These articles (Proschan 1963; Nelson 1972) are two of the most cited and influential articles in the reliability literature. Their impact on reliability and, more broadly, survival and event-history analysis has been substantial.

The main objectives of Nelson (1972) were to describe and present applications of hazard plotting, which had been developed by Nelson (1969, 1970) in earlier articles. These articles gave a nonparametric estimator of the cumulative hazard function of a failure time distribution, now known as the Nelson-Aalen estimator (e.g., Andersen, Borgan, Gill, and Keiding 1993, sec. 4.1), on which hazard plotting is based. This work has turned out to be one of the cornerstones of modern survival analysis and event-history analysis.

By the 1960s, probability plots were widely used to assess the validity of parametric distributions and to obtain graphical estimates of parameters. Probability plots are simplest and most effective for distributions that can be "linearized" (Meeker and Escobar 1998, chap. 6). For the Weibull distribution, for example, the survivor function $S(x) = \Pr(X > x)$ satisfies the relationship

$$\log[-\log S(x)] = \beta \log x - \beta \log \alpha,$$  

where $\alpha > 0$ and $\beta > 0$ are unknown parameters [see Nelson 1972, eq. (20)]. This suggests taking a nonparametric estimate $\hat{S}(x)$ and plotting $\log[-\log \hat{S}(x)]$ against $\log x$; a roughly linear plot is expected if the Weibull model is appropriate. For a random sample $x_1, \ldots, x_n$, the empirical survival function (ESF) $\hat{S}(x) = n(x)/n$ is such an estimate, where $n(x) = \sum I(x_i > x)$ is the number of observations greater than $x$. Because $\hat{S}(x)$ changes value only at the ordered observations $x(1) < \cdots < x(n)$, it is customary to plot only points corresponding to the $x(i)$'s.

The motivation for Nelson’s work (1969, 1970, 1972) was that, when samples of failure-time data are subject to arbitrary forms of right censoring, the order statistics $x(i)$ are not all known and the ESF is unobtainable. In 1958, Kaplan and Meier (1958) derived their now famous nonparametric “product-limit” estimate $\hat{S}_{KM}(x)$ to deal with this situation, and one approach is simply to use $\hat{S}_{KM}(x)$ in place of the ESF for probability plots (e.g., Lawless 1982, sec. 2.4). Nelson took a different approach, and presented a nonparametric estimator of the cumulative hazard function $H(x) = -\log \hat{S}(x)$. This takes the form

$$\hat{H}_{NA}(x) = \sum_{i: x_i \leq x} \delta_i/n(x_i),$$  

where for $i = 1, \ldots, n$ the value $x_i$ is either a failure time or censoring time, $\delta_i = 0$ if $x_i$ is a censoring time and 1 if it is a failure time, and $n(x_i) = \sum_{j=1}^n I(x_j \geq x_i)$ is the number of failure or censoring times greater than or equal to $x_i$. The estimator (2) was actually also proposed in a 1972 master’s thesis by Ø. Ø. Aalen (see Andersen et al. 1993, sec. 1.2.1) and discovered independently by Altschuler (1970). It is now usually referred to as the Nelson–Aalen (NA) estimator (e.g., Andersen et al. 1993, sec. 4.1).

Nelson (1972) and the two preceding articles were innovative and far reaching in several respects. In the article, Nelson discussed the use of (2) for probability plots: If $S(x)$ can be linearized, so can $H(x)$. He also showed how it can be applied to problems of multiple failure modes, or competing risks (see also Altschuler 1970 and Nelson 1970). These were great practical contributions, and it has become standard practice in survival analysis to estimate cumulative hazard functions (CHF’s) and to plot NA estimates both for description and as a diagnostic device. Direct, untransformed plots of $H_{NA}(x)$ are very useful; in particular, a roughly linear plot indicates consonance with an exponential distribution, and from nonlinear shapes we gain information about whether the hazard function might be increasing, decreasing, or nonmonotonic.

In fact, the estimator (2) has had ramifications far beyond the context of the original article. The field of event-history analysis has evolved rapidly since 1970, and the NA estimator turns out to provide nonparametric estimates of cumulative intensity functions in Markov models for multiple events; Andersen et al. (1993, chaps. 4, 6) provided details and numerous examples of applications. An extension of the NA estimator features prominently in multiplicative hazard or intensity-based regression models for failure times or multiple events (Andersen et al. 1993, sec. 7.2). Nelson (1988) showed that the NA estimator also provides a robust estimate of the cumulative mean function for recurrent events, and this has recently been extended to provide robust regression methods (Lawless and Nadeau 1995; Nelson 1995).

As with many brilliant ideas, the theoretical properties of the methodology surrounding the NA estimator were developed over several years. Breslow and Crowley (1974) and Aalen (1978) provided the first rigorous treatments; see Andersen et al. (1993, chap. 4) for details of these and other developments.
contributions. This development provided tools for constructing nonparametric confidence intervals for the CHF and for testing the equality of failure time distributions and of models for multiple events (Andersen et al. 1993, chap. 5). The methodology is embedded in survival and event-history analysis software in packages such as S-PLUS and SAS.

There were other interesting results shown by Nelson (1972)—for example, the indication that left-censored data could be handled by reversing the time scale, and a discussion of different types of censoring mechanisms and applications. They too endure as contributions that foresaw important later developments.

Proshan (1963) also wrote an exceptional article. Like Nelson (1972), he provided a practical discussion of methods that were based largely on earlier works. The main objective was to show that failure-time distributions that have a decreasing hazard function (or failure rate, as it is called in the article) can arise as a mixture of exponential distributions with different mean times to failure. This theme was illustrated through the analysis of the time intervals ("gaps") between successive failures of the air-conditioning equipment in a fleet of 13 Boeing jet airplanes, it being argued that the observed data were consistent with independent exponential gaps within airplanes but that data pooled across planes indicated a decreasing hazard function.

Two things have contributed to the lasting influence of the article. One is the discussion of mixture models leading to decreasing hazard functions, which has been of considerable interest in fields such as demography, economics, and reliability (e.g., see Arjas, Hansen, and Thyregod 1991; Aalen 1994). The other is that the article was one of the first to feature data on multiple units of a repairable system, and the dataset given has been continually referred to and reanalyzed since 1963; an early example was given by Cox and Lewis (1966, see pp. 49, 237). In their book on repairable systems reliability, Ascher and Feingold (1984, p. 144) remarked that "these data are almost certainly the most widely cited failure data in the reliability literature ...

Persons analyzing repairable systems have debated the use of renewal models versus Poisson models, and Proshan (1963) also featured prominently in this. Proshan's analysis and much other work uses renewal processes, in which a system is "good as new" after repair, as a starting point. Ascher and Feingold (1984, chaps. 2 and 8) criticized the naive use of renewal models and promoted Poisson processes for applications in which time trends exist. The appropriate model, of course, depends on the setting, but it is clear that time trends may be incorporated into either Poisson or renewal frameworks (e.g., Proshan 1963; Cox 1972; Lawless and Thiagarajah 1996).

Nelson (1972) and Proshan (1963) have both had a lasting impact on reliability, survival, and event-history analysis. The reasons are, in retrospect, clear: They addressed important practical problems, presented useful methodology, and raised issues of enduring relevance. The fact that both stimulated developments well beyond the setting for the original article is further testimony to their influence.

REFERENCES