Normalized Margins and Perceptron Algorithm

Non-linear Feasibility Problems
Given n points x_1, ..., x_n ∈ R^d and labels y_1, ..., y_n ∈ {−1, 1}, find a separating hyperplane w such that:
\[ y_i(w^T x_i) \geq 1 \quad \text{for all } i \]
and
\[ \|w\|_2 = 1. \]

Algorithm 1: Perceptron Algorithm

1. Initialize w = 0.
2. For t = 1, 2, ..., do
   a. Let x_t, y_t be a misclassified point.
   b. Update w: w ← w + y_t x_t.
3. Stop.

Smoothed Normalized Kernel Perceptron (NKP)

Normalized (Kernel) Margin
Let \( F_K \) be a RKHS and \( K(x, y) = \langle x, y \rangle \).

Algorithm 2: Normalized Kernel Perceptron

1. Initialize w = 0.
2. For t = 1, 2, ..., do
   a. Let x_t, y_t be a misclassified point.
   b. Update w: w ← w + y_t K(x_t, ·).
3. Stop.

Normalized Kernel Perceptron

Von-Neumann (VN) and Gordan’s Theorem

Gordan’s Theorem
For any \( v_1, v_2 \) in \( R^d \), there exists a \( w \neq 0 \) such that:
\[ v_1^T w \geq 0 \quad \text{and} \quad v_2^T w \geq 0. \]

Von-Neumann-Gilbert Algorithm

Algorithm 3: Von-Neumann-Gilbert Algorithm

1. Solve \( \min_{w \in F_K} \langle w, y \rangle \) for some y.
2. Update w:
   a. Add y to w if \( y^T w > 0 \).
   b. Remove y from w if \( y^T w < 0 \).
3. Stop.

Iterated Smoothed NKP-VN

Algorithm 4: Iterated Smoothed NKP-VN

1. Initialize w = 0.
2. For t = 1, 2, ..., do
   a. Solve \( \min_{w \in F_K} \langle w, y \rangle \) for some y.
   b. Update w:
      i. Add y to w if \( y^T w > 0 \).
      ii. Remove y from w if \( y^T w < 0 \).
3. Stop.