Confidence Intervals and a Peek at the Future

Confidence intervals

Confidence intervals are like the confidence statements we have been making about survey results.

Example. You take a random sample of 36 athletes who turn out for men's basketball at all major US Universities. Your sample averages 75 inches tall with a SD of 3 inches.

To construct an approximate 95% confidence interval using the CLT:

1. Calculate the *standard error of the mean*:

$$SE = SD/\sqrt{N} = 3/\sqrt{36} = 0.5$$

2. Using the 68–95–99.7 rule, the interval is

$$\begin{aligned} \overline{X} - 2 \cdot SE \quad &\text{to} \quad \overline{X} + 2 \cdot SE \\ 75 - 2 \cdot (0.5) \quad &\text{to} \quad 75 + 2 \cdot (0.5) \\ 74 \quad &\text{to} \quad 76 \end{aligned}$$

3. So we can say with 95% confidence that the mean height of all athletes who turn out for men's basketball at major US universities is somewhere between 74 and 76 inches.

Notes:

- For 68% confidence, use $\overline{X} \pm 1 \cdot SE$
- For 99.7% confidence, use $\overline{X} \pm 3 \cdot SE$
- For other confidences, use $\overline{X} \pm z \cdot SE$, where z comes from a table or from interpolation.
- Sometimes $2 \cdot SE$ is called a *"margin of error"*, like sample surveys.

What does “confidence” mean?

Exact interpretation:

> *The recipe on the previous page for 95% intervals ($\overline{X} \pm 2 \cdot (SE)$) will produce a numerical interval that traps the population mean, in 95% of all SRS's taken from that population.*

Approximate interpretation:

> *The chances are about 95% that the population mean is in the interval you calculated, using the recipe.*

Where are we going?

- In some situations there is a more exact answer than the CLT (68–95–99.7 rule) can give us. This calls for the "t-distribution", which we will talk about next week.

- Confidence intervals can be used to test hypotheses about data.

- IDEA:

  - A hypothesis about a population gives a prediction about the sample mean in a sample from the population
  - Compute a confidence interval from the sample.
  - If the confidence interval makes the prediction look wrong, this undermines the hypothesis.

Example:

- In the “SSHA” example, one “hypothesis” about the data was that older students would have the same attitudes about school (higher SSHA scores) as regular college-age students.
- What is a confidence interval for mean SSHA score in the population of older students?

- What does this say about the hypothesis?

- Next week and the following week we will formalize this idea, called “hypothesis testing” in statistics.