Computer Lab Exercise – Lab #9 – Partial Solutions

Question #1:

		Had Coffee? No	Had Coffee? Yes	Total
Had Dessert?	No	0.10	0.40	0.50
	Yes	0.20	0.30	0.50
Total		0.30	0.70	1.00

Question #2: Below is a generic Venn Diagram for problems with two simple events. Write a word or two describing each event in this problem, and fill in the probabilities below, using the joint probability table.

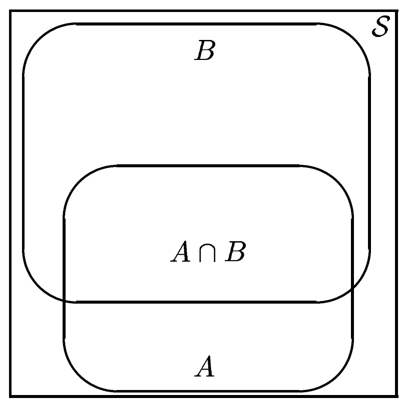
Event A, in words: Had Dessert.

Event B, in words: Had Coffee.

$P(A) =$ 0.50

$P(B) =$ 0.70

$P(A \cap B) =$ 0.30



♣ **Question #3:** The part of the diagram that should be shaded is the part of A at the bottom, that is not also in $A \cap B$. The probability that should be circled in the table is the 0.20 in the lower left corner of the table.

Question #4: $P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{\boxed{0.30}}{\boxed{0.70}} =$ 0.43

$P[B|A] = \frac{P[A \cap B]}{P[A]} = \frac{\boxed{0.30}}{\boxed{0.50}} =$ 0.60

♣ **Question #5:** You get the same answers whether you use the conditional probability calculation, or the column percent and row percent calculations, because the joint probability table just organizes the probabilities you need into a convenient form: row percents and column percents *are* conditional probabilities.

You Pay$(-)$	You Win$(+)$	Net Gain x	Probability $P(X=x)$
\$1.00	\$2.00	\$1.00	0.10
\$1.00	\$5.00	\$4.00	0.10
\$1.00	\$0.00	$-$\$1.00	0.80

Table 1: Net Gains for a lottery.

♣ **Question #6:** The expected value of your net gain is

$$(\$1.00)(0.10) + (\$4.00)(0.10) + (-\$1.00)(0.80) = -\$0.30$$

Question #7: If you played this lottery many times, on average you could expect to lose about 30 cents per play.
Question #8: The variance of your net lottery gain is

$$\begin{aligned} & (\$1.00 - [-\$0.30])^2(0.10) + (\$4.00 - [-\$0.30])^2(0.10) + (-\$1.00 - [-\$0.30])^2(0.80) \\ = \;& (1.69)(0.10) + (18.49)(0.10) + (0.49)(0.80) \\ = \;& 2.41 \end{aligned}$$

so the SD is $\sqrt{2.41} = 1.55$.
♣ **Question #9:** The only ticket with a net gain of at least \$3.00 is the \$5.00 payoff ticket (net gain \$4.00) so P[net gain at least \$3.00] $= 0.10$.
Question #10: P[net gain less than \$3.00] $= 1 - P$[net gain at least \$3.00] $= 1 - 0.10 = 0.90$.
Question #11:

$n!$	**Formula**	**Value**	**Meaning**
5!	$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$	120	Number of ways to arrange 5 objects
6!	$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$	720	Number of ways to arrange 6 objects
7!	$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$	5040	Number of ways to arrange 7 objects
8!	$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$	40320	Number of ways to arrange 8 objects

Question #12: $\binom{4}{1} = \frac{4!}{1!3!} = 4.$
This is the number of ways to get exactly 1 heads in 4 tosses.

$$\binom{8}{6} = \frac{\boxed{8!}}{\boxed{6!}\,\boxed{2!}} = \frac{(40320)}{(720)(2)} = 28.$$

This is the number of ways to get exactly 6 heads in 8 tosses.
♣ **Question #13:** $\binom{8}{6}(0.15)^6(1-0.15)^{8-6} = (28)(0.15)^6(0.85)^2 = 0.00023.$
Question #14: $\binom{8}{0}(0.15)^0(0.85)^6 = (1)(1)(.85)^6 = 0.38$
Question #15: $\mu_X = np = 8 \cdot (0.15) = 1.2$; $\sigma_X = \sqrt{np(1-p)} = \sqrt{8 \cdot (0.15) \cdot (0.85)} = 1.01.$