STAT 220: Sample Final Solutions


1) Investigators are performing a controlled double-blind study to examine the effects of a new iron supplement. They randomly selected a male treatment group, a male control group, a female treatment group, and a female control group. The treatment groups were given the iron supplement; the control groups were given a placebo. Then the red blood cell counts for all subjects were measured. For men and women respectively, the average red blood cell count for the treatment group was compared to the average red blood cell count for the control group.

Why did they study men and women separately? What problem does this study design avoid?

Splitting up women and men avoids the problem of gender confounding the results. Men and women have different red blood cell counts as well as different levels of iron. We want to try to prevent those differences from masking the possible difference associated with the iron supplement.

2) Calculate the mean, median, and mode of the following data:
   -3, 9, 10, -1, 10, 0, -1, -4, 2, -8, 7, 2, 0, 2, -2

First we sort the data: -8, -4, -3, -2, -1, -1, 0, 0, 2, 2, 7, 9, 10, 10.
We have 15 numbers total.
The mean: \[
\frac{-8+(-4)+(-3)+(-2)+(-1)+(-1)+0+0+2+2+2+7+9+10+10}{15} = 1.533
\]
The median: the middle of the 15 sorted numbers is in the 8th position: 0
The mode: the most common number is 2.

We add six to the original numbers. What is the new mean? The new median?

Adding six to all the numbers just picks them up and shifts them over 6.
The mean and median are also shifted by 6.
New mean = 7.533; new median = 6

We multiply the original numbers by \(\frac{1}{2}\). What is the new mode?

Multiplying the numbers by the same value doesn’t change the frequency of the unique values. The new mode is just \(2 \times \frac{1}{2} = 1\).
3) We surveyed a SRS of 500 UW students during finals week and asked them how many hours they studied per day and how many hours of sleep they got each night.
Let $x$ be the amount of daily study time (in hours). \( \text{avg } x = 6.4; \text{ SD}_x = 1.2 \)
Let $y$ be the amount of nightly sleep time (in hours). \( \text{avg } y = 6.5; \text{ SD}_y = 1.5 \)
The correlation $r$ between $x$ and $y$ is -0.6.
Both $x$ and $y$ are normally distributed. Their scatter diagram is football-shaped.

a) Find the probability that a student is studying fewer than 4 hours a day.
$P(X < 4)$?
We need to standardize:
\[
    z = \frac{4 - 6.4}{1.2} = -2
\]
We look up -2 in the table and get an area of 0.9545 between -2 and 2.
$P(X < 4) = P(Z < -2) = 0.50 - \frac{0.9545}{2} = 0.02275$

b) Find the regression line for predicting nightly sleep time from daily study time.
Our regression line equation is $y = mx + b$.
\[
m = \frac{r \cdot \text{SD}_y}{\text{SD}_x} = \frac{-0.6 \cdot 1.5}{1.2} = -0.75
\]
We use the point of averages to get the intercept.
\[
b = \bar{y} - m\bar{x} = 6.5 - (-0.75) \cdot 6.4 = 11.3
\]
Our line is: $y = 11.3 - 0.75x$

How much sleep do you predict that someone who studies 9 hours a day will get?
We plug $x = 9$ into our line. $y = 11.3 - 0.75 \cdot 9 = 4.55$.
4.55 hours a night.

c) For those who are studying 8 hours a day, what is the probability that they are sleeping more than 8 hours a night?
This is a normal curve in a vertical strip problem.
We want to examine the $y$-values for only those people who have $x = 8$.
They are distributed normally because our scatter diagram is football-shaped/homoscedastic.

We need their mean and SD. We use our regression line and our vertical strip SD formula.
\[
y^* = 11.3 - 0.75 \cdot 8 = 5.3
\]
\[
SD_{y^*} = \sqrt{1 - (-0.6)^2} \cdot 1.5 = 1.2
\]
Now we ask $P(Y > 8|X = 8)$?
We standardize:
\[
z = \frac{8 - 5.3}{1.2} = 2.25
\]
We look up 2.25 in the table and get an area of 0.9756 between -2.25 and 2.25
$P(Y > 8|X = 8) = P(Z > 2.25) = 0.50 - \frac{0.9756}{2} = 0.0122$
4)

You have two fair dice, one red and one black.
You roll the red die first, then the black die.
(Hint: write out the sample space)

We write out the pairs as (Red, Black)
\[(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)\]

What is the probability that exactly one of the dice is a three?

We can count the number of ways in the sample space:
\[(3,1) (3,2) (3,4) (3,5) (3,6) (1,3) (2,3) (4,3) (5,3) (6,3)\]

\[P(\text{exactly one of the rolls is 3}) = \frac{10}{36} = 0.278\]

We could also write: \(P(\text{exactly one roll is 3}) = P(\text{first roll is 3 AND second roll is not 3}) \lor P(\text{first roll is not 3 AND second roll is 3})\)

\[P(\text{exactly one roll is 3}) = \frac{1}{6} \cdot \frac{5}{6} + \frac{5}{6} \cdot \frac{1}{6} = \frac{10}{36} = 0.2778\]

What is the probability that at least one of the dice is a three?

So either one of the dice is a three (part a) or both dice are three’s. We have the first part. There’s only one way to get the second part: (3,3).

So \(P(\text{at least one of the dice is a three}) = P(\text{one of the dice is a three}) + P(\text{both dice are three’s}) = \frac{10}{36} + \frac{1}{6} - \frac{11}{36} = 0.3056\)

If we roll a three on the red die, what is the probability that the roll of the black die is less than the roll of the red die?

If we roll a three on the red die, our only possible rolls are: (3,1) (3,2) (3,3) (3,4) (3,5) (3,6).

On two of these rolls, the roll of the black die is less than the roll of the red die: (3,1) (3,2)

\[P(\text{roll of black die less than roll of red die } \mid \text{ roll a 3 on the red die}) = \frac{2}{6} = 0.333\]

What is the probability that the roll of the black die is less than the roll of the red die?

The previous part was a conditional probability. This one is not.

We can count up how many of our possible rolls satisfy this event.
\[(2,1) (3,1) (3,2) (4,1) (4,2) (4,3) (5,1) (5,2) (5,3) (5,4) (6,1) (6,2) (6,3) (6,4) (6,5)\]

\[P(\text{roll of black die is less than the roll of the red die}) = \frac{15}{36} = 0.417\]
5) The probability of a person wearing their seatbelt while driving is 0.8. After a car accident, a police officer will determine whether or not the driver was wearing his/her seatbelt. One cop has a particularly bad week and has to report to the scene of 10 car accidents.

What is the probability that exactly six drivers were wearing their seatbelts?

This is a binomial probability problem. You’re either wearing your seatbelt or you’re not. Wearing your seatbelt is independent from one car accident to the next.

Success = Wearing your seatbelt $P(S) = 0.8$

Failure = Not wearing your seatbelt $P(F) = 0.2$

$P(X = 6) = \binom{10}{6} 0.8^6 \cdot 0.2^4 = 0.088$

What is the probability that at least one driver was not wearing his/her seatbelt?

Now we’re looking at our “success” as not wearing our seatbelts.

Success = Not wearing your seatbelt $P(S) = 0.2$

Failure = Wearing your seatbelt $P(F) = 0.8$

$P(Y \geq 1) = 1 - P(Y = 0)$

$P(Y = 0) = \binom{10}{0} 0.2^0 \cdot 0.8^{10} = 0.107$

$P(Y \geq 1) = 1 - 0.107 = 0.893$

6) We are interested in determining the average number of children in a household in the state of Washington. In each county, we sample three cities. In each of those cities, we sample 1,000 households and ask them how many children live primarily in that household.

Is this a simple random sample? Why or why not? If not, what type of sample is this?

This is not a simple random sample. This is multi-stage cluster sampling. A SRS would be a random sample of households in the entire state. This sampling scheme is selecting random samples from multiple levels of selected areas.
7) At a carnival, there is a game with a large wheel with 5 sections. One section has a star on it; the other four are blank. You pay $2 to spin the wheel. If the wheel stops on the star section, you win $6 (i.e. you’re + 6). If it stops on any other section, you lose (i.e. you’re -2).

Set up a box model for the possible outcomes of each spin of the wheel.

\[ |6 - 2 - 2 - 2 - 2| \]

We make 20 draws with replacement from the box (i.e. spin the wheel 20 times).

Find the expected value and the SE for the average of the draws.

First we find the EV and the SD for the box.

\[ EV = \frac{6+(-2)+(-2)+(-2)+(-2)}{5} = -0.4 \]

\[ SD = \sqrt{\frac{(6-(-0.4))^2+4\cdot(-2-(-0.4))^2}{5}} = 3.2 \]

Now we find the EV and the SE for the average of 20 draws.

\[ EV = -0.4 \text{ (same as EV for box).} \]

\[ SE = \frac{SD}{\sqrt{n}} = \frac{3.2}{\sqrt{20}} = 0.716 \]

What is the chance that on average you make money? (i.e. the probability that the average is greater than zero)

The average of 20 draws follows a normal distribution with a mean of -0.4 and a SE of 0.716. We want P(avg > 0). First we standardize:

\[ z = \frac{0 - (-0.4)}{0.716} = 0.559 \]

We look up 0.559 in the table and get an area of about 0.42 between -0.559 and 0.559.

\[ P(\text{avg} > 0) = P(Z > 0.559) = 0.50 - 0.42 = 0.29 \]

About how many negative draws do we expect to see? (i.e. the number of times we lose)

We are now only interested in whether or not a draw is negative. We need to change our box.

\[ |0 1 1 1 1| \]

We can count the number of negative numbers we expect by finding the EV of the sum of 20 draws from our 0 - 1 box.

\[ \text{EV of the new box} = \frac{4}{5} = 0.8. \]

\[ \text{EV for the sum of 20 draws} = 20 \cdot 0.8 = 16. \]
8) A national children’s advocacy group would like to test whether or not consistently eating breakfast in the morning is associated with higher academic performance. In one school district, each child is labeled by whether or not they are breakfast-eaters. They randomly sample 300 breakfast-eaters and 200 non-breakfast-eaters. All sampled children take the same scholastic aptitude test. The breakfast-eaters average a score of 67 with a SD of 12; the non-breakfast-eaters average a score of 63 with a SD of 8. What does the advocacy group conclude?

We're comparing the average scores of two sampled groups.

Group 1: Breakfast-eaters

- \( n_1 = 300 \)
- \( \text{Avg } 1 = 67 \)
- \( SE_1 = \frac{12}{\sqrt{300}} = 0.693 \)

Group 2: Non-Breakfast-eaters

- \( n_2 = 200 \)
- \( \text{Avg } 2 = 63 \)
- \( SE_2 = \frac{8}{\sqrt{200}} = 0.566 \)

\( H_0: \) Avg 1 = Avg 2 \Rightarrow Avg 1 - Avg 2 = 0  \\
\( H_a: \) Avg 1 > Avg 2 \Rightarrow Avg 1 - Avg 2 > 0

Our two-group z test statistic:

\[
z = \frac{67 - 63}{\sqrt{0.693^2 + 0.566^2}} = \frac{4}{0.895} = 4.469
\]

We look up 4.469 and see that \( P(Z \geq 4.469) \approx 0 \). Our p-value for our test statistic is almost zero. We definitely reject the null hypothesis. We have evidence that breakfast-eaters score higher on average on the scholastic aptitude test.
9) (Hypothetical.) A high school’s grading policy is based on a curve. The final numerical grades in each class are sorted from highest to lowest. The top 20% are given A’s, the next 25% are given B’s, the next 30% are given C’s, the next 15% are given D’s, and the remaining 10% are given F’s.

A new teacher gives the following grades in her English class: 10 A’s, 8 B’s, 12 C’s, 3 D’s, and 2 F’s. Is her grade distribution significantly different from the high school’s policy?

This is a chi-square test. We have a label for each student.

$H_0$: the grades follow the high school policy’s distribution

$H_a$: the grades follow a different distribution

What we observe:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8</td>
<td>12</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

The class has 35 students total.

What we expect:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 · 0.20 = 7</td>
<td>35 · 0.25 = 8.75</td>
<td>35 · 0.30 = 10.5</td>
<td>35 · 0.15 = 5.25</td>
<td>35 · 0.10 = 3.5</td>
<td></td>
</tr>
</tbody>
</table>

Our chi-square test statistic:

$$
\chi^2 = \frac{(10 - 7)^2}{7} + \frac{(8 - 8.75)^2}{8.75} + \frac{(12 - 10.5)^2}{10.5} + \frac{(3 - 5.25)^2}{5.25} + \frac{(2 - 3.5)^2}{3.5} = 3.171
$$

We have $5 - 1 = 4$ degrees of freedom. We look along the 4th row of the chi-square table and see that our $p$-value is between 0.70 and 0.50. We do not reject the null hypothesis. We do not have evidence that the grades differ from the high school’s policy.

Sidenote: you would probably prefer for the sampled students to be randomly sampled from all of the new teacher’s classes rather than from just one class - worry about clustering effect.