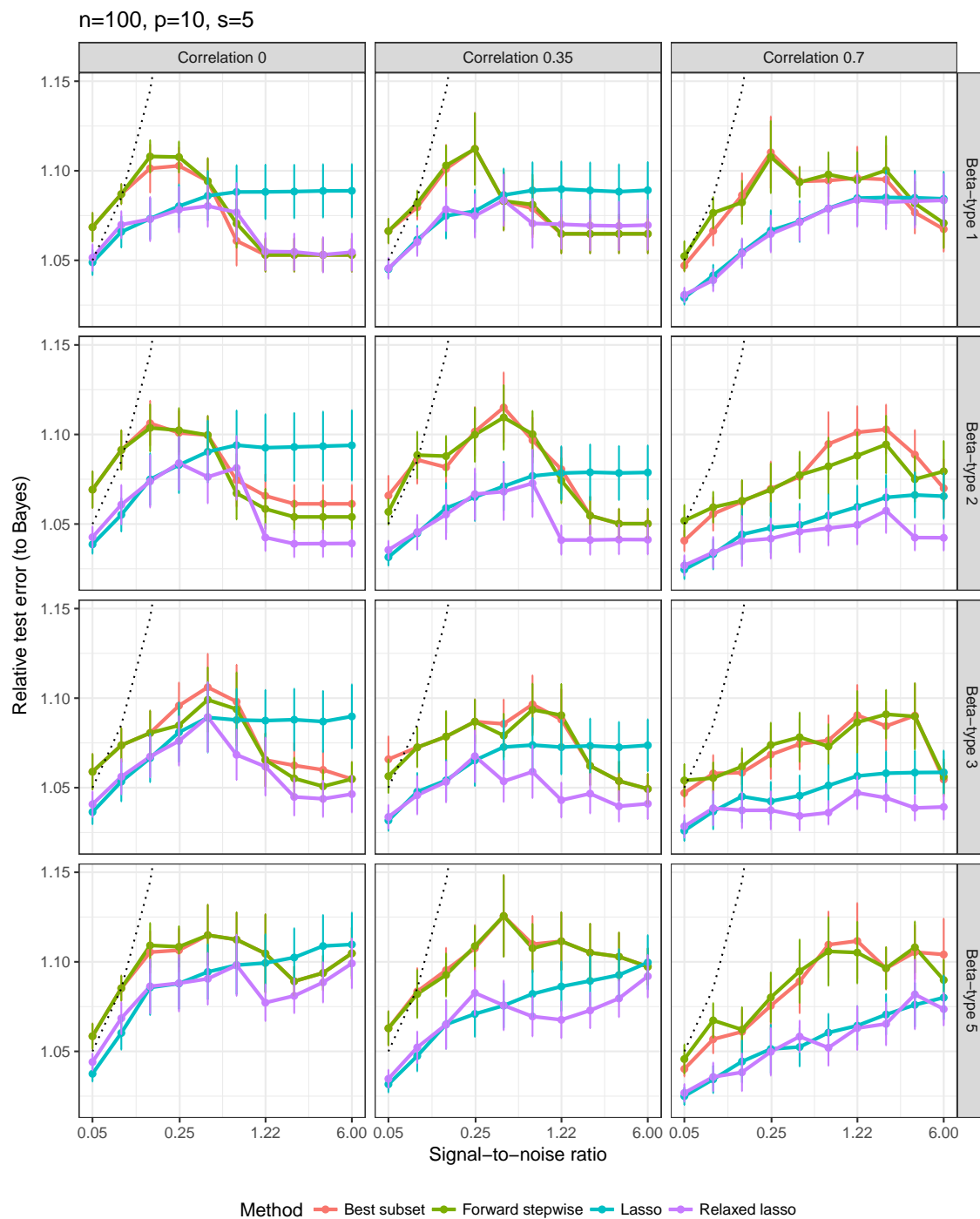
# A Validation tuning

## A.1 Low setting: $n = 100, p = 10, s = 5$

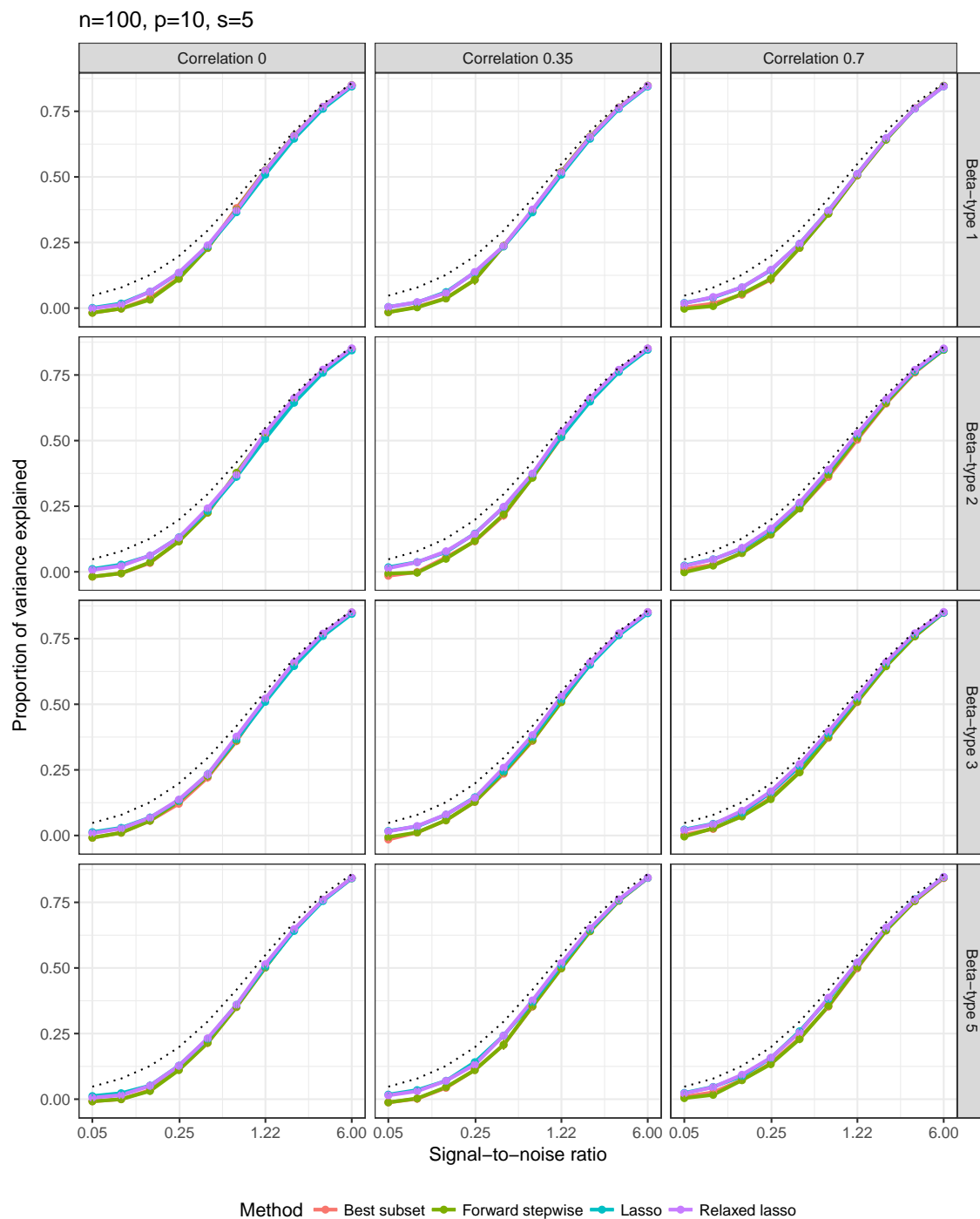
### A.1.1 Relative risk (to null model)



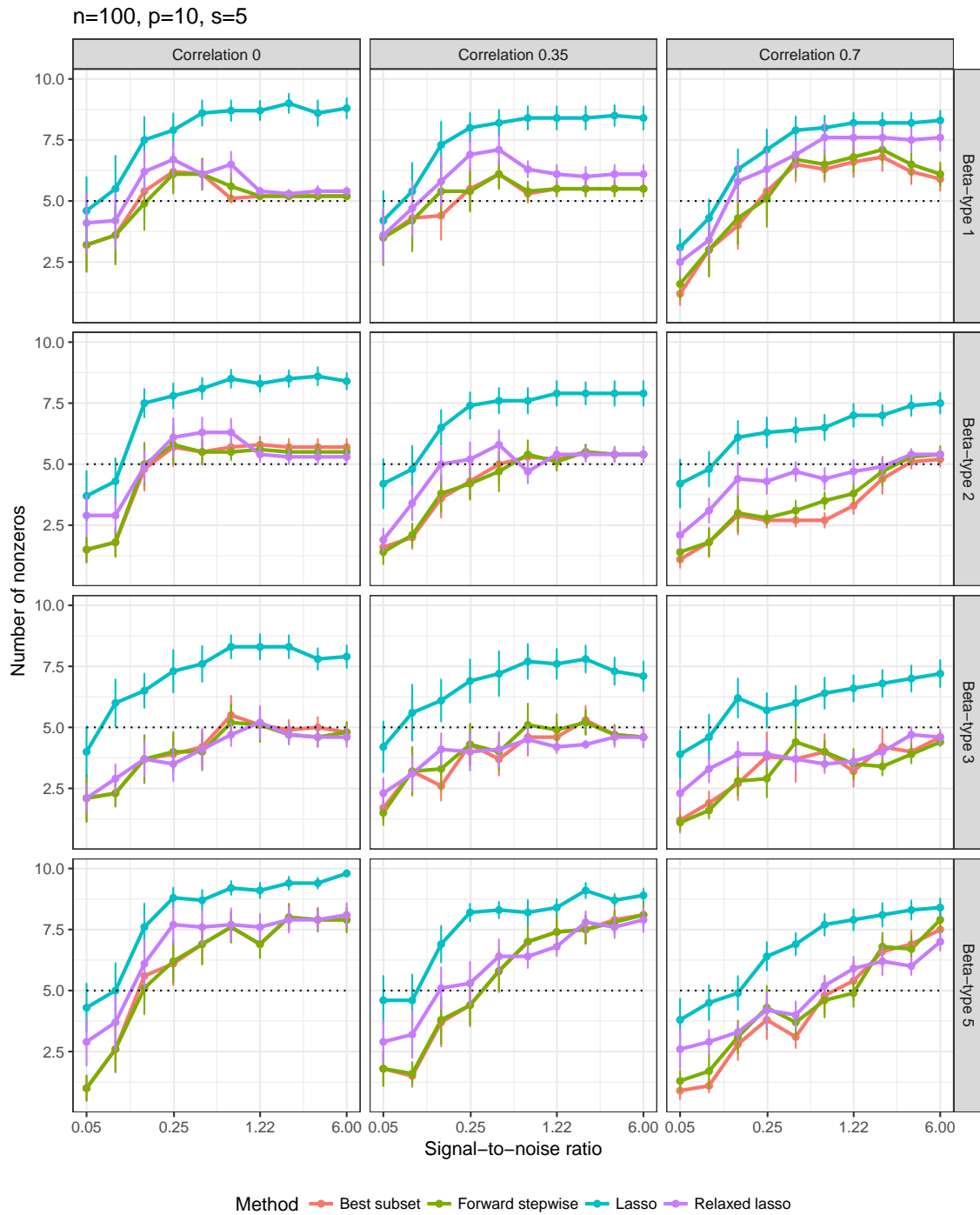
## A.1.2 Relative test error (to Bayes)



### A.1.3 Proportion of variance explained

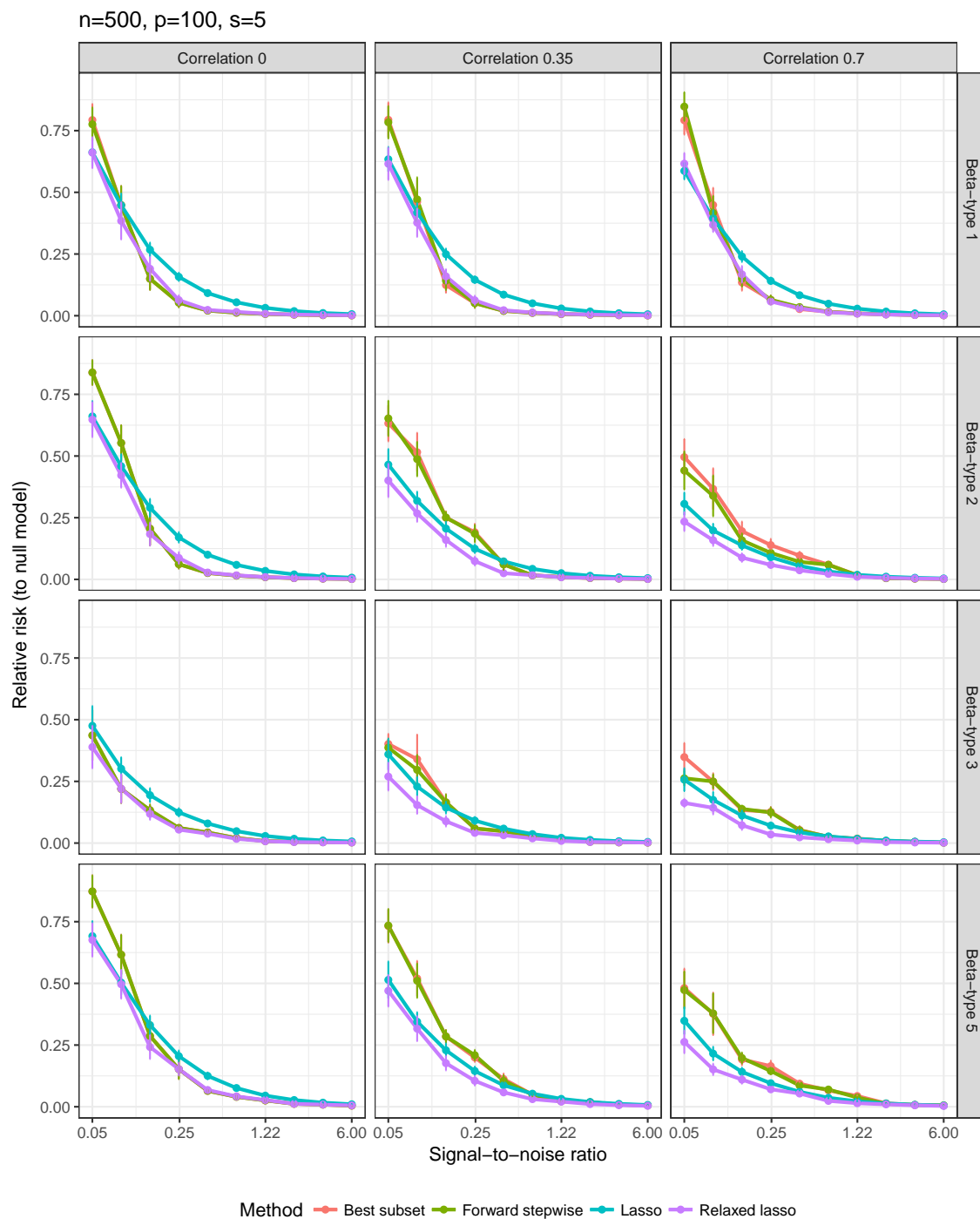


### A.1.4 Number of nonzero coefficients

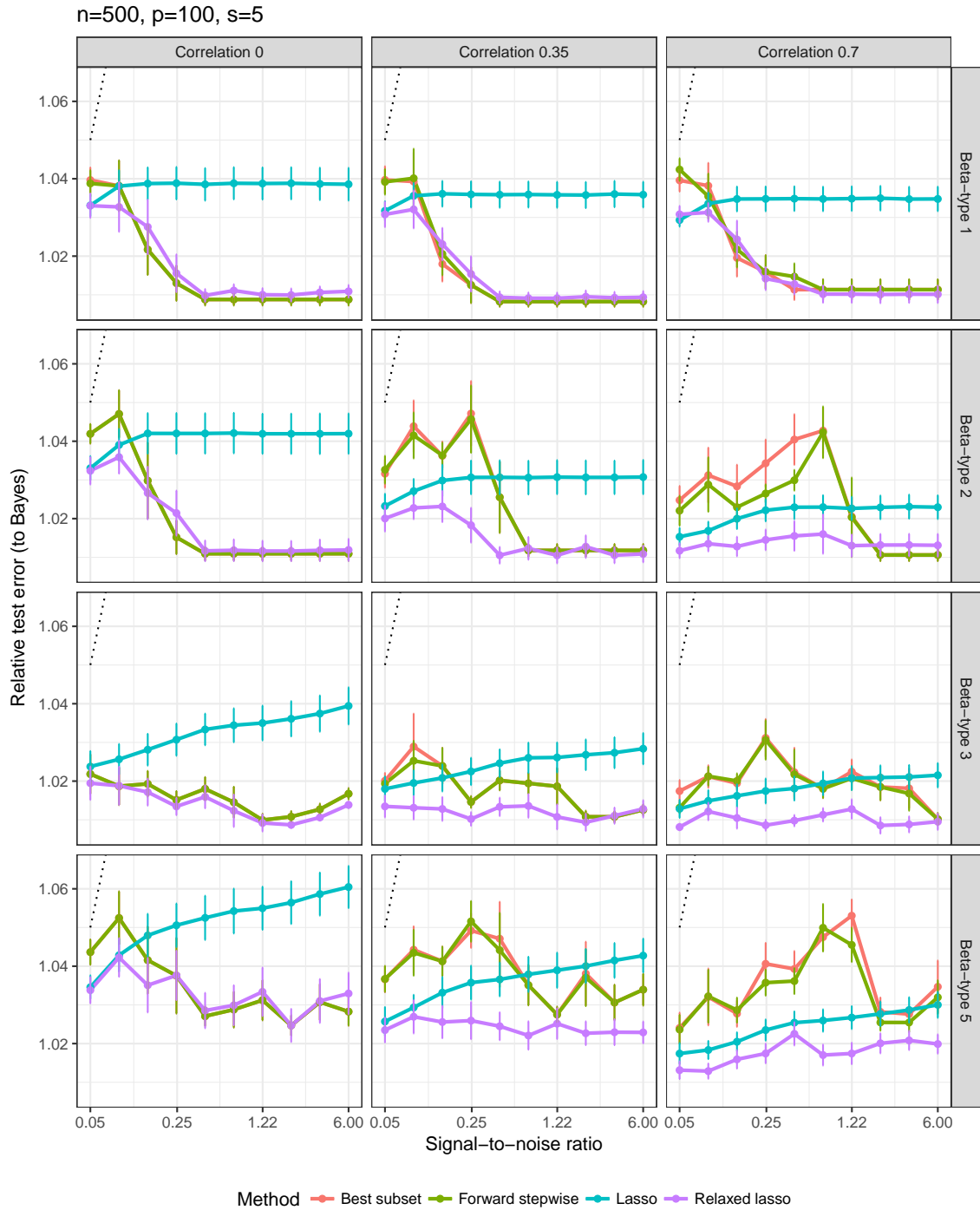


## A.2 Medium setting: $n = 500, p = 100, s = 5$

### A.2.1 Relative risk (to null model)

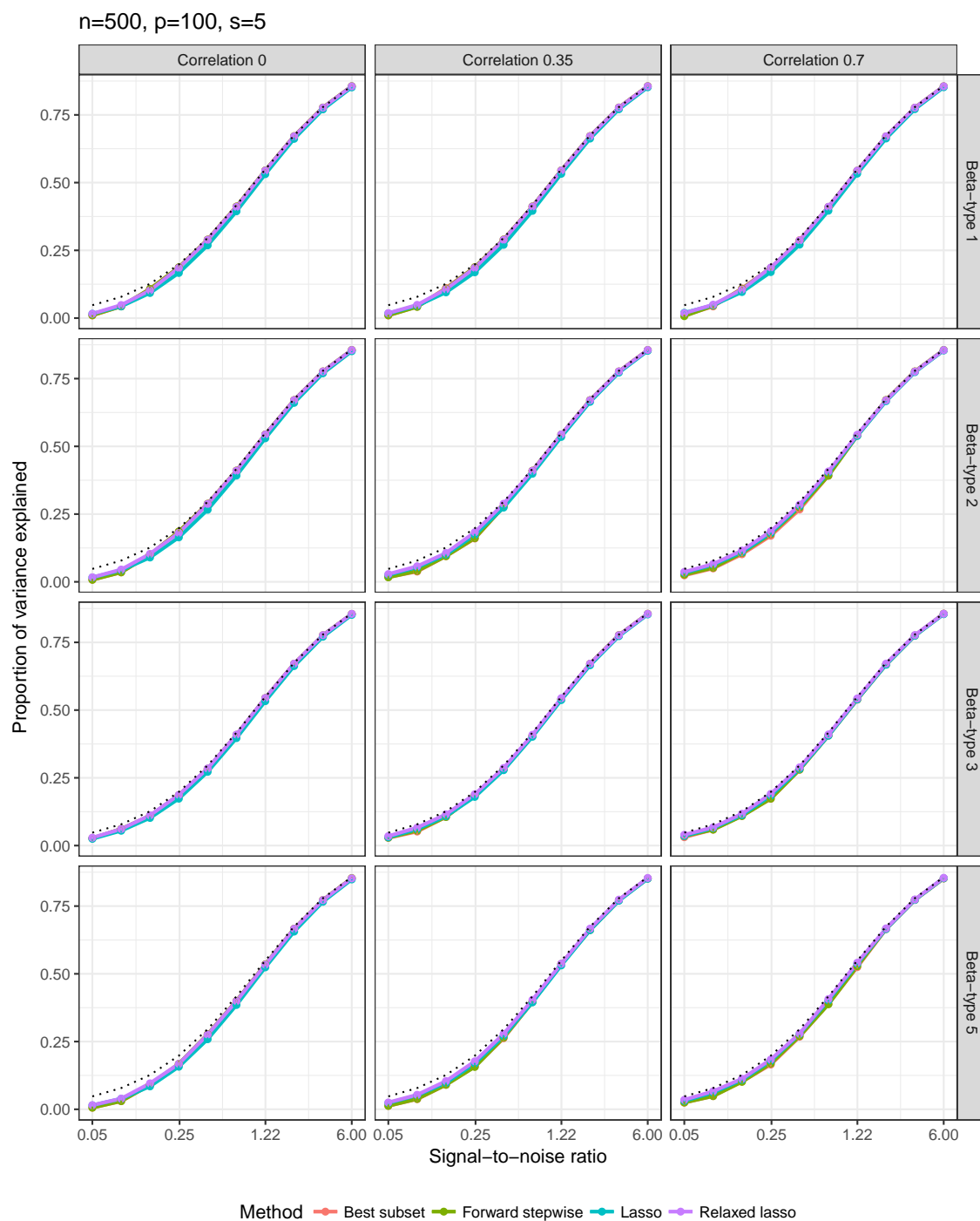


## A.2.2 Relative test error (to Bayes)

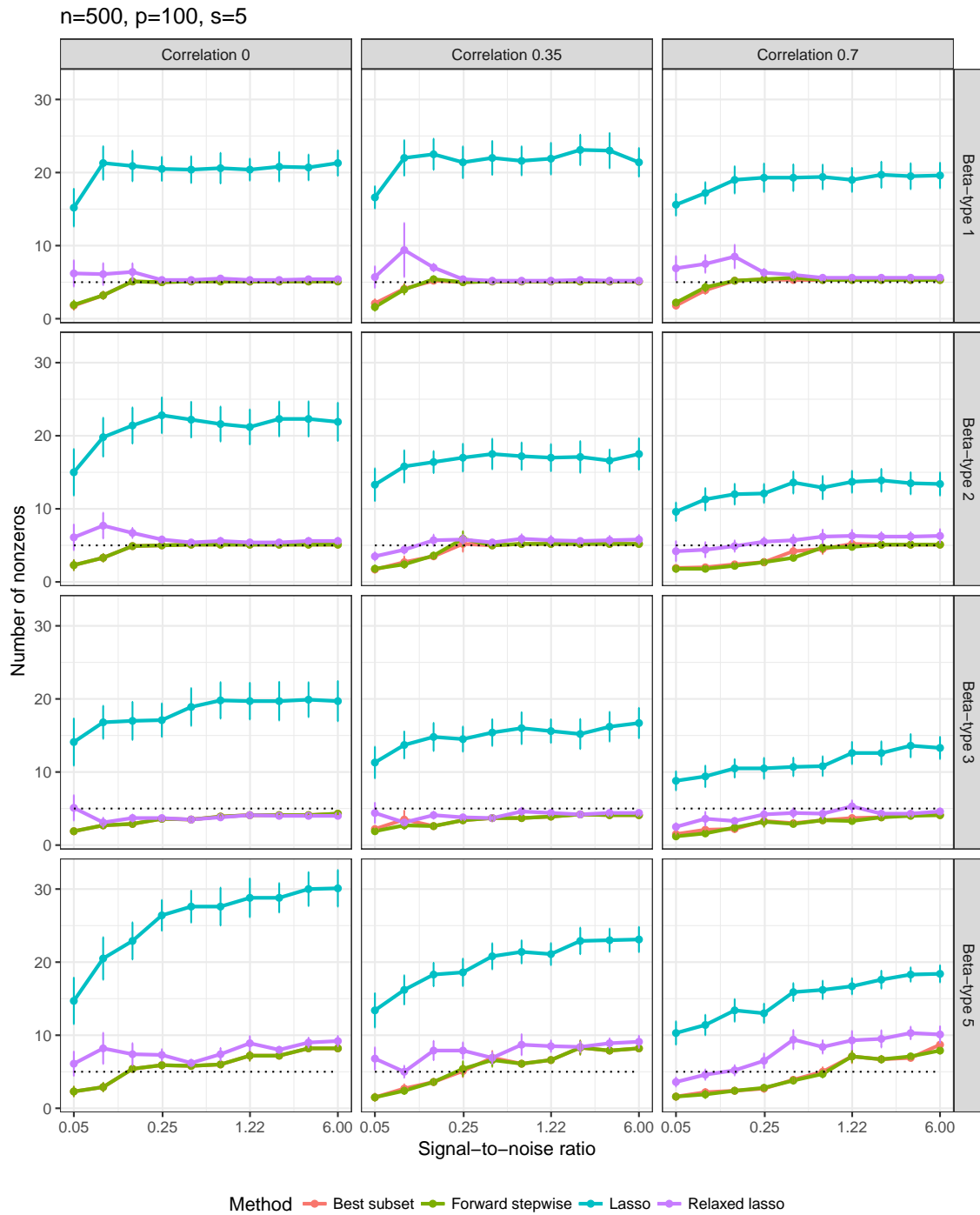




### A.2.3 Proportion of variance explained

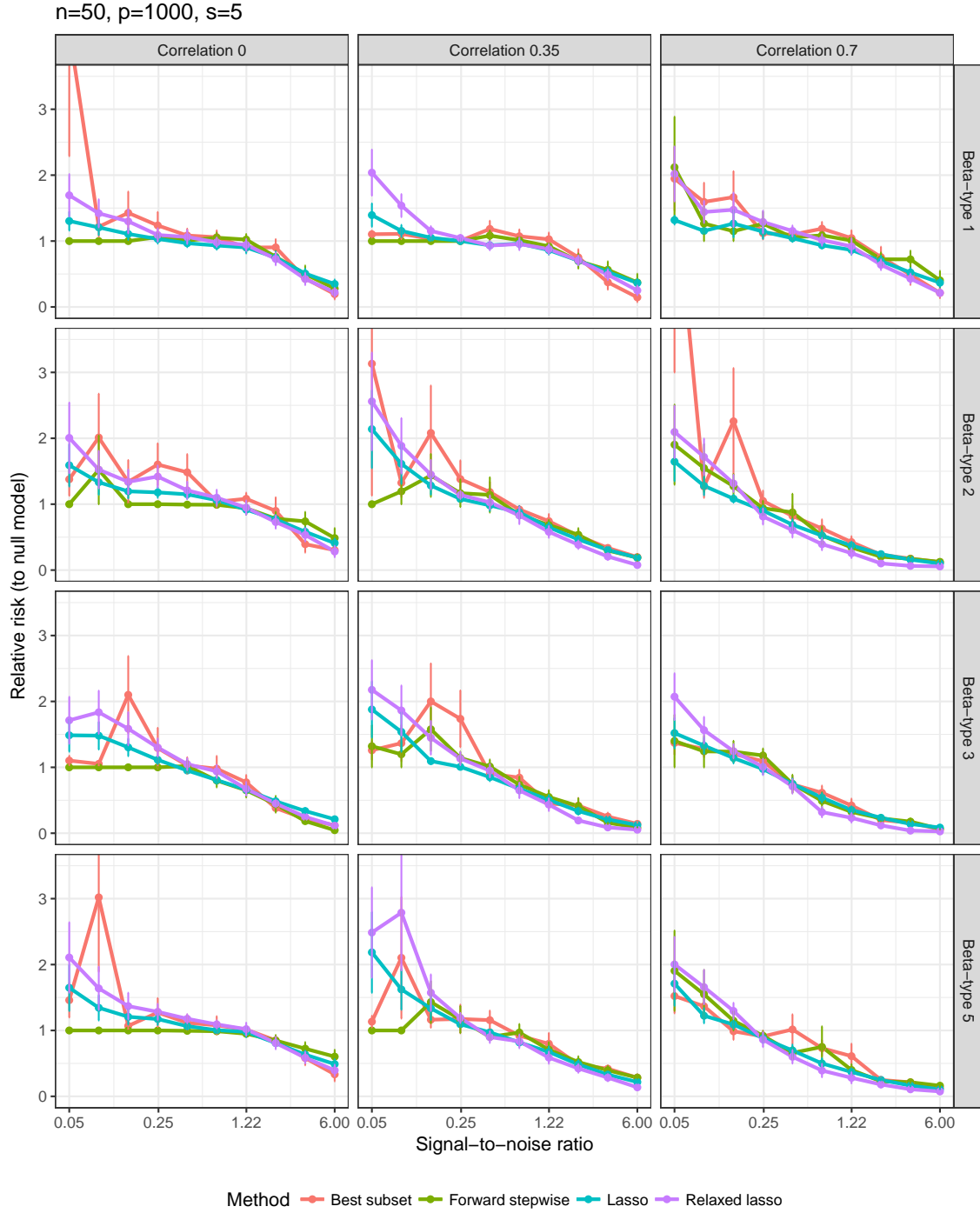


## A.2.4 Number of nonzero coefficients

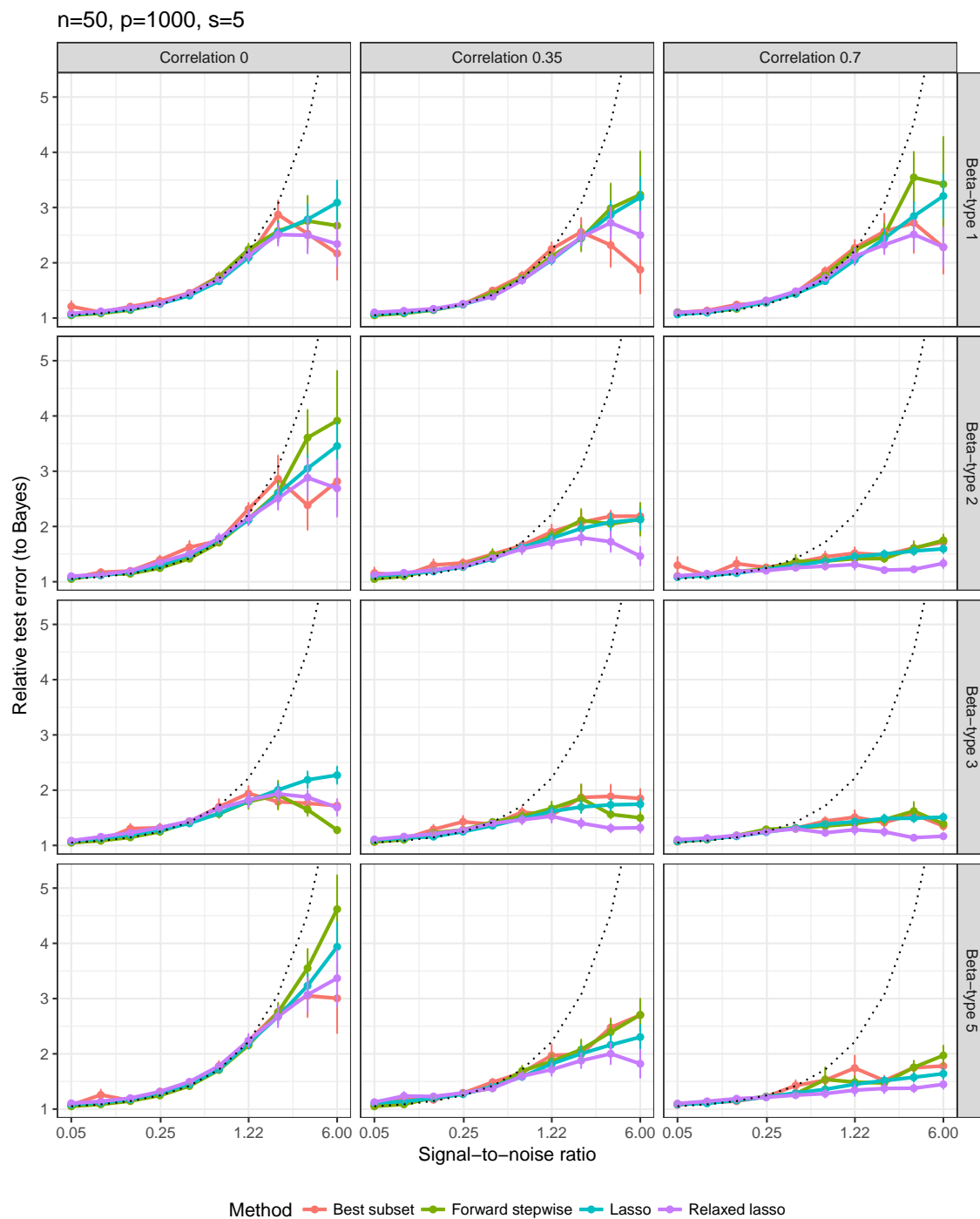


### A.3 High-5 setting: $n = 50, p = 1000, s = 5$

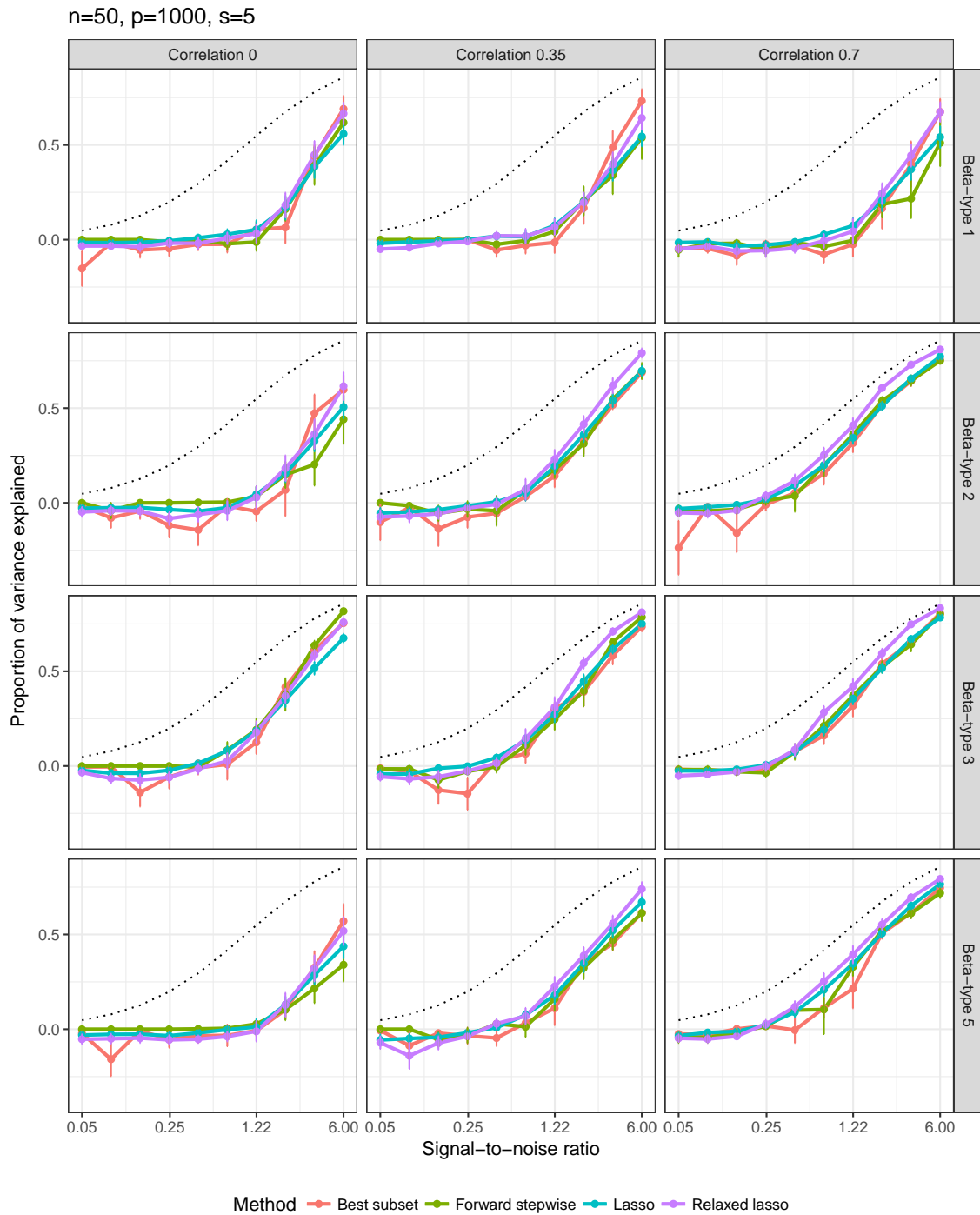
#### A.3.1 Relative risk (to null model)



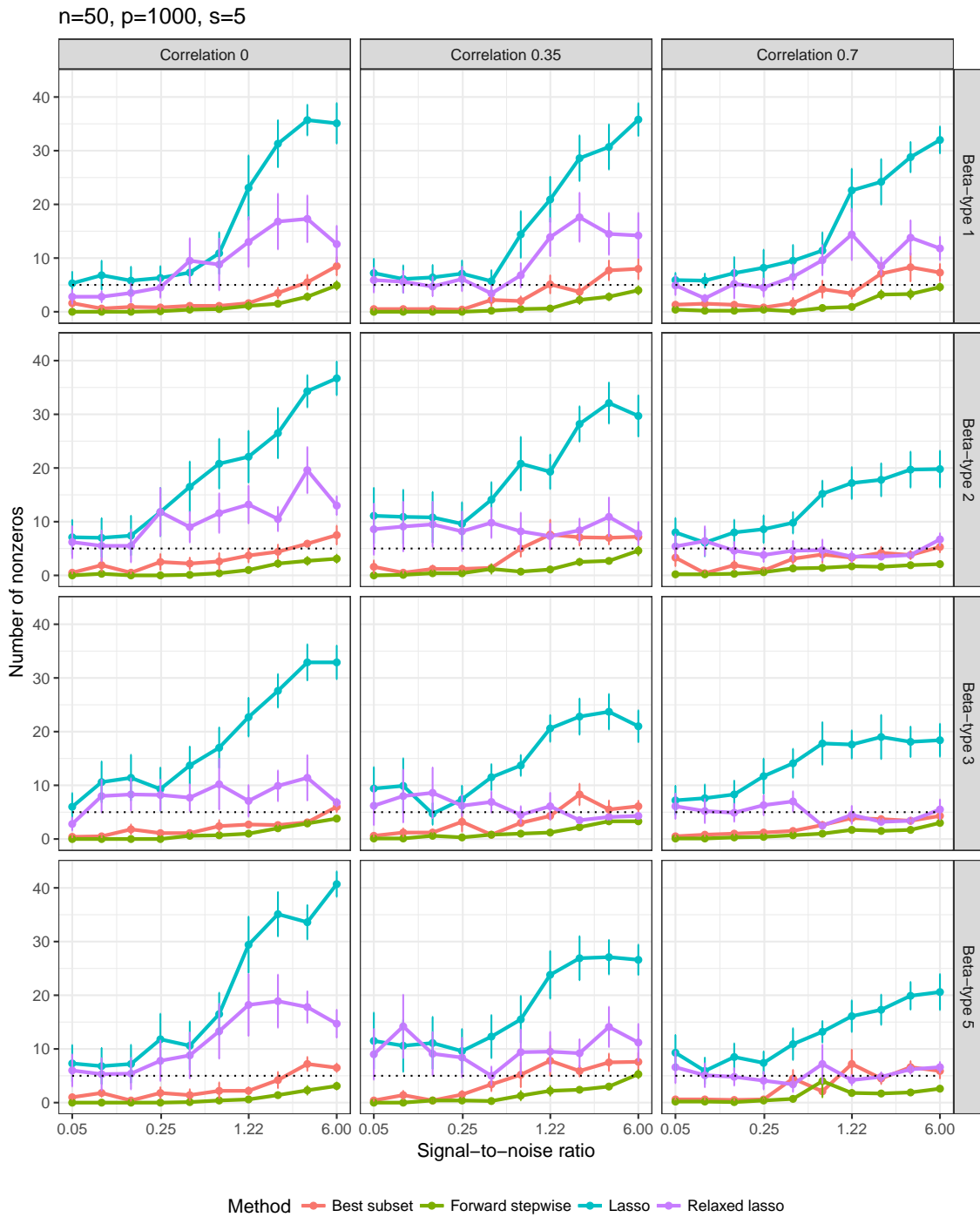
### A.3.2 Relative test error (to Bayes)



### A.3.3 Proportion of variance explained

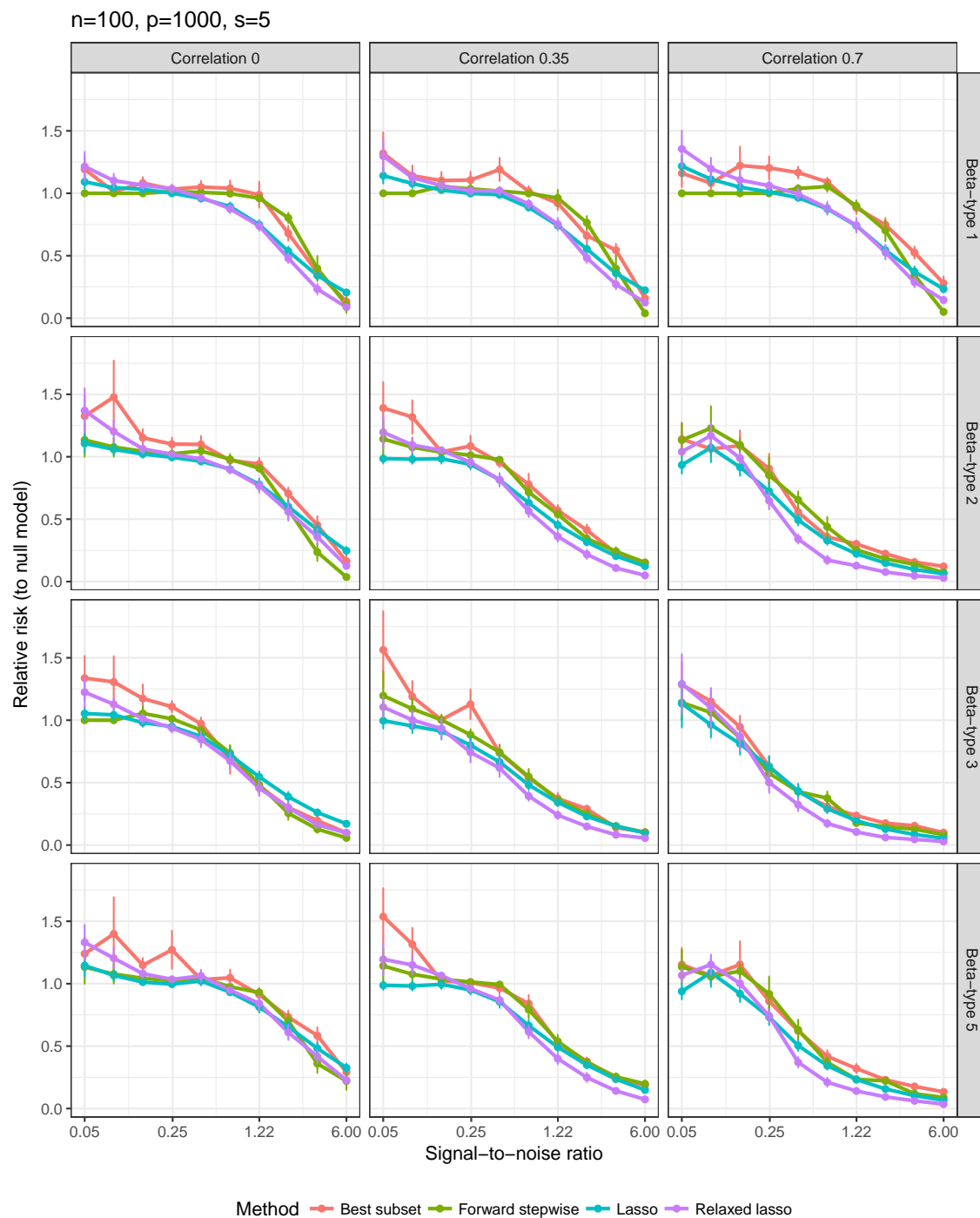


### A.3.4 Number of nonzero coefficients

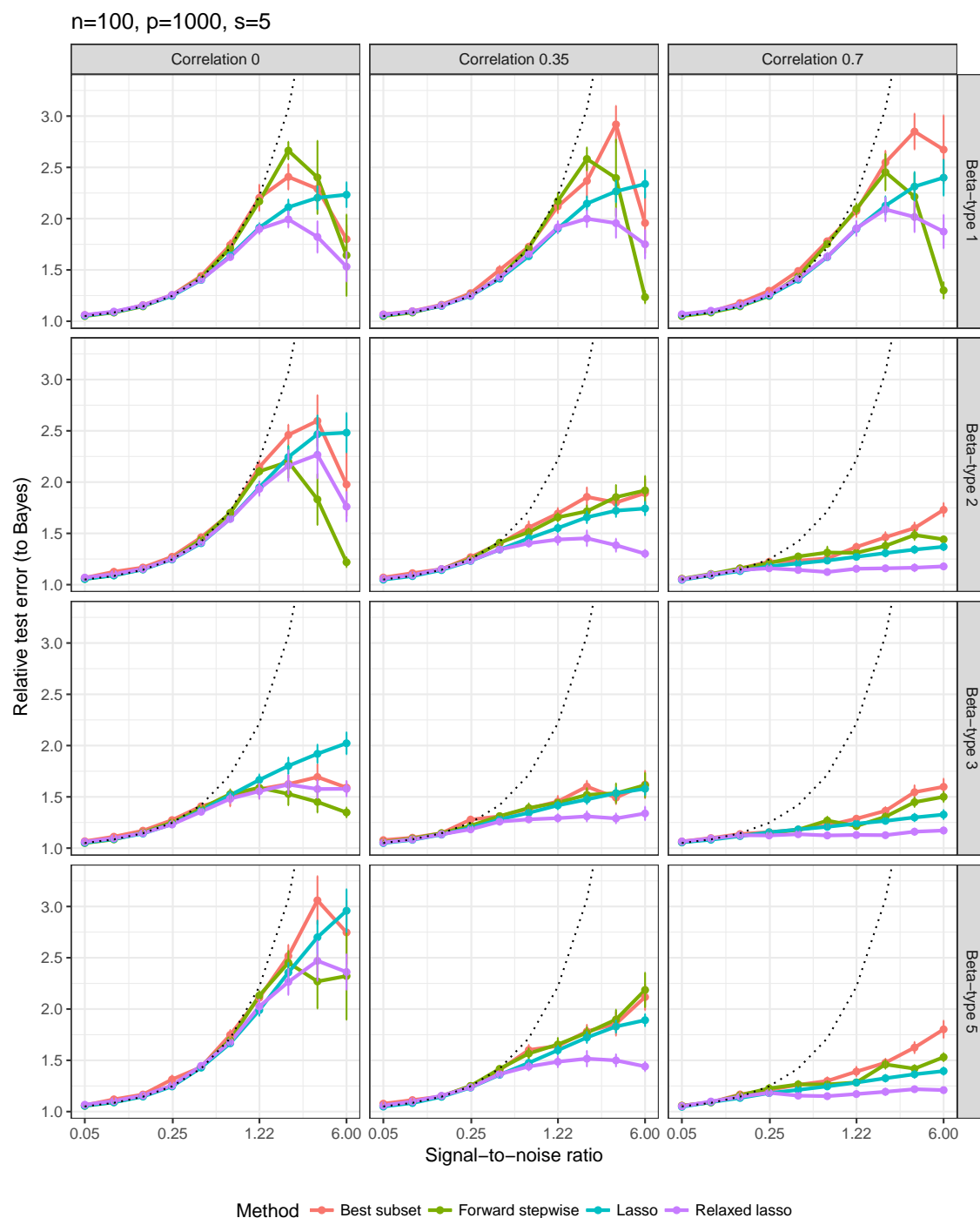


## A.4 High-10 setting: $n = 100, p = 1000, s = 10$

### A.4.1 Relative risk (to null model)

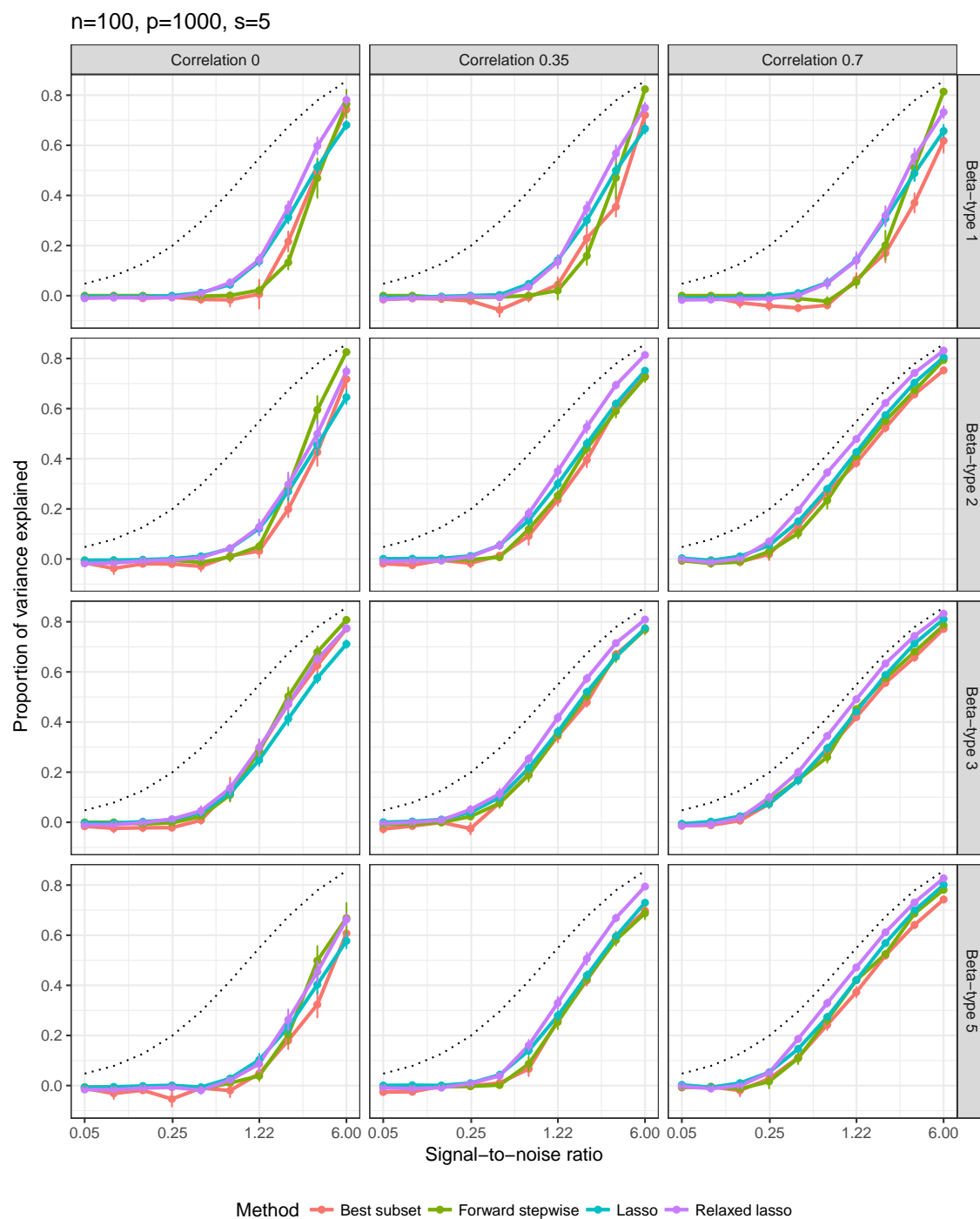


## A.4.2 Relative test error (to Bayes)

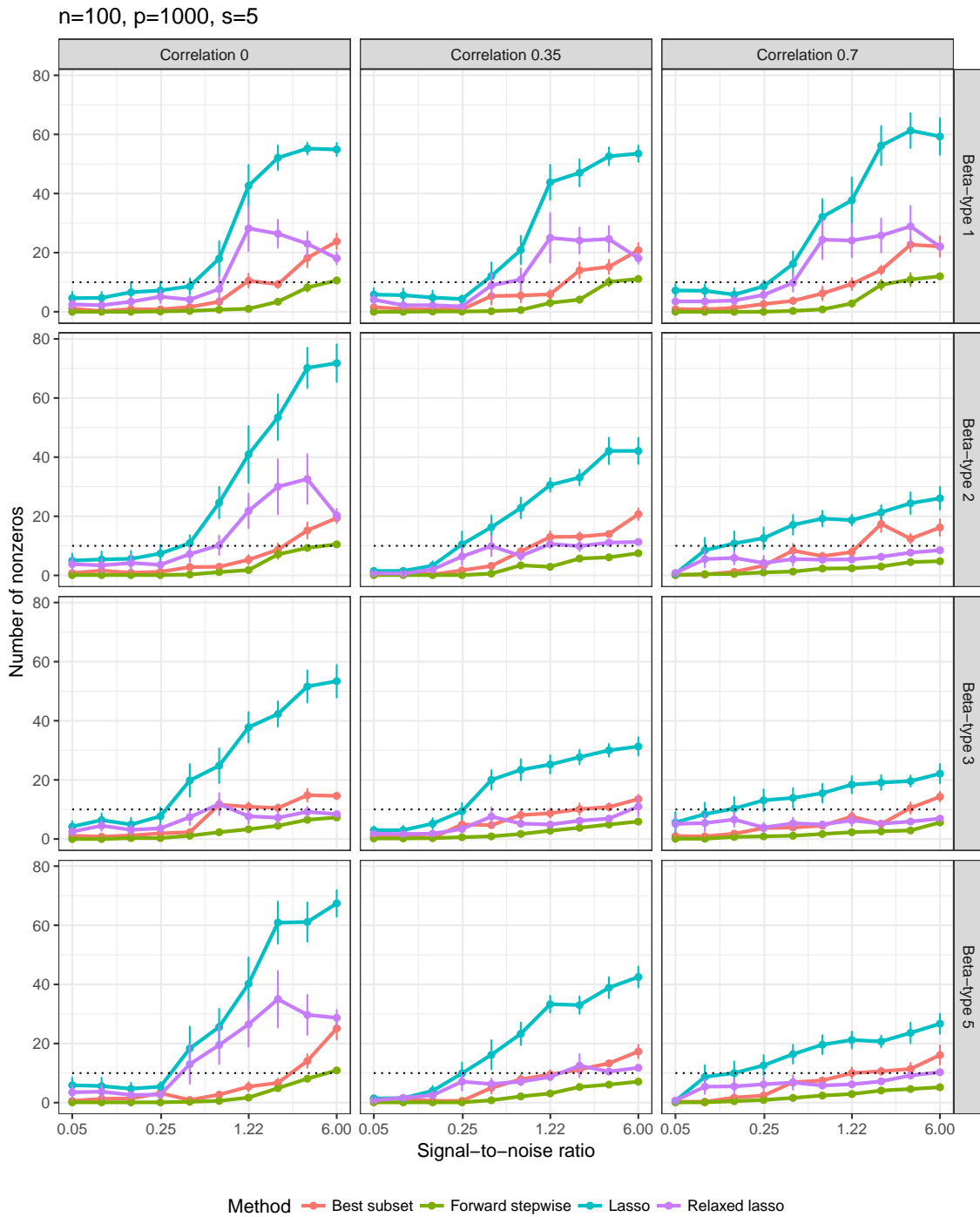




### A.4.3 Proportion of variance explained



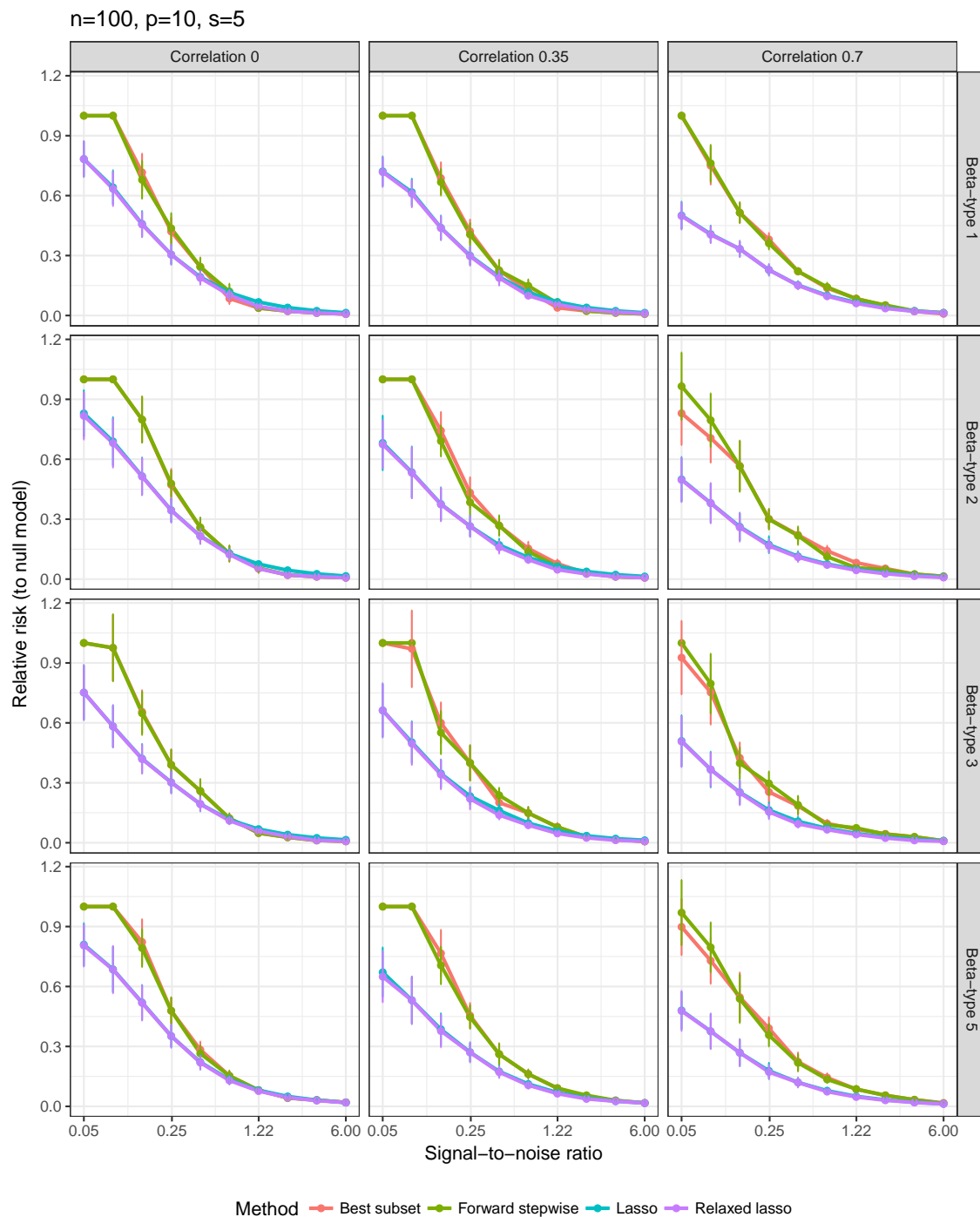
### A.4.4 Number of nonzero coefficients



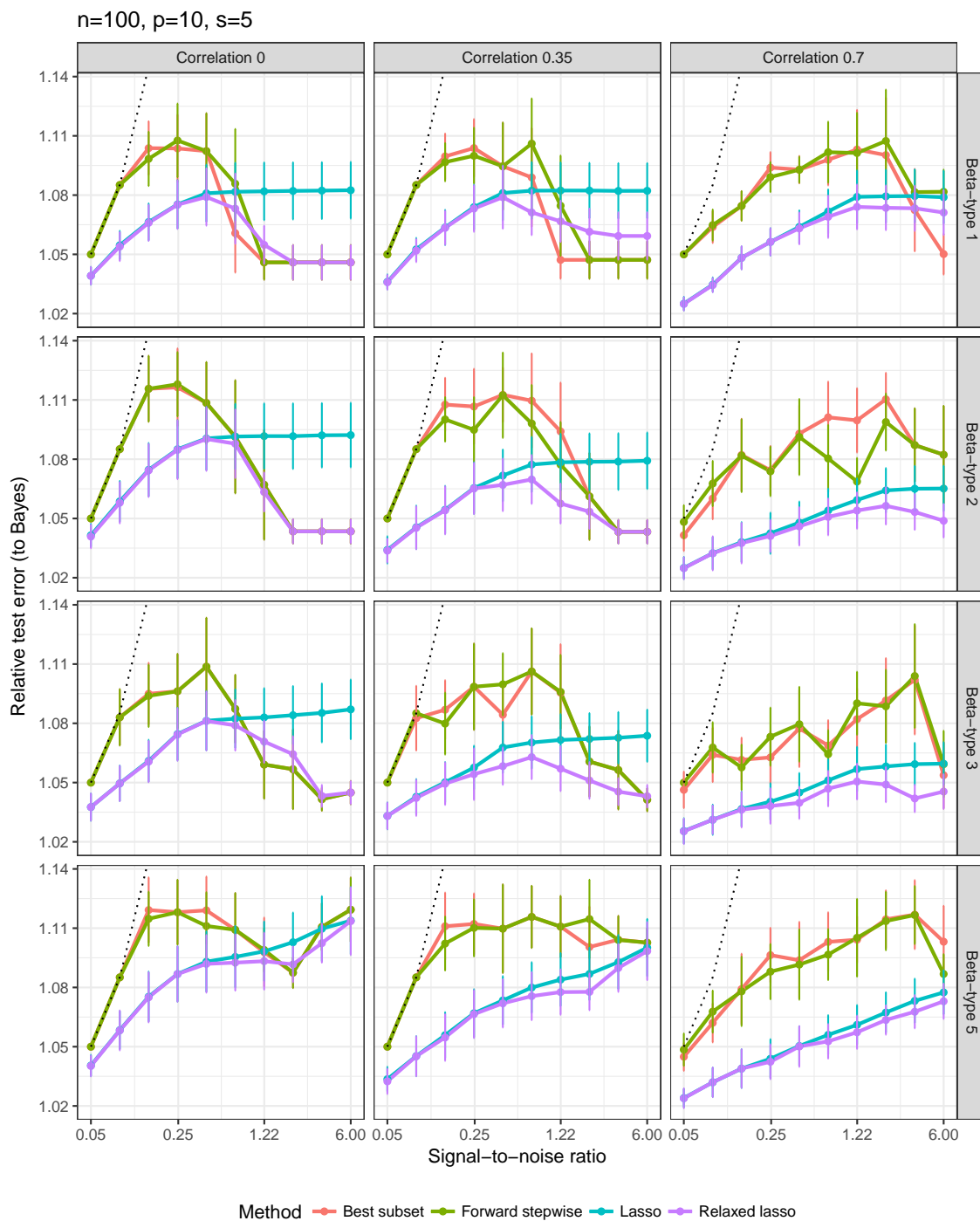
## B Oracle tuning

B.1 Low setting:  $n = 100, p = 10, s = 5$

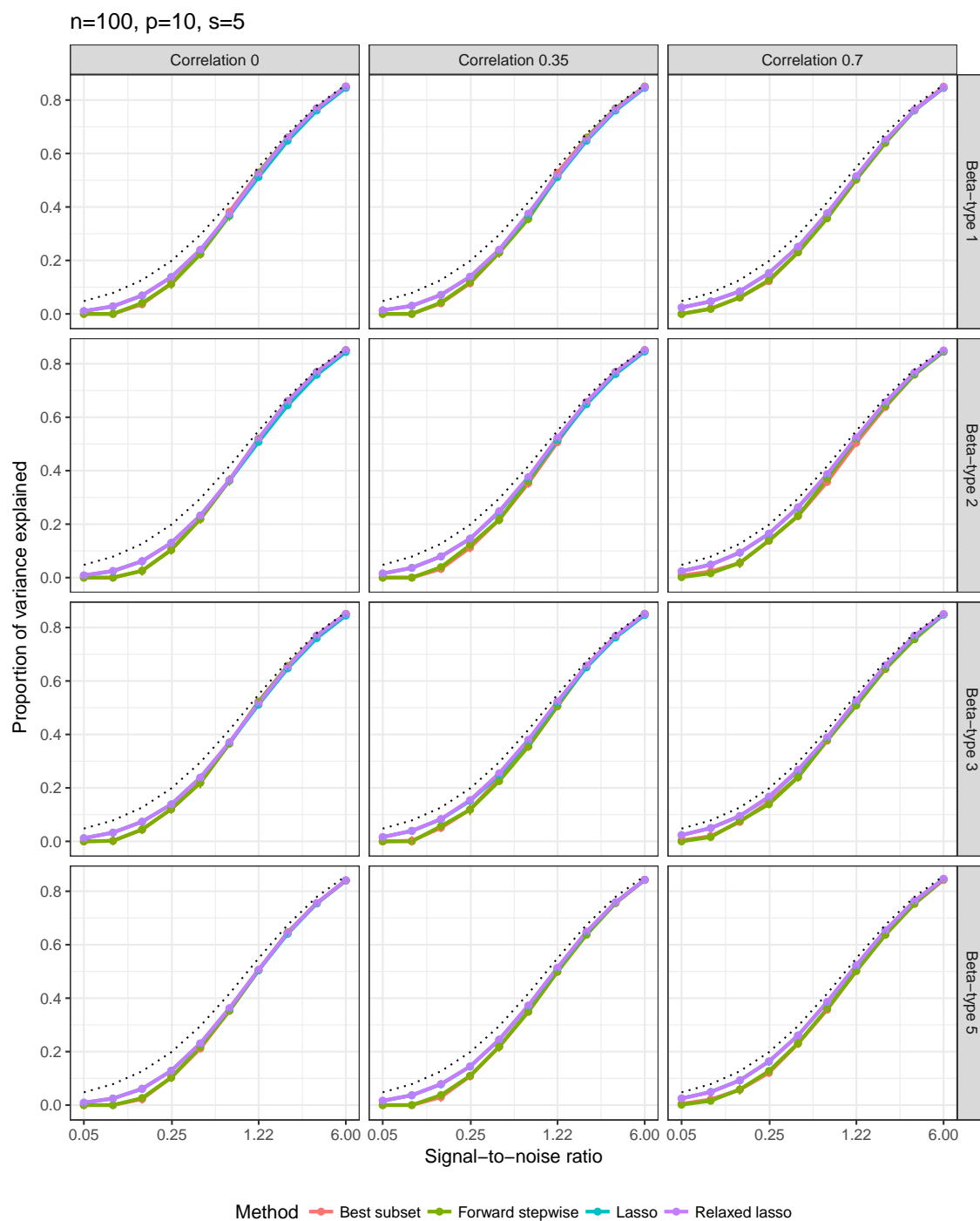
B.1.1 Relative risk (to null model)



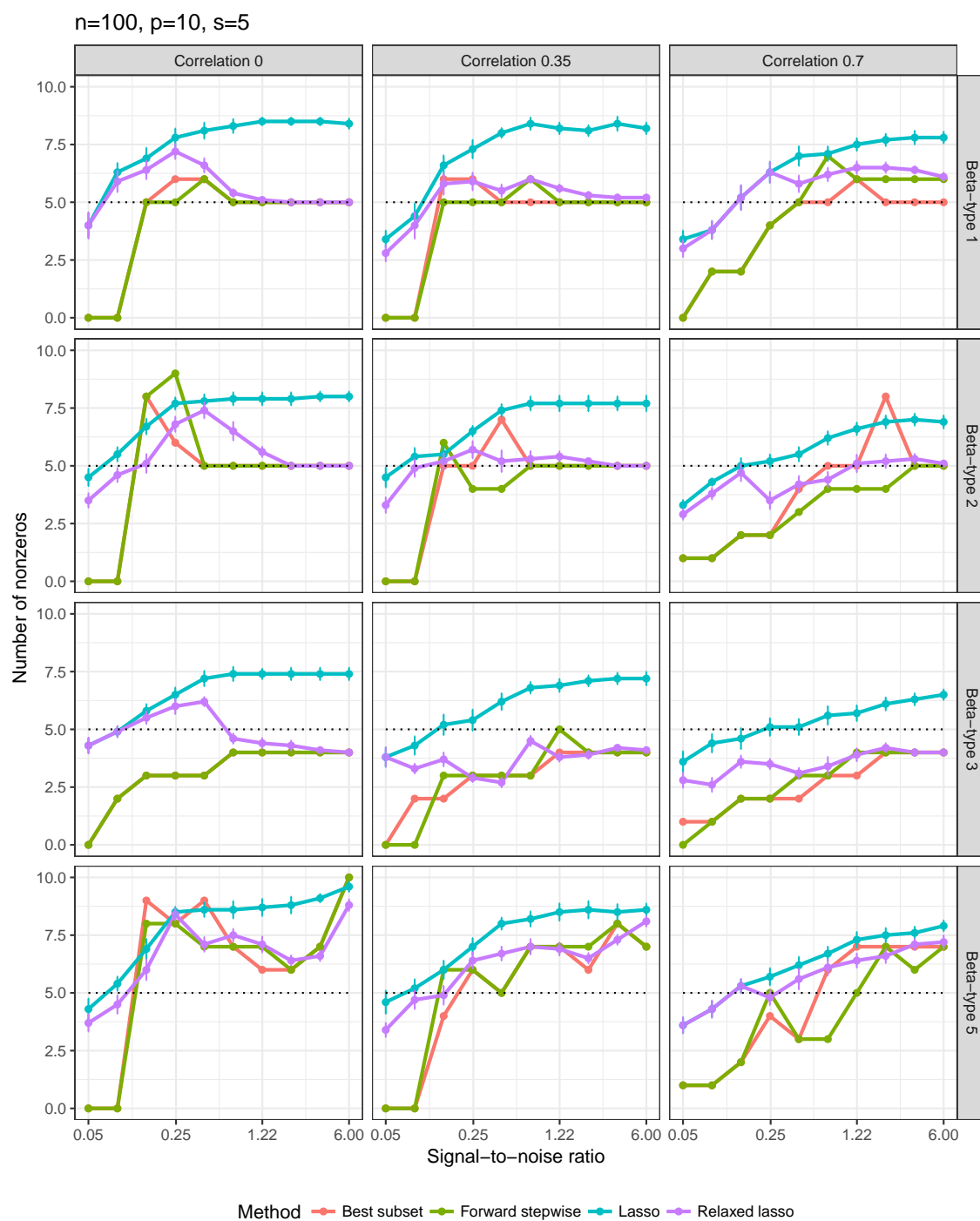
## B.1.2 Relative test error (to Bayes)



### B.1.3 Proportion of variance explained

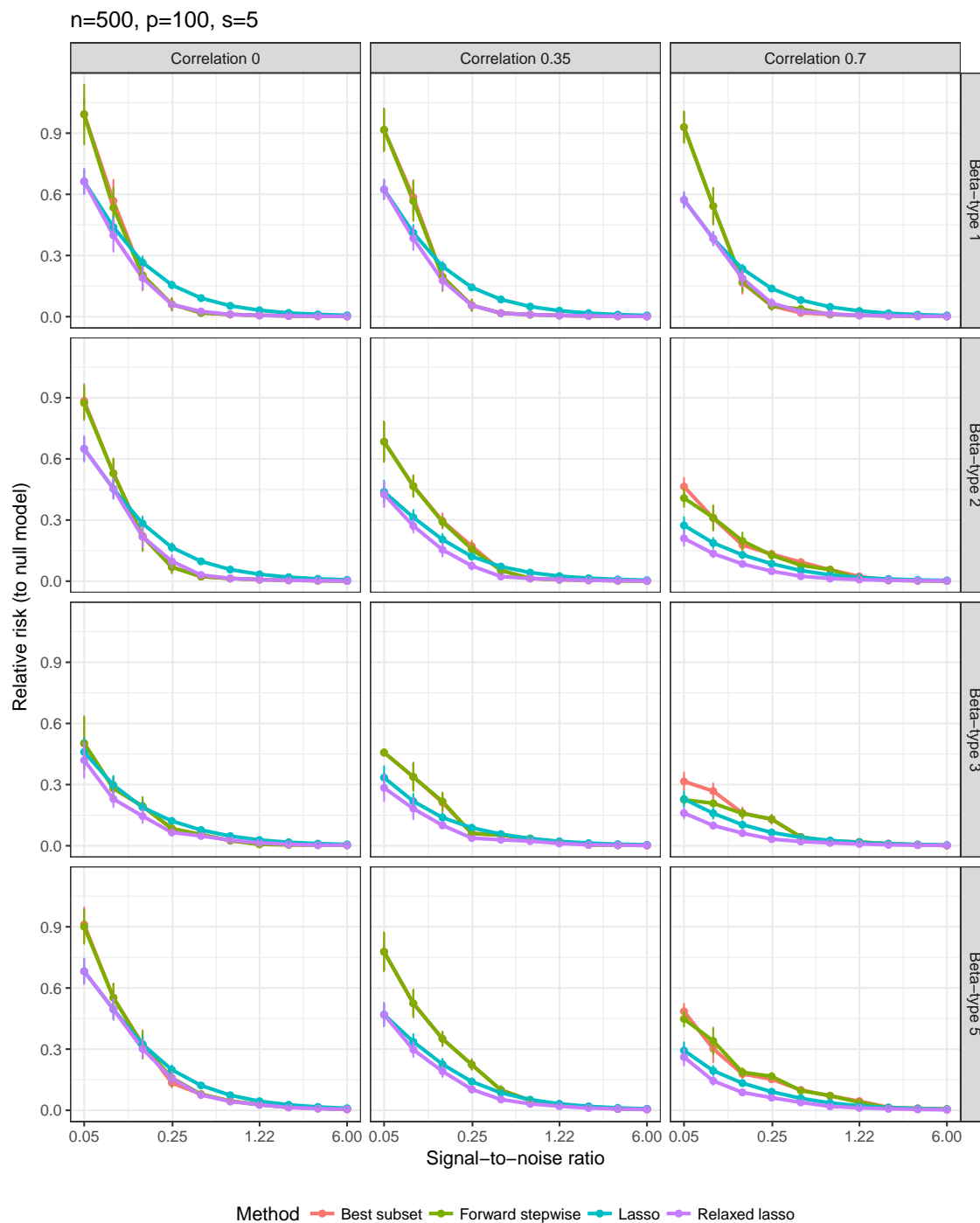


## B.1.4 Number of nonzero coefficients

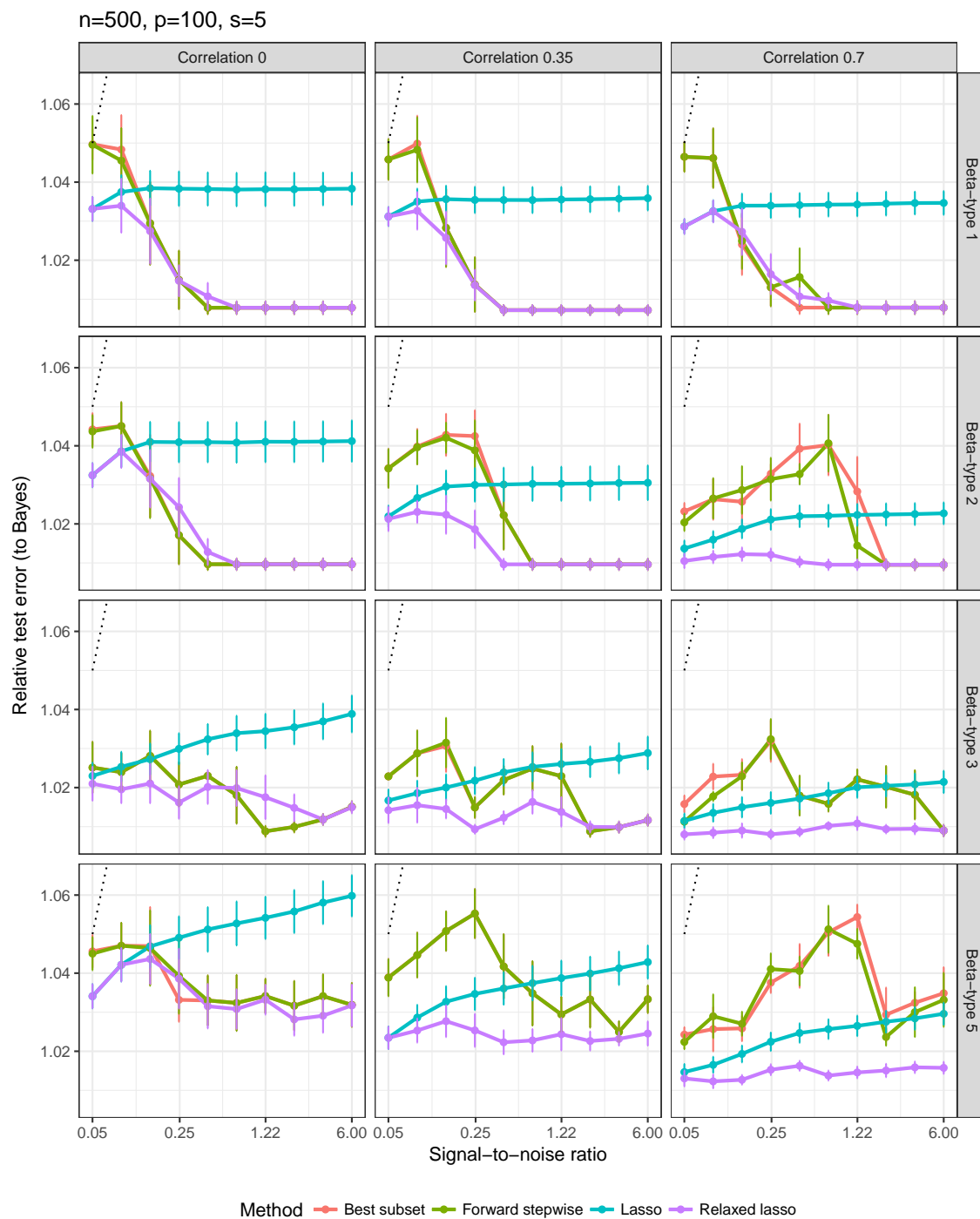


## B.2 Medium setting: $n = 500, p = 100, s = 5$

### B.2.1 Relative risk (to null model)

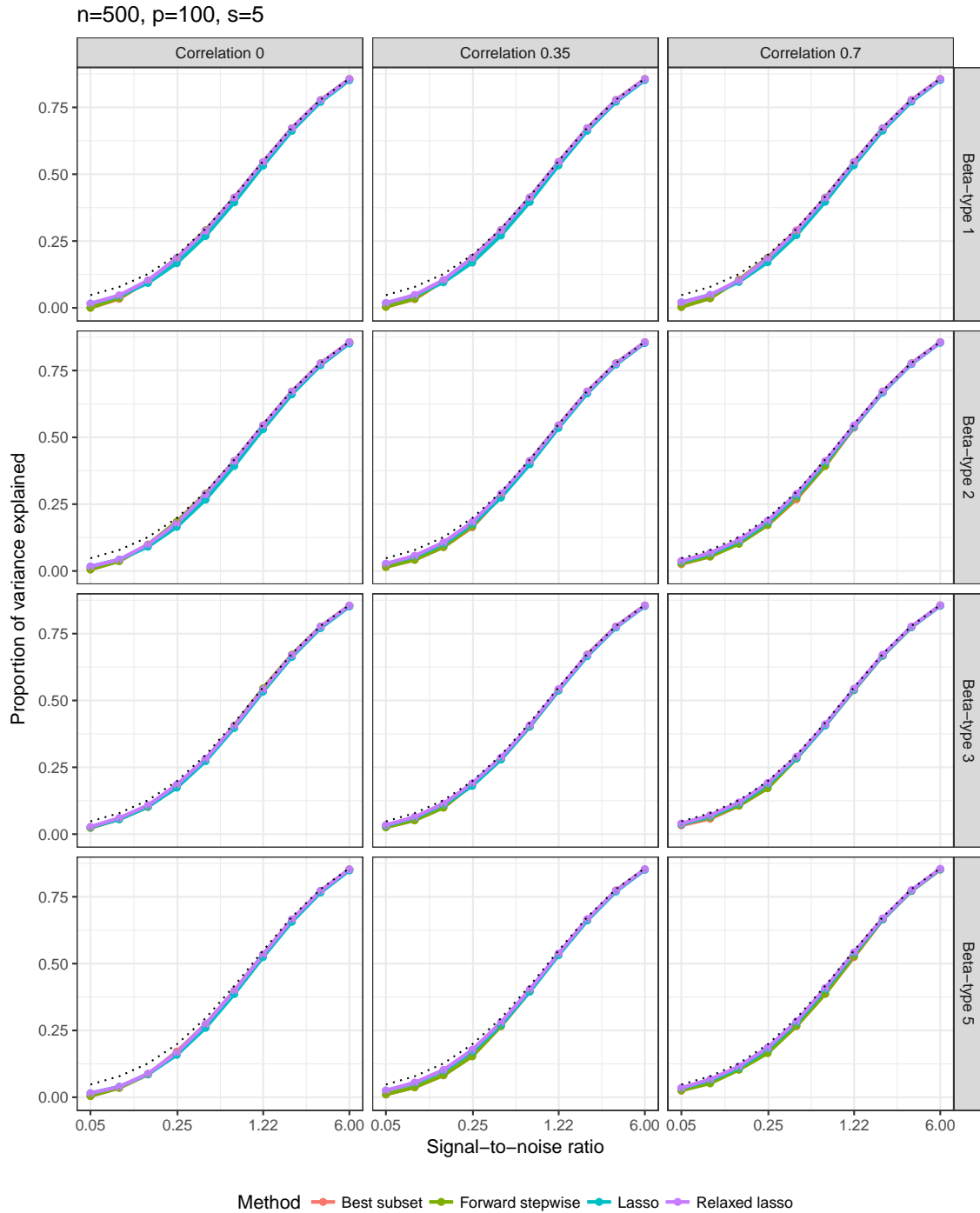


## B.2.2 Relative test error (to Bayes)

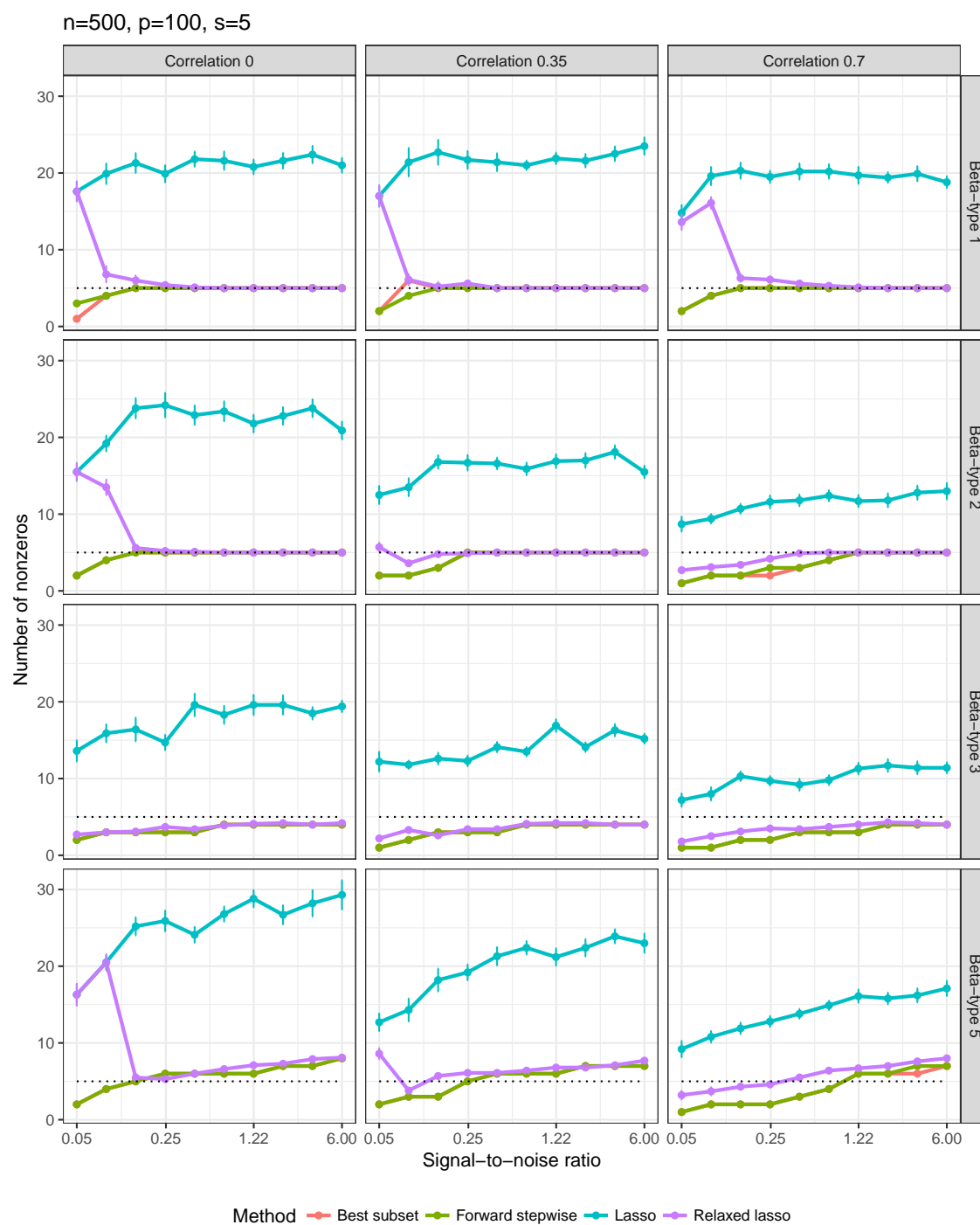




### B.2.3 Proportion of variance explained

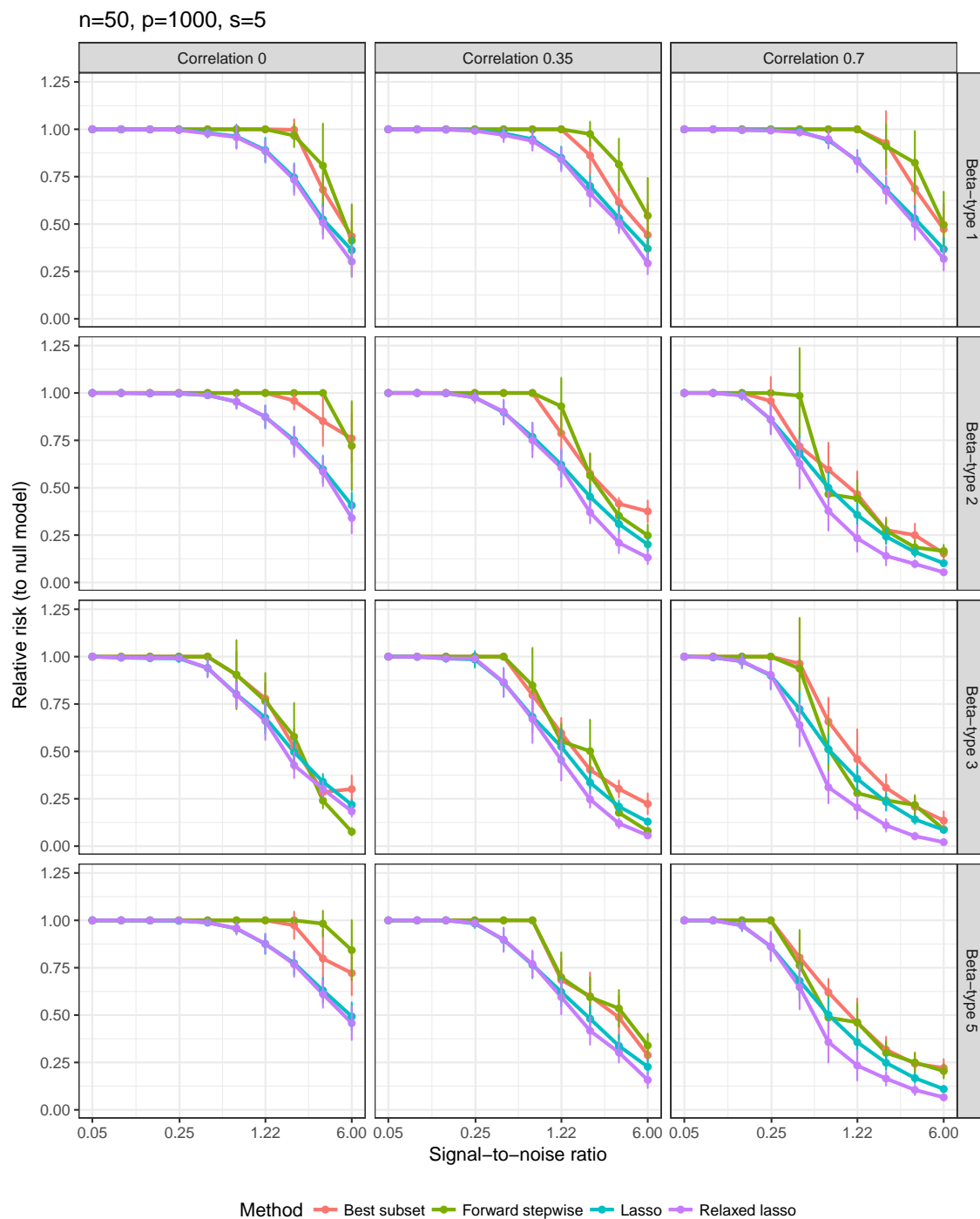


## B.2.4 Number of nonzero coefficients



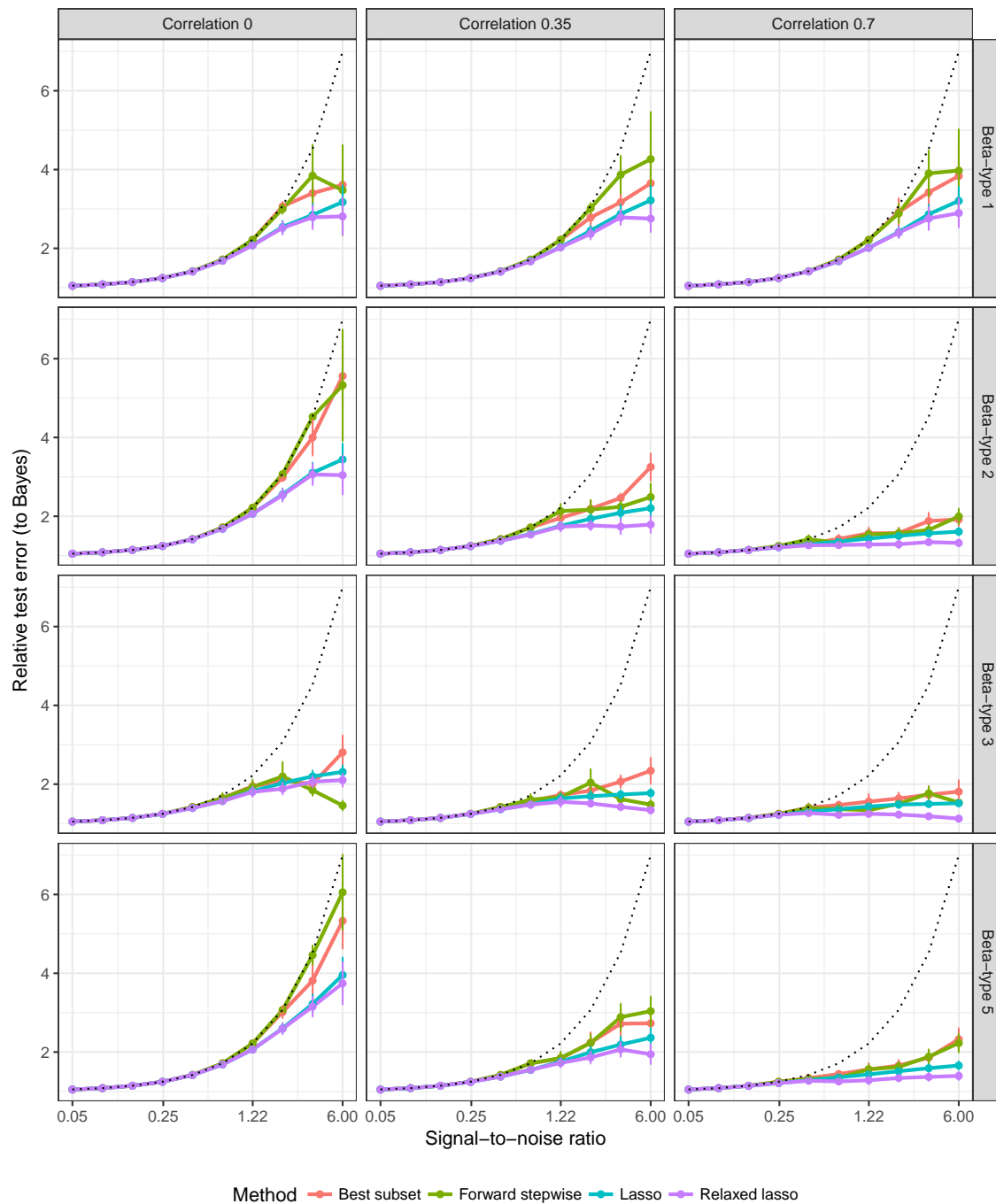
### B.3 High-5 setting: $n = 50, p = 1000, s = 5$

#### B.3.1 Relative risk (to null model)

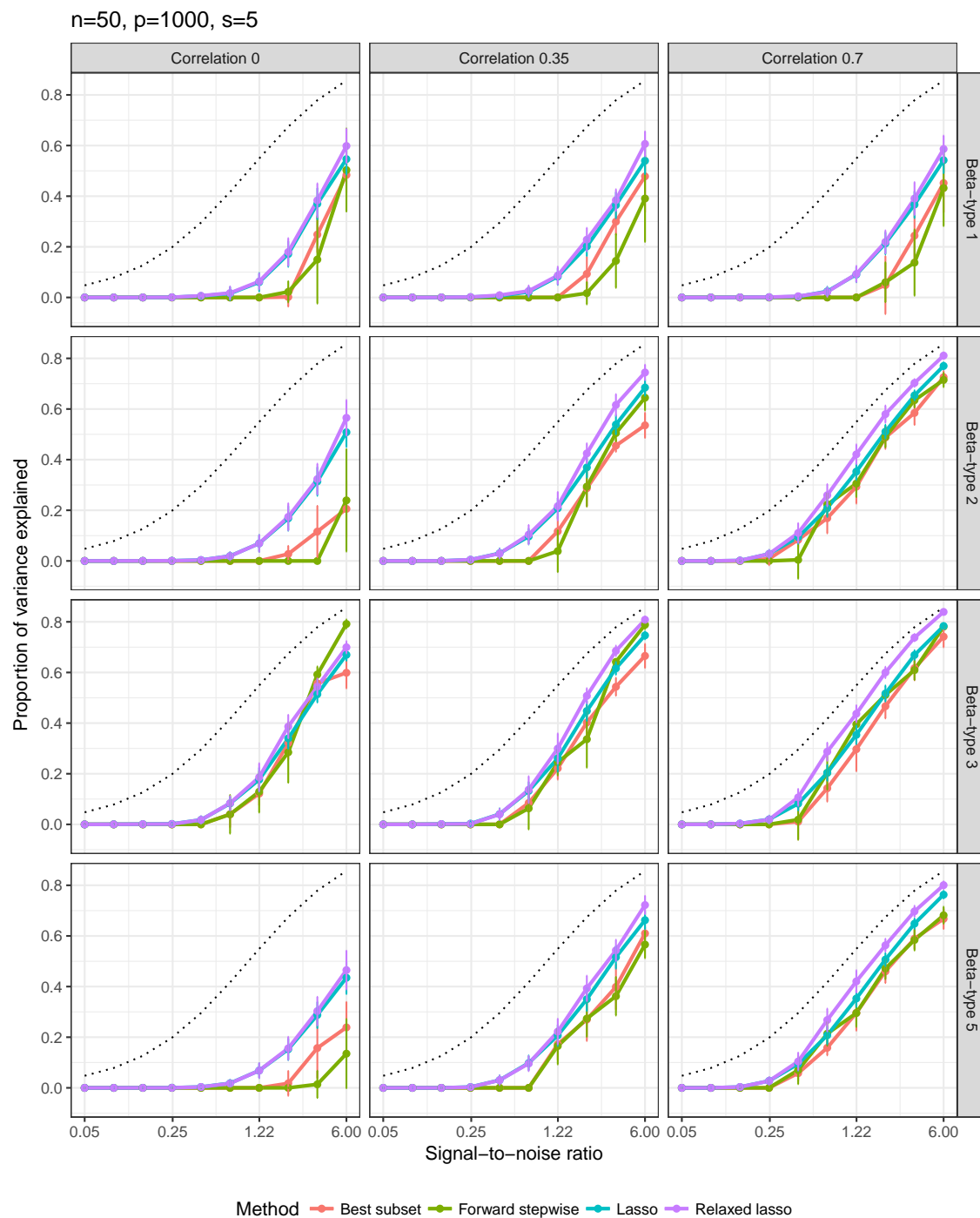


### B.3.2 Relative test error (to Bayes)

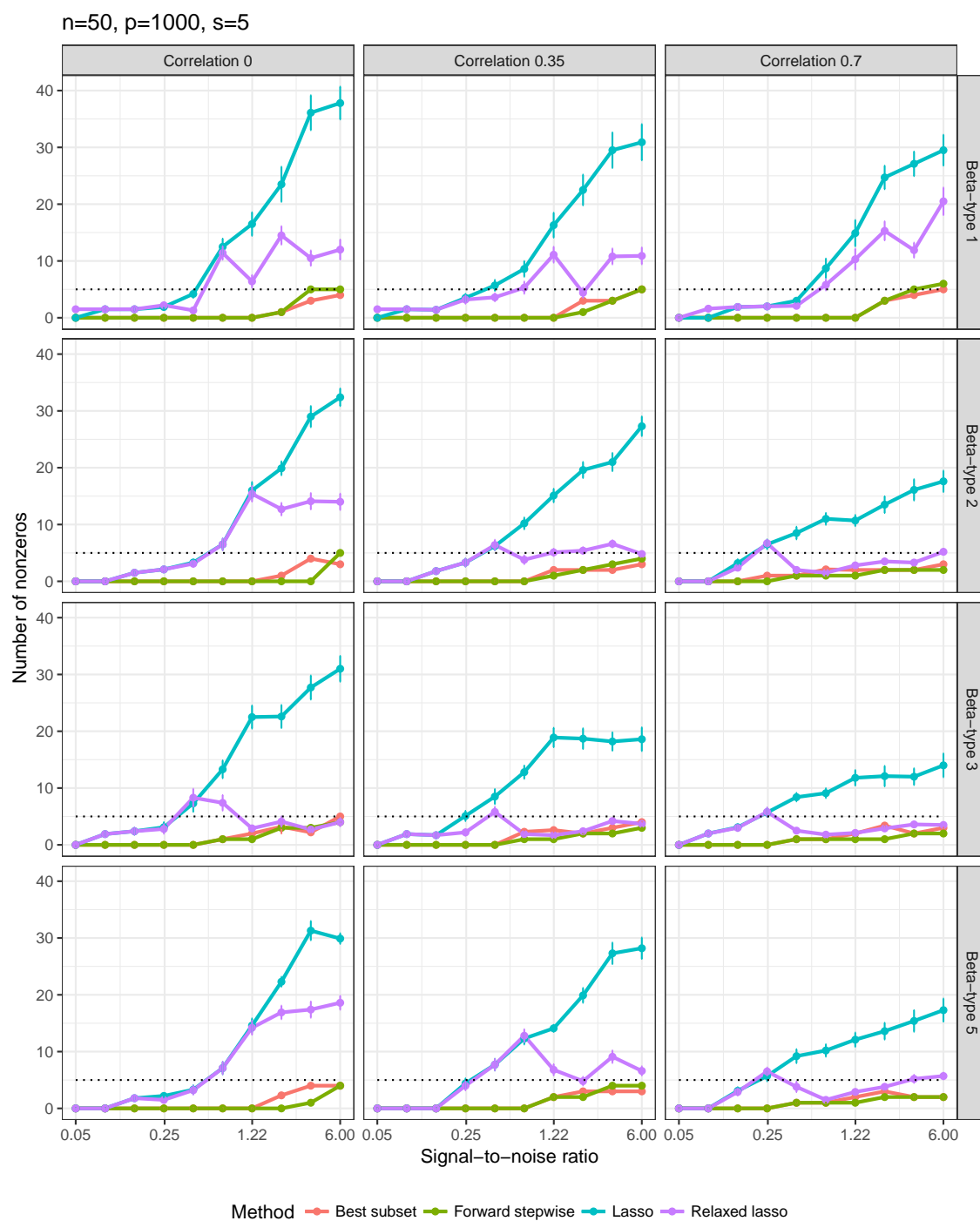
$n=50, p=1000, s=5$



### B.3.3 Proportion of variance explained

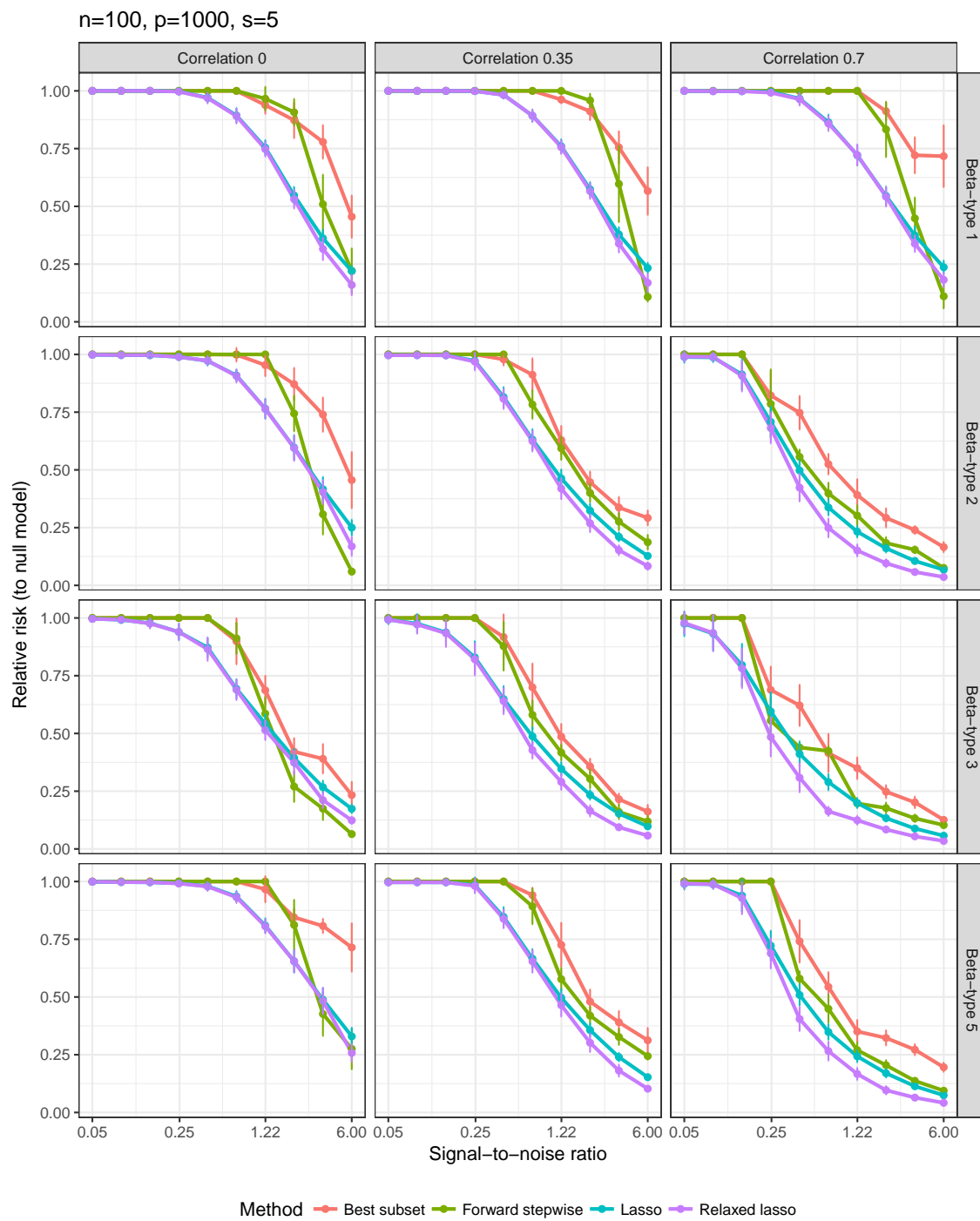


### B.3.4 Number of nonzero coefficients

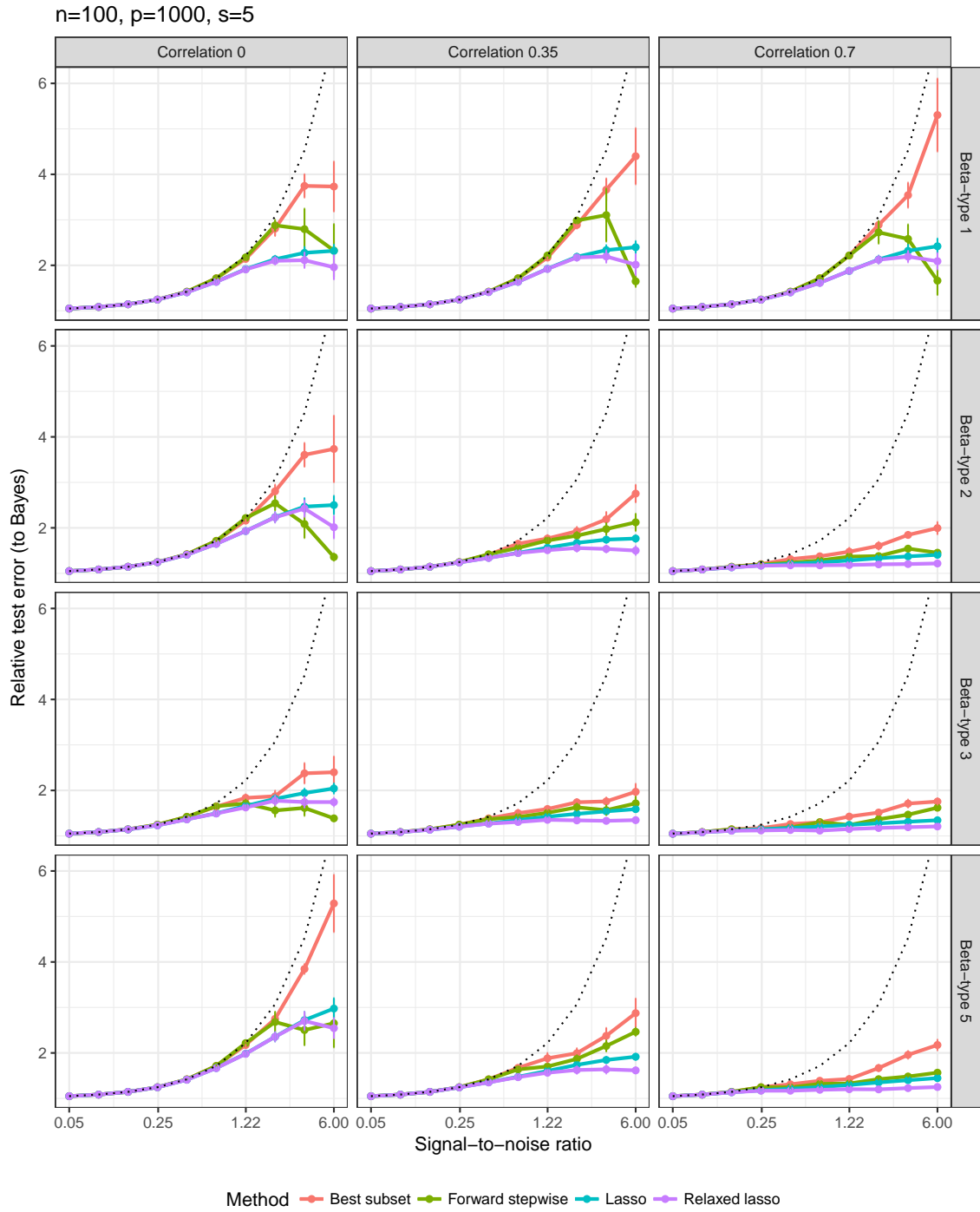


## B.4 High-10 setting: $n = 100, p = 1000, s = 10$

### B.4.1 Relative risk (to null model)

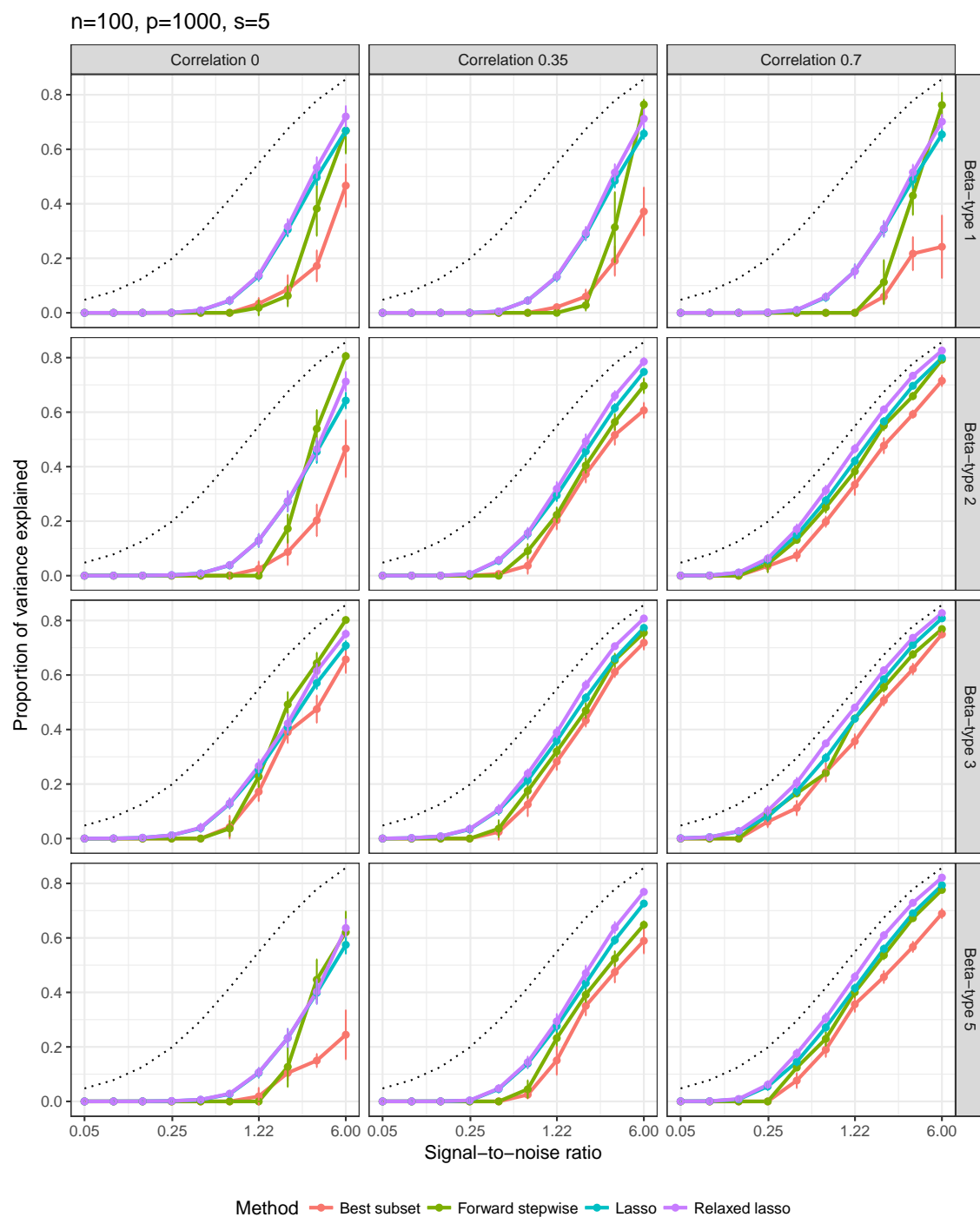


## B.4.2 Relative test error (to Bayes)





### B.4.3 Proportion of variance explained



## B.4.4 Number of nonzero coefficients

$n=100, p=1000, s=5$

