36-771 Fall 2018

Martingales 2 : Confidence sequences

The one and only homework

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There will be 8 questions in total, each worth 5 points.

Question 1 *Consider tossing a sequence of coins with probability p, and keeping track of the sum S_n (+1 for heads, -1 for tails).*

(a) *Write down the asymptotically pointwise valid CLT confidence interval for p.*

(b) *Write down any pointwise valid Chernoff-based confidence interval (mention which one you used) for p.*

(c) *Write down a uniformly valid* linear *confidence sequence (mention which one you used) for p.*

(d) *Write down a uniformly valid* curved *confidence sequence (mention which one you used) for p.*

Question 2 *For each of the previous bounds, run a simulation with 10^5 coins (of any bias you choose) to estimate the probability that the confidence interval for p is wrong at some time between 1 and 10^5 (repeat a 1000 times to get an accurate estimate of the probability).*

Question 3 *Play around with the inverted stitching method to come up with your own unique 1-subGaussian boundary that has crossing probability at most 0.1 before intrinsic time 10^9.*

Question 4 *If you had to use your a 1-subGaussian confidence sequence before a finite time, say 10^6, which of the following would you use? (a) a stitching boundary, (b) a normal mixture boundary, (c) the inverted stitching boundary. Justify your answer.*

Question 5 *Consider a series of iid coin tosses (+1 for H with probability p, -1 for T with probability q), and let S_n be the running sum. Prove that $(q/p)^{S_n}$ is a martingale with respect to the natural filtration. Find the value of C for which $C^n\lambda^{S_n}$ is a martingale, where λ is some positive constant.*

Question 6 *Prove that for any sum S_t of independent increments with finite variance, we have $\Pr(\exists t \in \mathbb{N} : S_t - t\mu > 1000\sqrt{t}) = 1$.*

Question 7 *What is the difference between a p-value and an always-valid p-value? What is the use of the latter and how do you construct it?*

Question 8 *What is the relationship of the normal mixture confidence sequence to the sequential probability ratio test (SPRT)?*