# 36-220 Lab #9
## Hypothesis Testing

Please write your name below, tear off this front page and give it to a teaching assistant as you leave the lab. It will be a record of your participation in the lab. Please remember to include whether you are in Section A or B. Keep the rest of your lab write-up as a reference for doing homework and studying for exams.

## Name:

## Section:

- The symbol ♣ at the beginning of a question means that, after you answer that question, you should raise your hand and have either the TA or lab assistant review your answer. Once they have reviewed your work they will place a check in the appropriate space in the table below. The purpose of this check is to be sure you have answered the question correctly.
- **You should try to complete as much of the lab exercise as possible. We understand that students work at different paces and have tried to structure the exercise so that it can be completed in the allotted time. If you work systematically through the handout and still don't complete every question don't worry. The important thing is that you understand what you are doing. Nonetheless, you are encouraged to complete the lab on your own.**

| **Check-Problem ♣** | Instructor's Initials |
|---|---|
| Question 8 | |
| Question 11 | |
| Question 14 | |

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# 1 Transformationing Data So It Looks Normal

Go to the course website and download "width.MTW" to your desktop. This records with width (in microns) of metal wires produced during a particular chip-manufacturing process.

1. Select **File** > **Open Worksheet**. Select "Desktop" in the "Look In:" pull-down menu. Highlight "width.MTW". Click **Open**.
2. Forty random observations of wire thickness are in column C1.
3. Make a histogram and Normal probability plot of "Width." To make the probability plot, Select **Graph** → **Probability Plot**. In the variables field, select "Width". Click **Ok**.

**Question #1: Describe the histogram and probability plot. Do the data look like a sample from a Normal distribution?**

**Question #2: How well is the manufacturing process meeting its target of 2981 microns? Comment on over/under thickness of wires.**

**Question #3: Estimate the population mean wire thickness of the chip-manufacturing process.**

4. It's easiest to test hypotheses regarding the mean and variance when we can assume the data are normal. Here we are going to try to *transform* the original measurements of wire thickness so that the transformed data will look normal. Try the following two transformation:

   (a) Take square roots of the data ($Y_i = \sqrt{X_i}$). Put these in column C2 and name the column SQRT_WIDTH; To do this, select **Calc** → **Calculator**. Under "Store result in variable" type "C2". Under "Expression", type "sqrt('Width')". Click **Ok**.

   (b) Take the logarithm of the data ($Z_i = \ln(X_i)$). Note that ln is the "natural log", or log-base $e$, where $e = 2.71828\ldots$. Put these in column C3 and name the column LOG_WIDTH. To do this, select **Calc** → **Calculator**. Under "Store result in variable" type "C3". Under "Expression", type "loge('Width')". Click **Ok**.

   Create a histogram and probability plot for each of the two sets of transformed data.

   **Question #4: Which of the two transformations would you select if you wanted to assume normality? Why?**

5. **From this point on, use only the natural log transformed data.** Denote the transformed data by $Z_1, Z_2, ..., Z_{40}$. Let $\mu$ denote the population mean of log wire thickness.

   **Question #5: Calculate a point estimate of $\mu$.**

## 2 Hypothesis tests

We want to see if there is evidence to claim that the population mean log wire thickness is equal to 8. (Note that $8 \approx \ln 2981$.) We perform a hypothesis test at the $\alpha = .05$ significance level for whether or not $\mu$ is equal to eight. Because we do not know, beforehand, the population standard deviation, we

can't use an ordinary $z$-test based on the standard normal distribution. Instead, as discussed in the text and lectures, we must use a $t$-test, which compensates for our uncertainty about the standard deviation $\sigma$. This is a one-sample $t$-test, and the form of our test statistic is:

$$T = \frac{\overline{Z} - 8}{s/\sqrt{n}}$$

Here, $\overline{Z}$ represents the sample mean of the transformed data and $s$ is the sample standard deviation.

**Question #6: What is the null hypothesis? What is the alternative?**

1. To calculate the test statistic, choose **Stat** > **Basic Statistics** > **1-Sample T**. Select *LOG_WIDTH*. In the "Test Mean" field, input "8.0." Click **OK**.

   **Question #7: What is the value of the test statistic (T) in this case? Make a sketch of the distribution of T under the assumption that the null hypothesis is true, and add the value of your test statistic to it.**

♣ **Question #8: What is the rejection region for this test? Add it to the sketch above.**

**Question #9: State the conclusion of your hypothesis test.**

2. Copy down the confidence interval for $\mu$ from the screen.

   **Question #10: Would you reach the same conclusions regarding $\mu$ using this CI**?

3. The process engineer just returned from vacation, and tells you that the standard deviation of the transformed data, based on historic data, is $\sigma = 0.3$.

   ♣**Question #11: Based on this new knowledge, which of the following would change in your test: the hypotheses, the test-statistic, the rejection region, the significance level $\alpha$? Write down the updated values.**

# 3 Bootstrap Hypothesis Test

Suppose that random variables $X_1$, $X_2$, and $X_3$ are independent, and each has the exponential($\lambda$) distribution. The parameter $\lambda$ is unknown. As usual, let $\overline{X}$ denote the sample mean.
**Question #12: Would we be justified in assuming that $\overline{X}$ is normally distributed?**

We want to test the null hypothesis $H_0 : \lambda = 1$ versus the alternative hypothesis $H_a : \lambda > 1$.

**Question #13: What would constitute evidence in favor of the alternative in this case? Values of $\overline{X}$ much larger than one, or values of $\overline{X}$ much smaller than one? Why?**

You decide that you will reject the null hypothesis if $\overline{X}$ is less than 0.4. We want to estimate the probability of Type I error for this test using some *bootstrap simulations*. This type of approach is common in cases where the distribution of the test statistic cannot be determined exactly.
Here is the procedure to follow.

1. Open a new worksheet in Minitab.
2. Label the first column "X1," the second column "X2," and the third column "X3."
3. Fill in each of the first three columns with 1000 simulated random variables which have the exponential($\lambda = 1$) distribution. Use **Calc → Random Data → Exponential**.
4. Label the fourth column "Xbar" and fill it with the sample mean for each row. In other words, the first three columns are the values of $X_1$, $X_2$, and $X_3$; now put $\overline{X}$ in the fourth column. Use **Calc → Row Statistics**.
5. Make a density histogram of the data in the "Xbar" column.

We have simulated the distribution of the test statistic under the null hypothesis.

**♣ Question #14: Looking at the histogram, estimate the probability of Type I error if you reject the null hypothesis for values of $\overline{X}$ smaller than 0.4.**

Finally, we want to sort the values in the "Xbar" column from smallest to largest. To do this go to **Data** → **Sort**. Choose column "Xbar" as both the "Sort Column" and the "By Column." Click the button to save the results on "Column of current worksheet" and save the result in C5.

**Question #15: What is the 50th value in the sorted list?**

**Question #16: If you wanted your test to have approximately probability 0.05 of Type I error, what rejection region would you use? Hint: Think about your answer to the previous question.**
Using theoretical calulations that we did not discuss in this course, one can show that you should reject the null hypothesis if $\overline{X}$ is less than 0.2726.