1. Suppose you have a Markov chain with transition matrix:

\[
P = \begin{bmatrix}
0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.2 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.8 & 0 & 0.2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0.2 \\
0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0.2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(a) Draw the state diagram, identify the classes, and their periods.

(b) Compute the matrix \( P^{(\infty)} \). You can check your work using MATLAB but you need to describe all the steps to compute \( P^{(\infty)} \) analytically (i.e. by hand) to earn points. You can use MATLAB to compute \((I - T)^{-1}\) if you need it.

2. Recall the matrix \( V \) which has entries:

\[ v_{ij} = \mathbb{E}[\text{# of visits to } j | X_0 = i]. \]

Compute all entries of the matrix \( V \). Again you can check your work using MATLAB but you need to write down all the steps to compute \( V \) analytically.

3. Three people, named A, B, and C, play the following game: In the first round, A plays B while C watches. The winner of that round plays in the second round against C while the loser of the first round watches. In the third round, the winner of the second round plays against the person who sat out the second round. And so on: In the \( n \)th round, the winner of round \( n - 1 \) plays against the person that sat out round \( n - 1 \). This continues until a player wins two consecutive rounds; that player is the winner of the game. Assume that rounds are independent, and if A plays B, A wins the round with probability 0.6; if B plays C, B wins the round with probability 0.5, and if A plays C, A wins the round with probability 0.45.

(a) What is the probability of winning for each person?

(b) What is the expected number of rounds before the game ends?