CHAPTER 7
Continuous Probability Distributions

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Chapter 7 - Learning Objectives

• Differentiate between the normal and the exponential distributions.
• Use the standard normal distribution and z-scores to determine probabilities associated with the normal distribution.
• Use the normal distribution to approximate the binomial distribution.
• Use the exponential distribution to determine related probabilities.

Chapter 7 - Key Terms

• Probability density function
• Probability distributions
  - Standard normal distribution
    » Mean, variance, applications
  - Exponential distribution
    » Mean, variance, applications
• Normal approximation to the binomial distribution
Chapter 7 - Key Concept

• The area under a probability density function between two bounds, a and b, is the probability that a value will occur within the bounded interval between a and b.

The Normal Distribution

• An important family of continuous distributions
• Bell-shaped, symmetric, and asymptotic
• To specify a particular distribution in this family, two parameters must be given:
  – Mean
  – Standard deviation

Areas under the Normal Curve

Use the standard normal table to find:
• The z-score such that the area from the midpoint to z is 0.20. In the interior of the standard normal table, look up a value close to 0.20. The closest value is 0.1985, which occurs at
  \[ z = 0.52 \]
Areas under the Normal Curve

Use the standard normal table to find:
- The probability associated with $z$: $P(0 \leq z \leq 1.32)$.

Locate the row whose header is 1.3. Proceed along that row to the column whose header is .02. There you find the value .4066, which is the amount of area capture between the mean and a $z$ of 1.32.

Answer: 0.4066

Areas under the Normal Curve

Use the standard normal table to find:
- The probability associated with $z$: $P(-1.10 \leq z \leq 1.32)$.

Find the amount of area between the mean and $z = 1.32$ and add it to the amount of area between the mean and $z = 1.10^*$. 

$0.3643 + 0.4066 = 0.7709$

Areas under the Normal Curve - Dealing with Negative Z’s

- Note - Because the normal curve is symmetric, the amount of area between the mean and $z = -1.10$ is the same as the amount of area between the mean and $z = +1.10$. 
Areas under the Normal Curve

Use the standard normal table to find:

- The probability associated with $z = P(1.00 \leq z \leq 1.32)$.

Find the amount of area between the mean and $z = 1.00$ and subtract it from the amount of area between the mean and $z = 1.32$.

\[
0.4066 - 0.3413 = 0.0653
\]

Standardizing Individual Data Values on a Normal Curve

- The **standardized $z$-score** is how far above or below the individual value is compared to the population mean in units of standard deviation.
  
  - “How far above or below” = data value – mean
  - “In units of standard deviation” = divide by $\sigma$

- Standardized individual value

\[
z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}
\]

Standard Normal Distribution: An Example

- It has been reported that the average hotel check-in time, from curbside to delivery of bags into the room, is 12.1 minutes. Mary has just left the cab that brought her to her hotel. Assuming a normal distribution with a standard deviation of 2.0 minutes, what is the probability that the time required for Mary and her bags to get to the room will be:
  
  a) greater than 14.1 minutes?
  
  b) less than 8.1 minutes?
  
  c) between 10.1 and 14.1 minutes?
  
  d) between 10.1 and 16.1 minutes?
An Example, cont.

Given in the problem:
\[ \mu = 12.1 \text{ minutes}, \ \sigma = 2.0 \text{ minutes} \]

- a) Greater than 14.1 minutes
\[ P(x > 14.1) = P(z > 1.00) \]
\[ = 0.5 - 0.3413 = 0.1587 \]

\[ z = \frac{x - \mu}{\sigma} = \frac{14.1 - 12.1}{2.0} = 1.00 \]

- b) Less than 8.1 minutes
\[ P(x < 8.1) = P(z < -2.00) \]
\[ = 0.5 - 0.4772 = 0.0228 \]

\[ z = \frac{x - \mu}{\sigma} = \frac{8.1 - 12.1}{2.0} = -2.00 \]

- c) Between 10.1 and 14.1 minutes
\[ P(10.1 < x < 14.1) \]
\[ = P(-1.00 < z < 1.00) \]
\[ = 0.3413 + 0.3413 = 0.6826 \]

\[ z_{\text{lower}} = \frac{x - \mu}{\sigma} = \frac{10.1 - 12.1}{2.0} = -1.00 \]
\[ z_{\text{upper}} = \frac{x - \mu}{\sigma} = \frac{14.1 - 12.1}{2.0} = 1.00 \]
An Example, cont.
Given in the problem:
\( \mu = 12.1 \) minutes, \( \sigma = 2.0 \) minutes

- d) Between 10.1 and 16.1 minutes

\[
\begin{align*}
  z_{\text{lower}} &= \frac{x - \mu}{\sigma} = \frac{10.1 - 12.1}{2.0} = -1.00 \\
  z_{\text{upper}} &= \frac{x - \mu}{\sigma} = \frac{16.1 - 12.1}{2.0} = 2.00 \\
\end{align*}
\]

\[ P(10.1 < x < 16.1) = P(-1.00 < z < 2.00) = 0.3413 + 0.4772 = 0.8185 \]

Example: Using Minitab

- Problem: What is the probability that the time required for Mary and her bags to get to the room will be:
  a) greater than 14.1 minutes?
  In Minitab’s session window, type
  MTB > cdf 14.1 k1;
  SUBC> norm 12.1 2.
  MTB > let k2 = 1 - k1
  MTB > print k2
  and you will see the answer: 0.1587

  b) less than 8.1 minutes?

  MTB > cdf 8.1;
  SUBC> norm 12.1 2.
  with answer: 0.0228

Example: Using Minitab

- Problem: What is the probability that the time required for Mary and her bags to get to the room will be:
  c) between 10.1 and 14.1 minutes?

  MTB > cdf 14.1 k1;
  SUBC> norm 12.1 2.
  MTB > cdf 10.1 k2;
  SUBC> norm 12.1 2.
  MTB > let k3 = k1 - k2
  MTB > print k3
  with answer: 0.6826

  d) between 10.1 and 16.1 minutes?

  Similar to c), try it and you will see the answer: 0.8185

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The Exponential Distribution

\[ f(x) = \lambda e^{-\lambda x} \quad \text{where } \lambda = \text{mean and standard deviation} \]
\[ e = 2.71828, \text{ a constant} \]

- Probability: \[ P(x \geq k) = e^{-\lambda k} \]
- Application: Every day, drivers arrive at a tollbooth. If the Poisson distribution were applied to this process, what would be an appropriate random variable? What would be the exponential distribution counterpart to this random variable?