36-201 Spring 1999
Solutions to Homework 9

1. <u>`Moore, 7.20 (p.423-424)`</u>.

   (a) There is a pool of 10,000 numbers to select from: (0000, 00001, ..., 9999), so the probability of any four digit number is the same, 1 in 10,000. Of course, this applies to the numbers 2873 and 9999.

   (b) People think that 2873 is more likely than 9999 because 9999 looks less random. So, as a strategy, we may choose 9999 because although it has the same chances of winning, we will win a larger amount because fewer other people will chose it.

2. <u>`Moore, 7.25 (p.425)`</u>.

   (a) The gambler is wrong because each spin is independent from the others. So it doesn't matter what he got in the previous spins, the next spin is equally likely to give a red or a black.

   (b) The gambler is wrong because in this case, the draws are not independent (each time we deliver a card, there are less cards in the pack). Actually, the fact that he already got 4 red cards diminishes the probability that he receives another red card in the next draw because there are less red cards in the pack.

3. <u>`Moore, 7.27 (p.425)`</u>.

   Although dying while playing soccer is more likely to occur than dying from the presence of asbestos, people will probably prefer to ban asbestos and not soccer. That is because they feel that dying while playing soccer is under their control, while getting cancer (and dying) from the presence of asbestos is not, regardless of probabilities. Also, banning asbestos implies a change for a third party while banning soccer restricts peoples' own activities.

   <u>`Moore, 7.28 (p.425-426)`</u>.

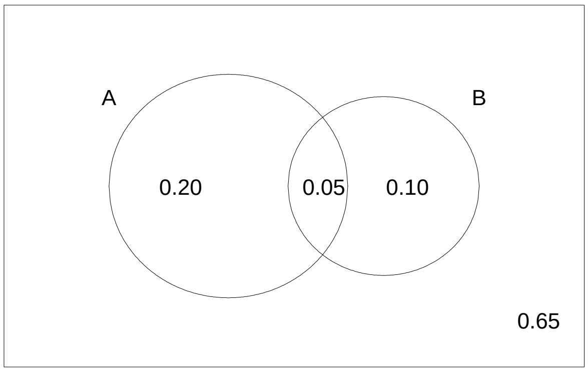
   (a) News media give more attention to airplane crashes because they are less common and more "sensational" (more people die at the same time).

   (b) That coverage makes us think that airplanes are more dangerous than cars because news media emphasize airplane crashes. We also tend to think that planes are more dangerous because they are not under our control, while driving a car looks like it is.

4. Siegel and Morgan, 21 (p.238-239).

(a) Define the events
A: *a student takes French*
B: *a student takes Statistics*.

i. We are told that 200 students out of 1,000 are taking French but not Statistics, that is a 20%. Also, 100 out of those 1,000 students are taking Statistics but not French, which is a 10%, and 50 are taking both classes, which is a 5%. So the Venn diagram looks like



iii. From the Venn diagram, we get

		French		
		Yes	*No*	
Stat	*Yes*	0.05	0.10	0.15
	No	0.20	0.65	0.85
		0.25	0.75	1.00

(b) We need $P(A \cap B^c)$ or P(French = Yes and Stat = No) $= 0.20$.

(c) We need $P(A)$ or P(French = Yes) $= 0.25$.

(d) We need $P(A \cap B)$ or P(French = Yes and Stat = Yes) $= 0.05$.

(e) We need $P(A \cup B)$ or
P(French = Yes and Stat = Yes)$+P$(French = Yes and Stat = No)$+P$(French = No and Stat = Yes)
= 0.05 + 0.10 + 0.20 = 0.35.

(f) We need $P(A^c) = 0.75$ or P(French = No) $= 0.75$.

(g) We need $P(A^c \cap B^c)$ or P(French = No and Stat = No) $= 0.65$.

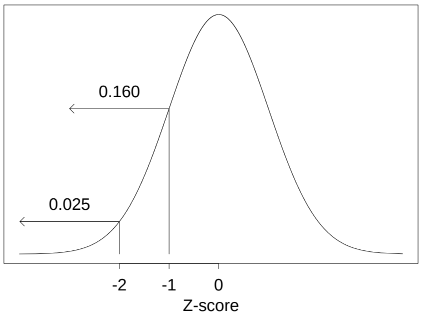
5. Siegel and Morgan, 21 (p.289).

(a) The Central Limit Theorem says that as the sample size gets larger, the distribution of the average gets closer to a normal distribution.

(b) The mean (or expected value) of the average income is the same as the mean of a single observation, that is, $32,000.

(c) The standard deviation of the average income equals the standard deviation of a single observation divided by the square root of the sample size, in this case, $9,000/5 = $1800.

(d) We first compute the Z-score of \$30,000 as $Z = (30,000 - 32,000)/1800 = -1.11$. So the probability of the average being below \$30,000 equals the probability of a standard normal being below -1.11.

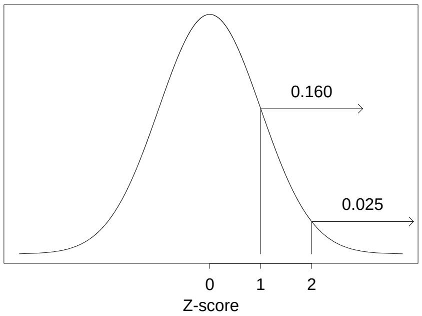


Using the 68-95-97.5 rule, we know that that probability must be between 0.025 and 0.160. Using linear interpolation we get

$$0.160 + \frac{0.160 - 0.025}{-1 + 2} \times (-1.11 + 1) = \underline{0.145}.$$

Alternatively, from the Table in Siegel and Morgan on page 262 we can also tell that the probability is <u>between 0.106 and 0.159</u>.

(e) The Z-score of \$35,000 is $Z = (35,000 - 32,000)/1800 = 1.67$. So the probability of the average being above \$35,000 equals the probability of a standard normal being above 1.67.

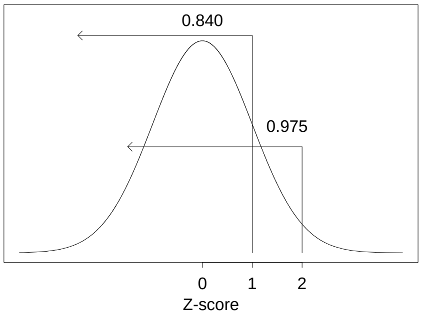


Using the 68-95-97.5 rule, we know that the probability must be between 0.025 and 0.160. Using linear interpolation we get

$$0.160 + \frac{0.160 - 0.025}{1 - 2} \times (1.67 - 1) = \underline{0.070}.$$

Alternatively, from the Table in Siegel and Morgan on page 262, we find that that probability of a standard normal being below 1.67 is 0.953. So, the probability of the average being above \$35,000 is 1 - 0.953 = <u>0.047</u>.

(f) The Z-score of \$34,000 is $Z = (34,000 - 32,000)/1800 = 1.11$. So the probability of the average being between \$30,000 and \$34,000 equals the probability of a standard normal being between -1.11 (from part(d)) and 1.11.



Using the 68-95-97.5 rule, we know that the probability of a standard normal being below 1.11 must be between 0.840 and 0.975. Using linear interpolation we get

$$0.840 + \frac{0.975 - 0.840}{2 - 1} \times (1.11 - 1) = 0.855.$$

So the probability of a standard normal being between -1.11 and 1.11 is the difference between 0.855 and 0.145 (from part(d)), that is <u>0.710</u>.