

Diamond-Pin Location

R. MOHAN

Indian Institute of Technology, Kharagpur (W.B.), India

A probabilistic model has been developed to estimate the extent of interference in loading components on to a diamond-pin locator of specified design. Then the reverse problem of determining the design-parameters (pin-diameters, and width of the diamond-pin) has been solved. The analysis shows that the difference between the mean diameters of the diamond-pin and the corresponding hole on the component should be kept greater than the difference between the mean diameters of the full-form pin and its corresponding hole. The results should be of use to the designers of the diamond-pin locators, and the basic approach of interest to the designers in general.

1. STATEMENT OF THE PROBLEM

Diamond-pin location is in common use in the design of jigs, fixtures and receiver gages. For machining the fork of the component of Fig. 1, for example, quite often the location is taken from the two previously machined holes (marked No. 1 and No. 2). In such a location there is the problem of interference between the holes and the pins in loading the components on to the fixture. With full-form cylindrical pins, this interference occurs in two ways. It always occurs if the hole diameter is smaller than the corresponding pin diameter, i.e. $D_1 < d_1$, $D_2 < d_2$ (Interference of Type I). Interference of Type II sometimes occurs if

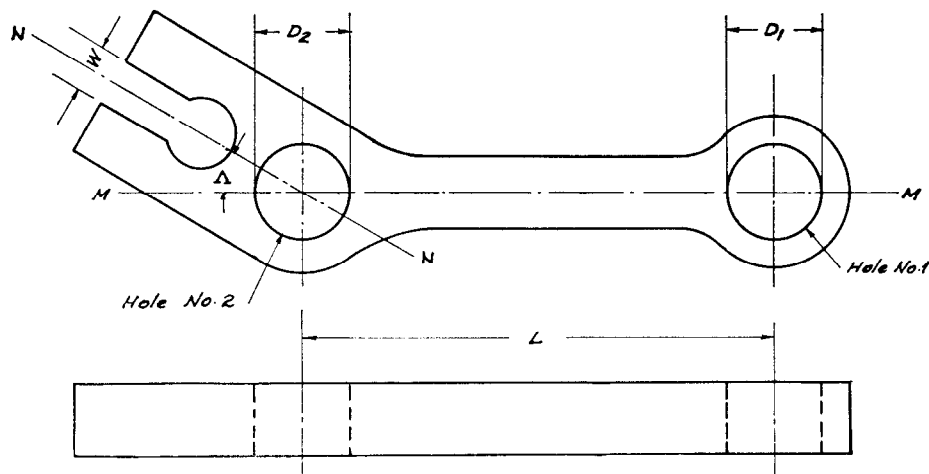


FIGURE 1—Machining of the fork-gap, W , needs diamond-pin location.

Received June 1966.

the distance L between the hole-axes does not equal the distance ℓ between the pin-axes (i.e. $L \neq \ell$)—on the outer sides of the pins when $L_0 < \ell_0$, and on the inner sides of the pins when $L_i > \ell_i$, Fig. 2. It is to be noted that 'interference of type II' can be meaningfully considered, only when 'interference of type I' is absent.

The first type of interference can be controlled by choosing proper pin diameters, e.g. by taking the high specification limit of the pin-diameter somewhat smaller than the low specification limit of the corresponding hole diameter. The second type of interference has quite a low probability of occurrence, in case the tolerance on L is less than that on D_1 and D_2 . But, as is usually the case, the tolerance on L is appreciably bigger than that on either D_1 or D_2 . If, in order to reduce the second type of interference, the diameters of the two pins d_1 and d_2 are made much smaller than the hole diameters D_1 and D_2 respectively (incidentally, the first type of interference is also reduced by this measure), the variations might objectionably increase on some other toleranced dimensions obtained in the present machining operation, e.g. on angle Λ (Fig. 1) between NN (the mid-plane of the fork) and MM (the plane carrying the two hole axes). The problem is usually solved by giving diamond shape to one of the two locating pins on the fixture (Fig. 3). Both the pins are generally not diamond shaped because of some other requirements. For example, it may be required that the plane NN should be within a small distance e from the axis of the hole No. 2. (Fig. 4). If pin No. 2 is diamond-shaped, the variations in e will be more, and may result in excessive rejections on this count.

Now, if the chordal length of the cylindrical portion of the diamond-pin,

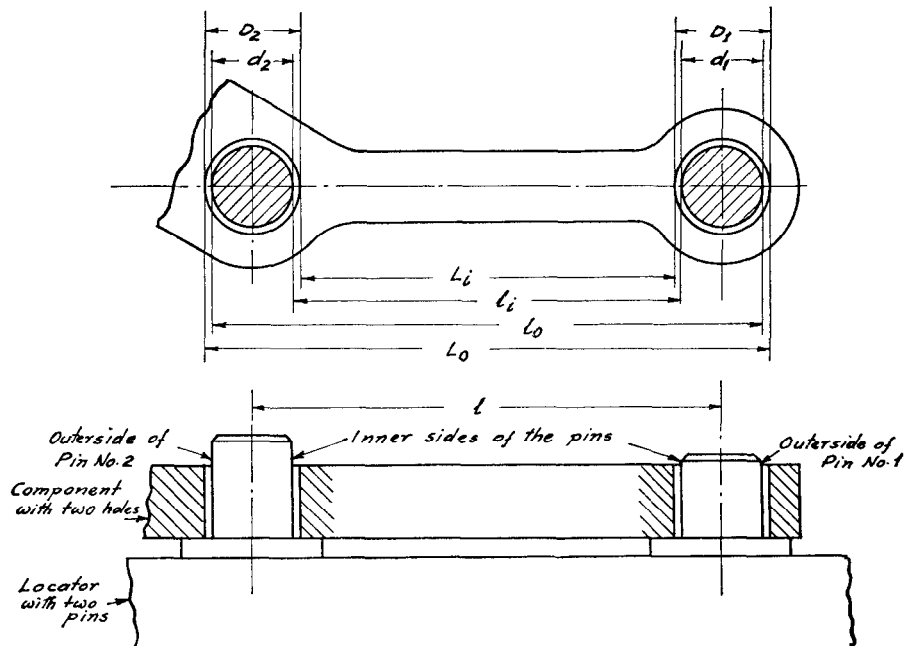


FIGURE 2—Locator with full-form cylindrical pins.

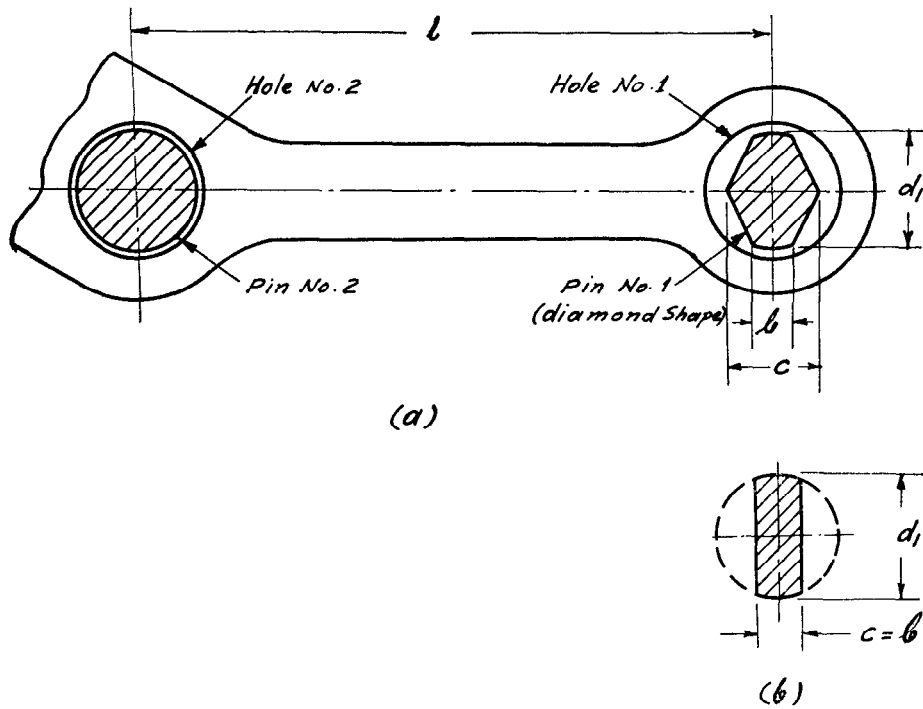


FIGURE 3—Diamond-pin location (a) design for big or slender diamond-pins;
(b) design for small and rigid diamond-pins.

called the *pin-width* could be made as small as one liked, the problem of the second type of interference would have been solved. But the pin-width, has to be kept large enough so that the wear rate on the guiding cylindrical portions of the diamond-pin may not be excessive; otherwise it will require frequent replacements of the pin. Then, what is the maximum possible pin-width, so that the fraction of the components that can be loaded on to the fixture is not less than

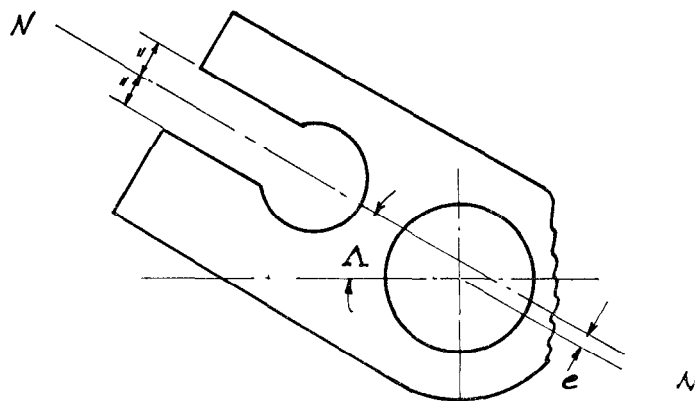


FIGURE 4— NN , the mid-plane of fork is required to be within a small distance, e , from the hole-axis.

a specified value? The present paper attempts to answer this question. For clarity, the symbols and notations used in this paper are first explained (Figs. 1-5).

- D_1 stands for the diameter of hole No. 1.
- d_1 stands for the diameter of pin No. 1.
- D_2 stands for the diameter of hole No. 2.
- d_2 stands for the diameter of pin No. 2.
- L stands for the distance between the axes of the holes (marked 1 and 2).
- ℓ stands for the distance between the axes of the locating pins on the fixture.
- b is the design value of pin-width, which is the same as the chordal length of the cylindrical portion on the diamond pin, (Fig. 5).

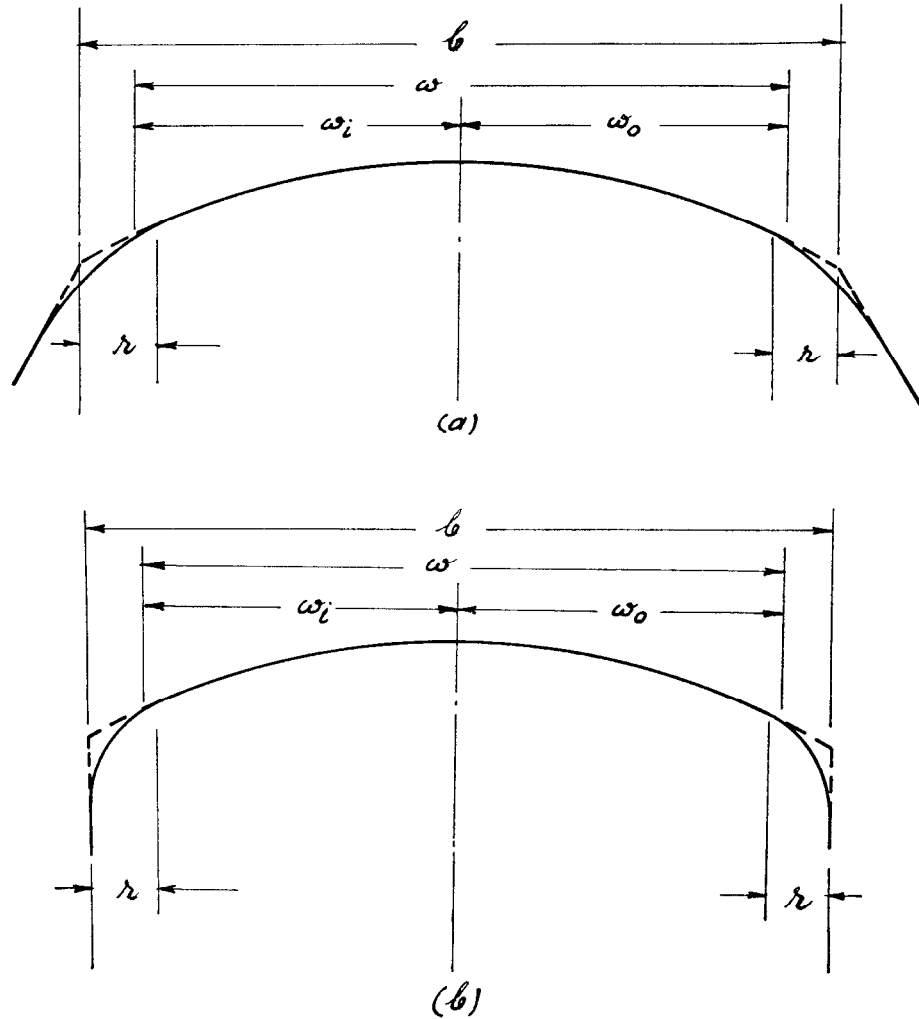


FIGURE 5—The Pin-width; (i) design for big or slender pins; (ii) design for small and rigid pins.

- w is the effective value of pin-width, which is obtained on the actual pin after removing the sharp corners. The magnitude of w is only slightly less than b , the extent of rounding off being of the order of a few thousandths of an inch, Fig. 5. A very small portion of the rounded corner may also come in contact with the hole-surface and participate in interference of Type II. Hence, w , strictly speaking, is slightly greater than the chordal length of the reduced cylindrical portion of diameter d_1 . But, for the purpose of analysis, the two will be considered equal, i.e. $w = b - 2r$. The error introduced by this approximation will be negligibly small for all practical purposes.
- c is the width of the diamond-pin in the plane MM . In case of small rigid-pins, sometimes c is taken equal to b , Figs. 3(b) and 5(b). This is not a critical dimension.
- Λ stands for the angle that NN (the mid-plane of the fork) makes with MM (the plane carrying the two hole-axes).
- x with its appropriate subscript, denotes the corresponding random variable, e.g. x_{D_1} denotes the diameter of hole No. 1 in a random work-piece.
- μ, σ with their appropriate subscripts, denote the mean and the standard deviation, respectively, of the corresponding random variable x , e.g. μ_{D_1} and σ_{D_1} are the mean value and the standard deviation of x_{D_1} .
- p the 'acceptance level', is the fraction of the workpieces (to be machined in the present operation) which it is possible to load on to the fixture.
- $q = (1 - p)$ is the 'rejection-level'.

The probability distributions for x_{D_1} , x_{D_2} , x_L and x_t are assumed to be fully known. For x_{d_1} and x_{d_2} we assume that the forms of the distributions and the corresponding values of standard deviations (σ_{d_1} , σ_{d_2}) are known.

The designer is required to determine the pin-diameters, and the pin-width b . The distance between the pin-axes is always kept equal to the mean distance between the hole-axes. In this selection, at present, he is guided by some empirical rules and by the feel of situation. Pin-diameters are chosen such that a positive minimum clearance (i.e., low specification limit of the hole diameter minus the high specification limit of the corresponding pin-diameter), of the order of several ten thousandths or several thousandths of an inch, is kept. This minimum clearance, in addition to taking care of the form-errors (deviations from cylindricity on holes and pins) and non-parallelism between the hole-axes and the pin-axes, usually ensures that the fraction rejected due to interference of type I is negligible, and some more accommodation is provided for variations on L . Table I (Bolotin and Kostromin 1959) provides a guide for selection of b . Another thumb-rule used by designers is to take $b = d_1/3$, where d_1 is the design value of the diameter of the diamond-pin. There are other rules as well. The result is usually a working solution; but it may be far from the best one. In any case, no indication of the acceptance level p is available.

In section 2 it has been explained how p can be evaluated for given effective pin-width w , when the probability distributions of x_{D_1} , x_{D_2} , x_L , x_t , x_{d_1} , and x_{d_2} are known. And, in section 3, it has been shown how the results of section 2

TABLE I
Selection of Pin-width b

Design Parameter	Diameter of the diamond-pin, d_1 , in mm .					
	From 4 to 6	From 6 to 10	From 10 to 18	From 18 to 30	From 30 to 50	Above 50
b in mm .	2	3	5	8	12	14
c in mm .	$d_1 - 1$	$d_1 - 2$	$d_1 - 4$	$d_1 - 6$	$d_1 - 10$	according to construction

could be utilised to obtain the maximum value of w for a given p by trial and error.

2. INTERFERENCE IN LOADING

The relative positions of pins and holes, and the conditions of the second type of interference are represented in Fig. 6. Let us consider two variables [Fig. 6(a)], x_0 and x_i given by

$$\begin{aligned} x_0 &= (x_L + \frac{1}{2}x_{D_1} + \frac{1}{2}x_{D_2}) - (x_t + \frac{1}{2}x_{d_1} + \frac{1}{2}x_{d_2}) \\ x_i &= (x_t - \frac{1}{2}x_{d_1} - \frac{1}{2}x_{d_2}) - (x_L - \frac{1}{2}x_{D_1} - \frac{1}{2}x_{D_2}) \end{aligned} \quad (1)$$

Interference on the outer sides of the full-form pins arises when x_0 is negative [Fig. 6(b)], and that on the inner sides when x_i is negative [Fig. 6(d)]. Equations (1) could be rewritten in a more convenient form as follows:

$$\begin{aligned} x_0 &= (x_L - x_t) + \frac{1}{2}[(x_{D_1} - x_{d_1}) + (x_{D_2} - x_{d_2})] \\ x_i &= (x_t - x_L) + \frac{1}{2}[(x_{D_1} - x_{d_1}) + (x_{D_2} - x_{d_2})] \end{aligned} \quad (2)$$

In case of the diamond-pin locator (i.e. one pin full and one pin truncated as shown in Fig. 3), the interference on the outer sides occurs when ${}_0x_w < w_0$ [Fig. 6(c)], and that on the inner sides when ${}_ix_w < w_i$ [Fig. 6(e)]. The variable ${}_0x_w$ is obtained by taking away the interfering portion ${}_0x_k$, of pin No. 1 from $x_{d_1}/2$, the outer side of pin No. 2 being just in contact with hole No. 2 [Fig. 6(b)]. Similarly ${}_ix_w$ is obtained.

Let A be the event $x_{D_1} < x_{d_1}$;
 B be the event $x_{D_2} < x_{d_2}$;
 C be the event ${}_0x_w < w_0$;
and D be the event ${}_ix_w < w_i$.

It is to be noted that the events A and B give rise to the first type of interference, and the joint events $(C\bar{A}\bar{B})$ and $(D\bar{A}\bar{B})$ give rise to the second type of interference, where $\bar{A}$ and $\bar{B}$ are events opposite to A and B respectively. These events are represented in the form of a Venn diagram in Fig. 7.

It is clear from the physical conditions of loading, and from equations (2), that the joint event CD is impossible if neither A nor B has occurred, i.e.

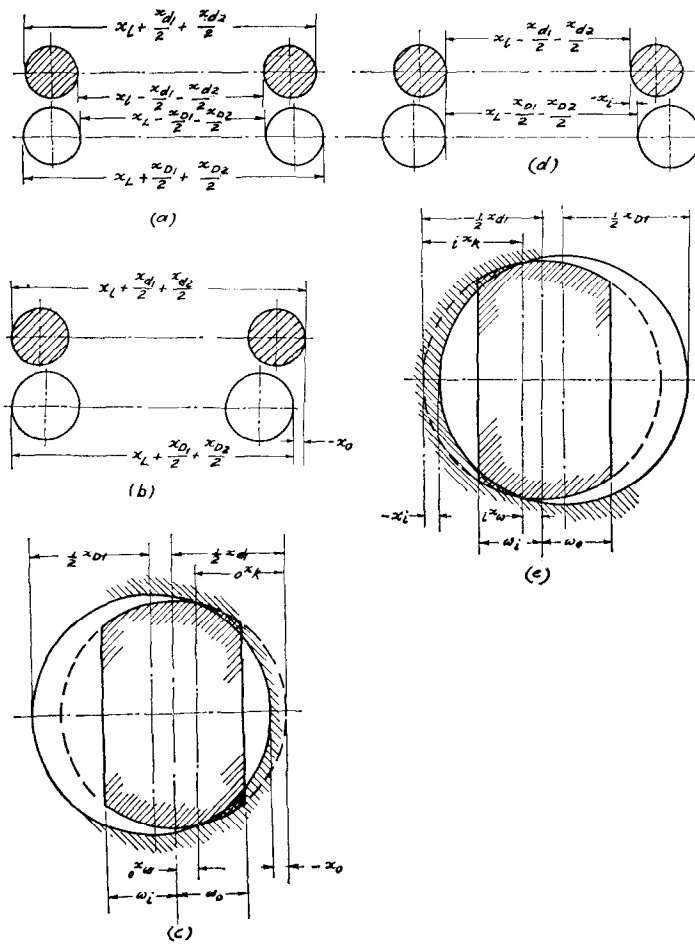


FIGURE 6—Conditions of Interference of Type II.

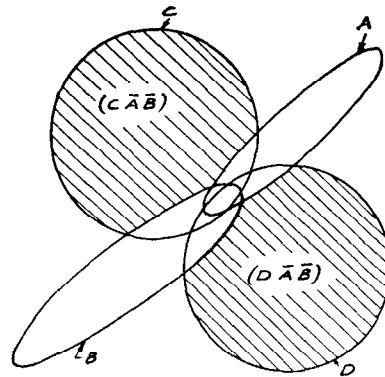


FIGURE 7—Representation of the various interference events.

$P\{CD/\bar{A}\bar{B}\} = 0$. In other words, as shown in Fig. 7, C and D intersect each other only inside A or B . Then, the probability that a random component can be loaded on to the fixture without interference is given by

$$p = P\{\bar{A}\bar{B}\bar{C}\bar{D}\} = 1 - P(A) - P(B) + P(AB) - P(C\bar{A}\bar{B}) - P(D\bar{A}\bar{B})$$

or,

$$(1 - p) = q = q_1 + q_2 - q_{12} + q_3 + q_4 \quad (3)$$

where,

$$\begin{aligned} q_1 &= P(A); & q_2 &= P(B); & q_{12} &= P(AB); \\ q_3 &= P(C\bar{A}\bar{B}); & q_4 &= P(D\bar{A}\bar{B}) \end{aligned} \quad (4)$$

q will be called the rejection-level.

It could be seen that for the location with full-form pins,

$$P(C) = P({}_0x_w < \tfrac{1}{2}x_{d_1}) = P(x_0 < 0)$$

and

$$P(D) = P({}_ix_w < \tfrac{1}{2}x_{d_1}) = P(x_i < 0).$$

Thus it becomes a particular case of the diamond-pin location, with $w_0 = w_i = d_1/2$.

Determination of q_1 , q_2 and q_{12} is usually simple. Thus, if x_{D_1} and x_{d_1} are uncorrelated (invariably true in practice), and have normal distributions, then the probability q_1 of event A is given by

$$P(x_{D_1} < x_{d_1}) = \int_{-\infty}^{-(\mu_{D_1} - \mu_{d_1})/(\sigma_{D_1}^2 + \sigma_{d_1}^2)^{1/2}} (\tfrac{1}{2}\pi)^{1/2} \exp(-t^2/2) dt. \quad (5)$$

Similarly, if x_{D_2} and x_{d_2} are uncorrelated (always true in practice) and have normal distributions, then the probability q_2 of event B is given by

$$P(x_{D_2} < x_{d_2}) = \int_{-\infty}^{-(\mu_{D_2} - \mu_{d_2})/(\sigma_{D_2}^2 + \sigma_{d_2}^2)^{1/2}} (\tfrac{1}{2}\pi)^{1/2} \exp(-t^2/2) dt. \quad (6)$$

The probability q_{12} of the joint event AB is then given by

$$\begin{aligned} P(x_{D_1} < x_{d_1}; x_{D_2} < x_{d_2}) &= \int_{-\infty}^0 \int_{-\infty}^0 \frac{1}{2\pi\sigma_{x_1}\sigma_{x_2}(1 - \gamma_{12}^2)^{1/2}} \exp \left[-\frac{1}{2(1 - \gamma_{12}^2)} \left\{ \left(\frac{x_1 - \mu_{x_1}}{\sigma_{x_1}} \right)^2 \right. \right. \\ &\quad \left. \left. - 2\gamma_{12} \left(\frac{x_1 - \mu_{x_1}}{\sigma_{x_1}} \right) \left(\frac{x_2 - \mu_{x_2}}{\sigma_{x_2}} \right) + \left(\frac{x_2 - \mu_{x_2}}{\sigma_{x_2}} \right)^2 \right\} \right] dx_1 dx_2 \end{aligned} \quad (7)$$

where $x_1 = (x_{D_1} - x_{d_1})$; $x_2 = (x_{D_2} - x_{d_2})$; and γ_{12} is the coefficient of correlation between x_1 and x_2 . In particular, if $\gamma_{12} = 0$,

$$\begin{aligned} P(x_{D_1} < x_{d_1}; x_{D_2} < x_{d_2}) &= P(x_{D_1} < x_{d_1}) \cdot P(x_{D_2} < x_{d_2}) \\ &= q_1 \cdot q_2 \end{aligned} \quad (7a)$$

It is to be noted that the variables x_{D_1} and x_{D_2} sometimes do deviate from

normality in some practical situations. If the deviation is mainly due to the effect of tool-wear, the distributions of x_{D_1} and x_{D_2} involve a rectangular component. In that case q_1 and q_2 could be obtained with the help of the expressions given by Bhattacharjee, Pandit, and Mohan [1963].

Determination of q_3 and q_4 needs further analysis. From (4) the probability q_3 of the joint event $(C\bar{A}\bar{B})$ is given by

$$P(C\bar{A}\bar{B}) = P({}_0x_w < w_0 ; x_{D_1} > x_{d_1} ; x_{D_2} > x_{d_2}).$$

To evaluate the joint probability, it is necessary to express ${}_0x_w$ as a function of known variables.

Now, the value of w_0 cannot exceed $x_{d_1}/2$. And ${}_0x_w$ is less than $x_{d_1}/2$ only for negative values of x_0 . When x_0 is negative, the geometry of Fig. 6(c) gives

$$(\frac{1}{2}x_{d_1} - {}_0x_w)(\frac{1}{2}x_{d_1} + {}_0x_w) = (\frac{1}{2}x_{d_1} - {}_0x_w + x_0)\{x_{D_1} - (\frac{1}{2}x_{d_1} - {}_0x_w + x_0)\}$$

that is

$$\begin{aligned} {}_0x_w &= (x_{d_1}/2) + \{x_0(x_{D_1} - x_0)/(x_{D_1} - x_{d_1} - 2x_0)\} \\ &= \frac{1}{2}x_{d_1} - \frac{1}{2}(x_{D_1} - x_0) \left[\frac{\{x_0 - \frac{1}{2}(x_{D_1} - x_{d_1})\} + \frac{1}{2}(x_{D_1} - x_{d_1})}{x_0 - \frac{1}{2}(x_{D_1} - x_{d_1})} \right] \end{aligned}$$

Substituting the value of x_0 from (2),

$${}_0x_w = \frac{1}{2}x_{d_1} - \frac{1}{2}(x_{D_1} - x_0) \left[1 + \frac{\frac{1}{2}(x_{D_1} - x_{d_1})}{(x_L - x_t) + \frac{1}{2}(x_{D_2} - x_{d_2})} \right]$$

Putting

$$\begin{aligned} X_1 &= (x_{D_1} - x_{d_1})/2; & X_2 &= (x_{D_2} - x_{d_2})/2; \\ X_3 &= (x_L - x_t); & X_4 &= X_2 + X_3, \end{aligned}$$

we have

$$\begin{aligned} {}_0x_w &= [x_{d_1} - (x_{D_1} - x_0)\{1 + (X_1/X_4)\}]/2 \\ &= [x_{d_1} - (x_{D_1} - x_0)\{1 + (1/y)\}]/2 \end{aligned} \quad (8)^*$$

where

$$y = X_4/X_1.$$

Expression (8) gives ${}_0x_w$ in terms of known variables. It does not seem to be possible, however, to obtain a simple expression for the probability q_3 of the joint event $(C\bar{A}\bar{B})$. This difficulty could be overcome by the use of some suitable approximations.

Let us consider

$${}_0x_w^* = -\mu_{d_1}/2y \quad (9)$$

* It may be noted that the above analysis holds good only when $0 < {}_0x_w < x_{d_1}/2$. Substituting the value of ${}_0x_w$ from (8), then $(x_{D_1} - x_0)/\{x_{d_1} - (x_{D_1} - x_0)\} < y < -1$ must be satisfied. Now, x_0 has to be negative and $x_{D_1} > x_{d_1}$ in the above expressions. Hence, the denominator of the extreme left member of the above inequality is negative and very small in magnitude compared to the numerator. Therefore, for all practical purposes, $-\infty < y < -1$.

as an approximation to ${}_0x_w$ given by (8); and

$$q_3^* = P\{C^*\bar{A}\} \quad (10)$$

as an approximation to $q_3 = P(C\bar{A}\bar{B})$, where

$$P(C^*) = P[{}_0x_w^* < w_0].$$

From expression (9) it is clear that for every value of y , there is one and only one value of ${}_0x_w^*$. Therefore

$$\begin{aligned} q_3^* &= P\{{}_0x_w^* < w_0; x_{D_1} > x_{d_1}\} \\ &= P\{-\mu_{d_1}/2y < -\mu_{d_1}/2Y; x_{D_1} > x_{d_1}\} \end{aligned}$$

where

$$Y = -\mu_{d_1}/2w_0 \quad (11)$$

Hence

$$q_3^* = P\{y < Y; x_{D_1} > x_{d_1}\}$$

Let E represent the event $y < Y$. Then the probability of $(E\bar{A})$ has to be evaluated.

Geary [1930] has shown that if $z = (b' + y')/(a' + x')$, where x' and y' are two normal variates distributed with means zero, standard deviations α and β respectively, and correlation coefficient γ ; and a' and b' are constants, then the function

$$t = (a'z - b')/(\alpha^2 z^2 - 2\gamma\alpha\beta z + \beta^2)^{\frac{1}{2}} \quad (12)$$

is normally distributed with mean zero and standard deviation unity provided $(a' + x')$ is unlikely to assume negative values.

In the present situation, y is a quotient of two normal variates X_4 and X_1 with known parameters and $\bar{A}$ is the event that X_1 does not take negative values. Also, in actual designs $P(\bar{A}) = 1$, for all practical purposes. Hence

$$P\{y < Y; x_{D_1} > x_{d_1}\} = P(y < Y)$$

Therefore

$$q_3^* = \int_{-\infty}^T (\frac{1}{2}\pi)^{\frac{1}{2}} \exp(-t^2/2) dt \quad (13)$$

where

$$T = (\mu_{X_1} \cdot Y - \mu_{X_4})/(\sigma_{X_1}^2 Y^2 + \sigma_{X_4}^2 - 2\gamma_{14}\sigma_{X_1}\sigma_{X_4}Y)^{\frac{1}{2}} \quad (14)$$

γ_{14} being the correlation coefficient between X_1 and X_4 .

It should be noted that the value of Y given by (11) leads to an underestimation of the value of $P(C\bar{A})$. On the other hand, $P(C\bar{A})$ is obviously larger than $P(C\bar{A}\bar{B})$. The two errors are of the same order, hence they balance each other to quite some extent; and their magnitudes being not large, the resultant error is small. Thus q_3^* , which can be easily obtained from (11), (13) and (14), provides a close approximation to q_3 .

Now, $(x_L - x_i)$ is almost invariably a symmetrically distributed variable with zero mean. In that case the value of q_4^* can be obtained from (11), (13),

and (14) replacing w_0 by w_i in (11). In particular, if the diamond-pin is symmetrical, as is usually the case $w_i = w_0$, so that $q_4^* = q_3^*$. In the rare cases, with non-symmetrical distribution of $(x_L - x_i)$, using the geometry of Fig. 6(e), one could proceed exactly as above and obtain q_4^* .

Thus, the proportion of components that cannot be loaded on to the fixture (the rejection level) is given approximately by

$$q^* = q_1 + q_2 - q_{12} + q_3^* + q_4^* \quad (15)$$

The value of q^* given by (15), the author feels, will be sufficiently close* to q given by (3) in almost all practical cases. This statement may be viewed in the light of the fact that, so far, in absence of such an analysis, nothing could be said even about the order of the rejection level, and one could only hope that it will not be too high.

The evaluation of q will now be illustrated with the help of numerical examples. In these illustrations the values of μ_{d_1} , μ_{d_2} and b are determined by a conventional method.

ILLUSTRATIVE EXAMPLES

Example 1. Fork of the component shown in Fig. 1 is to be milled. The holes No. 1 and No. 2 are already machined to give centre-distance L within $5.0'' \pm 0.005''$ and hole-diameters within $0.997'' \pm 0.002''$. From previous tool-room experience it is known that $\sigma_{d_1} = \sigma_{d_2} = 0.00004''$ and $\sigma_L = 0.00004''$. The variables x_{D_1} , x_{D_2} , x_{d_1} , x_{d_2} , x_L , and x_i can all be assumed to be independent normal variates, and the specification limits correspond to $\pm 3\sigma$ limits with respect to the mean values.

A minimum clearance of $\pm 0.00005''$ is chosen. Hence,

$$(\mu_{D_1} - 3\sigma_{D_1}) - (\mu_{d_1} + 3\sigma_{d_1}) = .0005''$$

and

$$(\mu_{D_2} - 3\sigma_{D_2}) - (\mu_{d_2} + 3\sigma_{d_2}) = .0005''$$

Solving, we get $\mu_{d_1} = \mu_{d_2} = .99438''$; also from Table I, for the given pin-diameter, $b = 8 \text{ mm.} = 0.315 \text{ in.}$

Now we proceed to find out the value of q^* corresponding to these design parameters. We have

$$\mu_{x_1} = (\mu_{D_1} - \mu_{d_1})/2 = 0.00131''.$$

and

$$\mu_{x_2} = (\mu_L - \mu_i) + [(\mu_{D_2} - \mu_{d_2})/2] = 0.00131'',$$

μ_i being taken equal to μ_L . Also

$$\sigma_{x_1} = [(\sigma_{D_1}/2)^2 + (\sigma_{d_1}/2)^2]^{\frac{1}{2}} = 10^{-4} \times (11.1511)^{\frac{1}{2}} \text{ inch}$$

$$\sigma_{x_2} = [\sigma_L^2 + \sigma_i^2 + (\sigma_{D_2}/2)^2 + (\sigma_{d_2}/2)^2]^{\frac{1}{2}} = 10^{-4} \times (289.0889)^{\frac{1}{2}} \text{ inch.}$$

* It is not safe to make any comment on the order of accuracy without simulating the exact expressions on the digital computer and in the absence of careful and extensive experiments. But, it is guessed that in most of the cases $|q_3^* - q_3|/q_3$ may not exceed .05.

In this situation we can take $r = 0.0025''$. Then the effective pin-width, $w = b - 2r = 0.310''$. Also, from considerations of symmetry $w_0 = w, = w/2$. From (11),

$$Y = -0.99438/0.310 = -3.2077$$

From (14),

$$T = -\frac{13.1 \times 3.2077 + 13.1}{[11.1511 \times (3.2077)^2 + 289.0889]^{\frac{1}{4}}} \times \frac{10^{-4}}{10^{-4}} = -2.743$$

From (13), using Biometrika Tables,

$$q_3^* = 0.0030. \text{ Also, } q_4^* = q_3^* = 0.0030.$$

From (5),

$$q_1 = \int_{-\infty}^{-3.339} (\frac{1}{2}\pi)^{\frac{1}{2}} \exp(-t^2/2) dt = 0.00042.$$

Also, $q_2 = q_1 = 0.00042$; $q_{12} = q_1 \times q_2 = (0.00042)^2 \simeq 0$. Hence, from (15)

$$\begin{aligned} q^* &= q_1 + q_2 - q_{12} + q_3^* + q_4^* \\ &= 0.00042 \times 2 - 0 + 0.0030 \times 2 \\ &= 0.00684. \end{aligned}$$

Example 2. The fork of the component of example 1 is to be ground in its finish-machining stage. After the milling of the fork, the two holes have been finish machined, and now the hole diameters vary within $1.0003'' \pm 0.0003$. All other conditions remain unchanged.

A minimum clearance of $+0.0005''$ is chosen as in example 1. It gives $\mu_{d_1} = \mu_{d_2} = 0.99938''$.

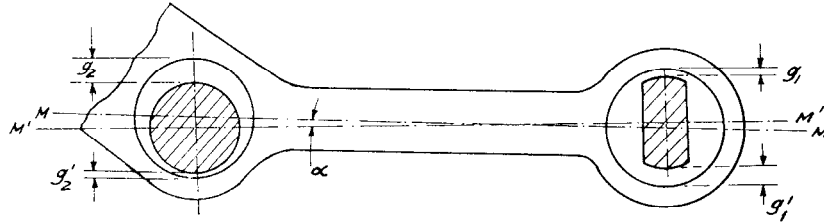
Again, from Table I, b is chosen equal to 8 mm. Also r is kept $.0025''$. Hence, the design-parameters for the diamond-pin locator remain the same as in the above example. Proceeding as before, $\mu_{x_1} = \mu_{x_2} = 0.00046''$, $\sigma_{x_1} = 10^{-4} \times (0.29)^{1/2}$ inch, $\sigma_{x_2} = 10^{-4} \times (278.2278)^{1/2}$ inch; $w = 0.310''$.

From (11), $Y = -3.2238$; and from (14) $T = -1.159$. Hence, $q_3^* = q_4^* = 0.1231$, and $q_1 = q_2 \simeq 0$, $q_{12} \simeq 0$. Therefore $q^* = 0.2462$.

From examples 1 and 2 it is clear that the design parameters chosen with the help of empirical rules can sometimes prove to be quite satisfactory; while in other situations, the rejection-level could be too high to be acceptable. This points to the necessity of having a more scientific method for determining the design-parameters. This is taken up in the next section.

3. DETERMINATION OF DESIGN PARAMETERS μ_{d_1} , μ_{d_2} , AND b

The designer is provided with the value of q and with the tolerance on some other dimension, Δ , for example. Tolerance on Δ , as will be presently seen, fixes the sum of μ_{d_1} and μ_{d_2} . The designer, by a process of trial and error, could then determine the values of μ_{d_1} , μ_{d_2} , and b that satisfy all the given conditions. A numerical example at the end of this section illustrates the method.

FIGURE 8—Non-coincidence of MM and $M'M'$ in actual loading.

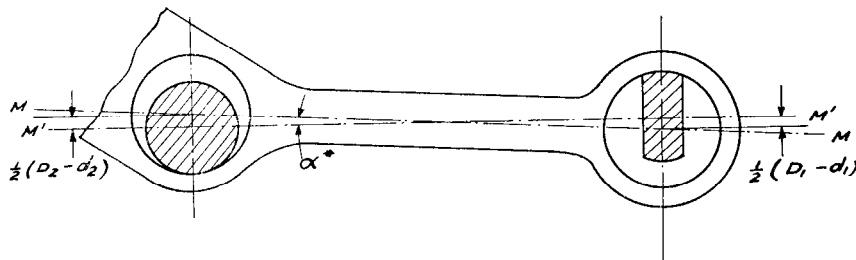
Variations in Δ occur due to various reasons, e.g. variations in the inclination α of the plane MM carrying the hole-axes to $M'M'$ carrying the pin-axes when components are loaded on to the fixture (Fig. 8), variations in the setting of the fixture on the machine tool, variations in the movements of the various moving units of the machine tool, etc. Only the first factor, viz. α , is affected by the selection of the design parameters presently under consideration, as such the tolerance on α (which will be tighter than the tolerance on Δ) will be assumed to be given.

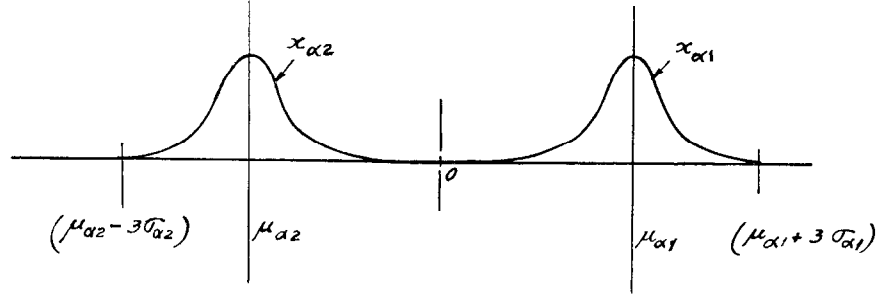
The angle α varies mainly due to variations in the clearances between the two holes and their corresponding pins. Also for the same combination of the fixed values of x_{D_1} , x_{d_1} , x_{D_2} , x_{d_2} , x_L and x_t , α may take different values in loading, Fig. 8, depending, upon the values that g_1 , g_1' , g_2 , g_2' happen to take. Quite often g_1 and g_2' are zero, Fig. 9, or g_1' and g_2 are zero.

Preliminary investigations reveal, that the distribution of α obtained by repeated loading of one and the same component on to the same fixture (i.e. for fixed values of clearances) is not consistent. It varies from operator to operator, and for the same operator from time to time. In view of this unpredictability, the designer should work on the safe side. For a given set of clearances, Fig. 9 shows the maximum possible inclination α^* . Again, α^* being a very small angle,

$$\alpha^* = \sin \alpha^* = \frac{(D_1 - d_1) + (D_2 - d_2)}{2L}$$

Considering that the probability of loading the component clockwise (as shown in Fig. 9) is equal to that of loading it anticlockwise, Fig. 10 represents the distribution of x_{α^*} , where x_{α^*} denotes the random variable corresponding

FIGURE 9—The extreme value of α ($= \alpha^*$).

FIGURE 10—The distribution of x_{α^*} .

to α^* . It is a mixture of two separate populations (with equal weightages) corresponding to x_{α_1} and x_{α_2} , such that

$$x_{\alpha_1} = + \frac{(x_{D_1} - x_{d_1}) + (x_{D_2} - x_{d_2})}{2x_L}; \quad x_{\alpha_2} = - \frac{(x_{D_1} - x_{d_1}) + (x_{D_2} - x_{d_2})}{2x_L}$$

It can be seen that the variations in L , in the practical working range, do not affect α to any appreciable extent. Hence

$$x_{\alpha_1} = + \frac{(x_{D_1} - x_{d_1}) + (x_{D_2} - x_{d_2})}{2\mu_L}; \quad x_{\alpha_2} = - \frac{(x_{D_1} - x_{d_1}) + (x_{D_2} - x_{d_2})}{2\mu_L} \quad (16)$$

If these distributions are normal, α^* will fall within the range (fig. 10)

$$\pm [(\mu_{D_1} - \mu_{d_1}) + (\mu_{D_2} - \mu_{d_2}) + \tau(\sigma_{D_1}^2 + \sigma_{d_1}^2 + \sigma_{D_2}^2 + \sigma_{d_2}^2)^{\frac{1}{2}}] / 2\mu_L \text{ radians}$$

with the probability

$$\int_{-\infty}^{\tau} (\frac{1}{2}\pi)^{\frac{1}{2}} \exp(-z^2/2) dz. \quad (17)$$

Thus $(\mu_{d_1} + \mu_{d_2})$ is expressed in terms of known (or given) quantities. Now the determination of μ_{d_1} , μ_{d_2} and b by trial and error, will be illustrated with the help of a numerical example.

Example 3. The design-parameters μ_{d_1} , μ_{d_2} and b will be obtained for the diamond-pin locator to be used for the operation explained in example 2 (section 2). The additional conditions are that 99% times α should lie within $\pm 0^\circ 1'$; and $q = .0027$.

$0^\circ 1'$ equals .00029 radian; and value of τ corresponding to the probability .99 is given from normal tables as 2.33. Hence from (17),

$$\pm .00029 = \pm [(\mu_{D_1} - \mu_{d_1}) + (\mu_{D_2} - \mu_{d_2}) + 2.33(\sigma_{D_1}^2 + \sigma_{d_1}^2 + \sigma_{D_2}^2 + \sigma_{d_2}^2)^{\frac{1}{2}}] / 2\mu_L$$

giving $\mu_{d_1} + \mu_{d_2} = 1.99806$ in.

A number of combinations of w , μ_{d_1} , and μ_{d_2} were tried to find out the best one satisfying all the given conditions. The procedure for the determination of q^* is identical to the one detailed in example 1 (section 2). The results are given in Table II. Combination No. 9 appears to be the best. However these values should not be used straightaway without giving due consideration to the effects of form-errors, non-parallelism of axes, and method of measurement.

TABLE II
Rejection-levels for Different Combinations of w , μ_{d_1} and μ_{d_2}

S No.	w inch	μ_{d_1} inch	μ_{d_2} inch	q_1	q_2	q_{12}	$q_3^* = q_4^*$	q^*
1	.300	.99868	.99938	0	0	0	.0299	.0598
2	.200	.99868	.99938	0	0	0	.0038	.0077
3	.200	.99806	1.00000	0	.0033	0	.0003	.0040
4	.220	.99809	.99997	0	.0011	0	.0011	.0033
5	.220	.99813	.99993	0	.0003	0	.00123	.0028
6	.220	.99811	.99995	0	.0006	0	.00114	.0029
7	.220	.99812	.99994	0	.0004	0	.00118	.0028
8	.220	.99815	.99991	0	.00015	0	.00132	.0028
9	.220	.99814	.99992	0	.0002	0	.00127	.0027

Thus, if pneumatic gage is used to check the hole-diameter, some sort of average size of a hole is given; while, from the viewpoint of interference and distribution of α , it is the minimum dimension on a particular hole which is of importance. Hence pin-diameters should be made smaller than the calculated sizes. Also, the extreme distribution of α has been used in obtaining the value of $(\mu_{d_1} + \mu_{d_2})$. In actual loading, α will not always take the extreme values ($\alpha^* = \alpha_1$ or α_2) given by (16). In addition to this, the difference between L and ℓ tends to restrict the magnitude of α . Both of these factors act in the direction of reducing the dispersion of α . This reduction might very well be of the order of 10–20%.

Keeping all the above facts in mind, the following values of the design-parameters are recommended for the situation of example 3;

$$\mu_{d_1} = .9978''; \quad \mu_{d_2} = .9996''; \quad b = w + 2r = .225''.$$

4. LOCATION FROM ONE HOLE AND PLANE SURFACES

The expressions obtained in section 2 could be easily modified to suit another situation where diamond-pin location is used, Fig. 11. The bottom surface (marked A) and the hole No. 1 of the component (shown chain-dotted) are previously machined. The hole H is now to be machined taking 3-point location from $A(S_1, S_2, S_3)$, 1-point from the unmachined surface $B(S_4)$ and the remaining 2-points from the hole No. 1. If a full form pin, keeping $d_1 \ll D_1$, is used, the component has to be carefully secured against the pin every time in the same direction. This necessitates an additional feature (e.g. a screw-arrangement), and, therefore, quite often diamond-pin is used instead. Diamond-pin provides accommodation for variations on L ; at the same time it limits the side-ways movement due to the small difference between D_1 and d_1 .

It is obvious that the expressions for this case are obtained by making $x_{D_2} = x_{d_2} = 0$ in the expressions for the fork of Fig. 1. In this case μ_{d_1} will be fixed up by some other requirement, e.g. it may be necessary that the axis of the hole H be within a small specified distance from the plane perpendicular to A and carrying the axis of the hole No. 1. Hence the designer has to find

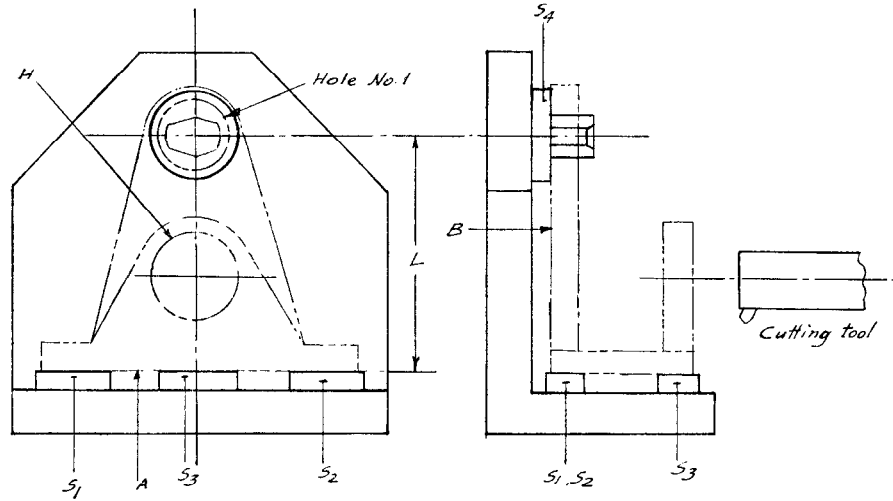


FIGURE 11—Location from one hole and two plane surfaces.

out b only; which will need a much smaller number of trials than those required for the case of two holes discussed in example 3 (section 3).

It is also to be noted that the event $B(x_{D_s} < x_{a_s})$ does not occur in this case. Hence $q_3 = P(C\bar{A})$, and $q_4 = P(D\bar{A})$. Thus the second approximation of expression (10) is not necessary for this case.

5. COMPARISON WITH ANALYSIS BASED ON 'ARITHMETIC ADDITION OF ERRORS'

Bolotin and Kostromin (1959) have discussed the problem of diamond-pin location from two holes; and have used a method based on the 'arithmetic sum of errors' approach'. The expression (9) of section 2 can be rewritten as

$${}_0x_w^* = -(\mu_{d_1}X_1/X_4)/2$$

Equivalent to this expression they give

$$b/2 = (D \cdot \Delta / \Sigma) / 2 \quad (9a)$$

where

D is the minimum diameter of the hole corresponding to the diamond-pin,

Δ is the minimum radial clearance on the diamond-pin,

$\Sigma = \delta_1 + \delta_2 - \Delta_1$;

where

δ_1 and δ_2 are half of the tolerance-ranges for L and ℓ respectively,

Δ_1 is the minimum radial clearance on the full-form pin.

The parallelism between $b/2$ and ${}_0x_w^*$, D and μ_{d_1} , Δ and X_1 , Σ and X_4 may be noted.

It can be easily seen that the values of the pin-width calculated from (9a) will usually be much different from those used in industry or the ones obtained with the help of the present analysis. Thus, for the situation of example 1 (section 2), it gives $b = .051''$ while $b = .315''$ is quite satisfactory.

Their expression equivalent to (16) of section 3, is

$$\alpha = \sin \alpha = (\Delta_2 + \Delta_3)/2L \quad (16a)$$

where Δ_2 and Δ_3 are the maximum clearances on the two pins. Expression (16a) gives the 'maximum angular error'—a typical way of dealing with tolerances and errors in 'arithmetic addition' method. The values of α obtained from expression (16a) will always be too conservative.

Evaluation of α and b , using the expressions (16a) and (9a) respectively, gives us a good insight into the working of the method of arithmetic sum of errors. The expression (16) involves only sum of random variables, hence (16a) will give values of α that are much on the safer side but not absurd; while (9a) can sometimes lead to absurd results as (9) contains a quotient of random variates.

6. CONCLUDING REMARKS

1. It is clear from (14) that an increase in μ_{x_1} is much more effective in reducing the probability of the second type of interference than the same increase in μ_{x_2} . Also, from (17), it could be seen that an increase in either of them has the same effect in increasing variations in α (and therefore in Δ). Thus, a reduction in the diameter of the diamond-pin is more advantageous than that in the diameter of the other (full-form) pin. Hence, μ_{d_1} should be chosen to keep $q_2 = P(x_{D_2} < x_{d_1})$ sufficiently low, and then all possible reduction be made in μ_{d_1} .

2. The extent of error introduced by the two approximations of section 2, while expected to be low, has not been ascertained in the present paper. The order of this error could be revealed by comparing the calculated values with those obtained by simulating the exact expression (8) on a digital computer. Again, the distribution of α should be more thoroughly investigated by conducting experiments in actual industrial conditions, and then one may use the distribution of α^* with some suitable correction coefficient. Also, the effect of form-errors, non-parallelism between the axes, and the method of measurement has not been taken into account in the present analysis. And, the paper does not provide any idea about the errors introduced in the estimates when the assumption of normality of x_{D_1} , x_{D_2} , etc. holds only approximately.

3. The basic approach of treating the various random inputs as random variables throughout the analysis, is general, and could be used to deal with a large number of situations in design. Such situations are not limited only to jig-fixture design, but arise in a very wide range of mechanical products, electrical and electronic devices, etc.

Such an approach could be of great use in the industrial setups where statistical methods are effectively used for controlling quality in production. The establishments where such control-techniques are not used will benefit much less, as

the distributions of the various input elements will not be consistent, and, as such, any assumptions about them will introduce appreciable errors in the estimates.

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