1. Consider random variables \(X, Y,\) and \(Z\). Assume the following facts:

\[
\begin{align*}
\mathbb{E}(X) &= 2 & \mathbb{E}(Y) &= 3 & \mathbb{E}(Z) &= 6 \\
\mathbb{E}(XY) &= 2 & \mathbb{E}(YZ) &= -3 & \mathbb{E}(ZX) &= 4 \\
\mathbb{E}(X^2) &= 7 & \mathbb{E}(Y^2) &= 10 & \mathbb{E}(Z^2) &= 32.
\end{align*}
\]

Define new random variables \(U, V,\) and \(W\) by:

\[
\begin{align*}
U &= X + Y + Z \\
V &= X + Y(1 + Z) \\
W &= (X + 2Y)(Y - 3Z) - X.
\end{align*}
\]

Calculate \(\mathbb{E}[U], \mathbb{E}[V]\) and \(\mathbb{E}[W]\).

2. Suppose that you have a random variable \(X\) which can take values in \(\{1, 2, \ldots, M\}\).

Show that you can write:

\[
X = \sum_{i=1}^{M} \mathbb{I}_{X \geq i}.
\]

Use this to relate the expected value \(\mathbb{E}[X]\) to tail probabilities of the form \(\mathbb{P}(X \geq i)\).

3. The unit interval \([0, 1]\) is divided into three line segments by cutting at two randomly chosen values \(X\) and \(Y\). Assume that \(X\) and \(Y\) are independent \(U[0, 1]\) random variables. Find the probability that the three line segments can be arranged to form a triangle. (You are allowed to move around the line segments freely in the plane.)

**Hint:** If the three line segments are to form a triangle, what is the longest that any side can be?

4. A miner is trapped in a mine with 4 doors. The first door leads to a tunnel that leads him out in 3 hours. The second door leads him back to the mine after 4 hours, the third leads him back after 2 hours and the last door leads him back after 1 hour. The miner picks the doors with probabilities \(\frac{1}{4}, \frac{1}{2}, \frac{1}{8}\) and \(\frac{1}{8}\) respectively. What is the expected length of time till the miner reaches safety?

**Hint:** Try conditioning on the first step.

5. Two players take turns shooting a target, player 1 hits the target with probability \(p\) on any turn and player 2 hits with probability \(q\). They stop shooting when the target is hit twice consecutively.

- Denote the expected number of shots taken in total when player 1 goes first by \(\mu_1\) and when player 2 goes first by \(\mu_2\). Calculate \(\mu_1\) and \(\mu_2\).
Suppose $h_1$ is the number of times the target is hit when player 1 goes first, and $h_2$ is the number of times the target is hit when player 2 goes first. Compute $h_1$ and $h_2$.

**Hint:** Again condition on the first step and use the law of total expectation. This time you might also need to condition on the second step to complete the calculation!