36-201: Statistical Reasoning

1. [12 points; 3 for each part]

(a) As you toss a fair coin (probability of heads = 0.5) more and more times, the *fraction* of tosses that are heads will typically get closer and closer to 1/2, if the tosses are independent of one another. (mark one box only)

☒ True.
☐ False.

(b) As you toss a fair coin (probability of heads = 0.5) more and more times, the *number* of heads will typically get closer and closer to half the number of tosses, if the tosses are independent of one another. (mark one box only)

☐ True.
☒ False.

(c) The population of students at a certain college spends an average of **2** hours outside of class on homework assignments, for every hour in class lecture or lab. Joe polls a random sample of 36 students in one of his classes, and he discovers that the average time these students spend outside of class on assignments is **2.6** hours per hour in class.

The number **2** is a (mark one box only)

☐ Statistic.
☒ Parameter.

(d) In part (c), the number **2.6** is a (mark one box only)

☒ Statistic.
☐ Parameter.

2. [26 points] This past Sunday morning I was making oatmeal (honest!) and I noticed the following story printed on the box:

> *"Great News! Town Confirms Oatmeal May Help Lower Cholesterol"*
>
> "100 people in Lafayette, Colorado volunteered to eat a good-sized bowl of oatmeal for 30 days to see if simple lifestyle changes—like eating oatmeal—could help reduce cholesterol. After 30 days, 98 lowered their cholesterol. With these great results, the people in Lafayette proved to themselves that simple lifestyle changes can make a real difference."

Following up on this study at `http://www.quakeroats.com`, I found that participants were recruited through "word-of-mouth buzz and ads in local papers", and that the average drop in cholesterol for the group was about 25 mg/dl (milligrams per deciliter). This result is consistent with the general knowledge that dietary changes and exercise can be effective in reducing cholesterol levels. The National Heart, Lung and Blood Institute recommends that healthy people keep total cholesterol to 200 mg/dl or less, and that above 240 mg/dl is unhealthy, so a reduction of 25 mg/dl sounds pretty good.

(a) [4 points] The study in this story was an (mark one box only)

- [X] Experiment. The treatment was decided beforehand.
- [ ] Observational Study.
- [ ] [Can't tell from the information given.]

(b) [6 points] A longer description of the study at `http://www.quakeroats.com` contains the passage "Mitch Kanter, Ph.D., director of clinical research for Quaker Nutrition, the division of Quaker Oats that analyzed the Lafayette data, said that factors other than oatmeal consumption contributed to the impressive numbers." Name any two plausible factors—*besides eating the oatmeal itself*—that could have contributed to the reduced cholesterol levels and explain why they are plausible (one sentence for each factor).

There are many possible answers. Here are some

- The subjects were sensitized to their eating habits and changed their diets in other ways that would reduce cholesterol.
- The subjects were sensitized to health habits (oatmeal is a "healthy food") and so they decided to increase their daily exercise level.
- The subject made some other lifestyle change, like quitting smoking, that the student thinks is associated with lowering cholesterol (student must write that he/she thinks there's a connection).
- Only subjects who were also taking a cholesterol-lowering medicine participated in the study (perhaps because they were generally motivated to reduce their cholesterol level, etc.).

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(c) [4 points] Who is likely to have conducted the study? (mark one box only)

- ☒ The Quaker Oats Company.
- ☐ An independent research organization.
- ☒ Can't tell from the information given.

This is not acceptable.

(d) [4 points] What can you *definitely* conclude, on the basis of this study alone, about the effects of eating oatmeal on lowering cholesterol levels? (mark one box only)

- ☐ Eating a bowl of oatmeal every day reduces cholesterol levels by a significant amount.
- ☐ Making people aware of their health habits by having them to eat oatmeal every day will cause them to change their lifestyles, resulting in lower cholesterol.
- ☐ The people who volunteered for this study were ready to change their lifestyles to reduce cholesterol levels.
- ☒ Maybe eating oatmeal or changing your lifestyle can reduce your cholesterol levels, but it's difficult to evaluate these claims from this study.

(e) [8 points] Design a simple study that would be better for determining whether eating a bowl of oatmeal every day reduces cholesterol levels, by answering the following questions (one sentence each):

I expect/hope that most answers are like the following; let me know if some are not...

- How should subjects in your study be chosen:

  Take a representative sample from the population of interest.
- What treatment(s) should be given:

  There should be at least two treatments, an "oatmeal" treatment and a "non-oatmeal" treatment.

  The non-oatmeal treatment could be a "no change in diet" treatment, or it could be an alternative diet item, like bagels or poptarts or whatever..., or it could be a "thou shalt not eat oatmeal" treatment, for example.
- How should treatment(s) be assigned to subjects:

  Randomly of course!
- How would you control for the confounding effects you listed in part (b) above:

  The answer should be relevant to the student's previous answers. Here are some answers that are relevant to my answers above.
  - Getting a representative sample and randomly assigning them to treatments solves the "self selection bias" problem.
  - To solve the problem of having the oatmeal sensitize the subjects to other lifestyle changes (diet, exercise, smoking, etc.), every treatment group should get lots of informational seminars on the benefits of lifestyle changes for reducing cholesterol, and the benefits of lowering your cholesterol.

3. [25 points] Telephone polling of a thousand people or so by random digit dialling is currently the most common method used by polling organizations such as Gallup, Harris, and the major news networks such as CBS and ABC. With the World Wide Web's (WWW) tremendous growth, however, an online poll could compile literally tens of thousands of opinions quickly and at a fraction of the cost of traditional telephone surveys. As part of a report on the potential uses of WWW-based surveys, the Pew Research Center for People and the Press (`http://www.people-press.org`) compared results of several different surveys conducted immediately following Clinton's televised address in mid-August 1998 admitting an improper relationship with Monica Lewinsky. The reversal of percentages favoring a Clinton resignation in the surveys in Table 1 is striking.

Should Clinton Resign?	America Online	— Trad. Telephone Polls — Gallup	ABC	CBS
Yes	52%	23%	36%	21%
No	48%	72%	61%	61%
Don't know/Other	0%	5%	3%	18%
Total	100%	100%	100%	100%
Sample Sizes	$(N = 118,565)$	$(N = 633)$	$(N = 508)$	$(N = 944)$

Table 1: America Online (AOL) results based on responses from 118,565 AOL users who voluntarily answered the question. Results from Gallup, ABC, and CBS are from national telephone surveys conducted at approximately the same time, following Clinton's televised (mid-August 1998) address.

(a) [10 points] Samples from telephone surveys can be treated as approximately SRS's. Compute a margin of error for the Gallup poll, and make a confidence statement about the percent of US adults who favored Clinton resigning at that time (fill in the blanks below, use the space provided for your work).

If you use $\hat{p} = 0.5$ in the SE calculation:	If you use $\hat{p} = 0.21$ in the SE calculation:
$SE = \sqrt{(0.5)(0.5)/633} = 0.01987$	$SE = \sqrt{(0.21)(0.79)/633} = 0.01619$
$MOE = 2 \cdot SE = 3.97\%$	$MOE = 2 \cdot SE = 3.24\%$
So interval runs from	So interval runs from
17.03% to 24.97%	17.76% to 24.24%

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(b) [5 points] The sample size for the CBS survey was almost twice that of the Gallup survey. *Check* *<u>all</u>* *that apply*:

- ☐ The larger sample size for the CBS survey will result in less biased survey results.
- X The larger sample size for the CBS survey will result in more precise survey results.
- ☐ You can't compare the bias and precision of the two surveys, because (fill in the blank):

  ____ .

(c) [5 points] The sample size for the America Online (AOL) survey was much larger than for the CBS survey. *Check* *<u>all</u>* *that apply*:

- ☐ The larger sample size for the AOL survey will result in less biased survey results.
- ☐ The larger sample size for the AOL survey will result in more precise survey results.
- X You can't compare the bias and precision of the two surveys, because (fill in the blank):

  <u>The AOL survey is not a random sample (e.g.)</u> .

(d) [5 points] Write down one plausible difference between the people who responded to the AOL survey and those who responded to the traditional telephone surveys, that helps explain the different survey results. (One sentence only.)

For example...

- The AOL'ers are a self-selected sample and hence not representative of the population of US adults, say.
- The AOL'ers are more well-to-do, techie-nerd types, etc., etc.

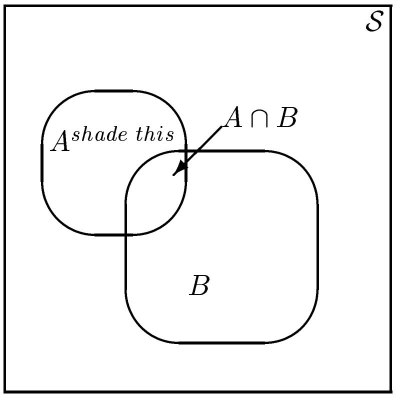
4. [25 points] In the general U.S. population, the rate of cases of HIV (the virus that causes AIDS) is roughly 30 people per 100,000, so the probability of a randomly sampled US citizen having HIV is roughly 0.0003. A medical test is being developed to detect the presence of the HIV virus; but as with every medical test, the test reads "positive" for both HIV carriers (true positives) and some non-carriers (false positives). The probability of a positive reading for this test is about 0.0009. Finally the probability that a randomly-selected person would both have HIV and test positive by this test is about 0.00029.

(a) [5 points] Here is a joint probability table for this problem. Fill in the missing cells.

		(B) Tests Positive?		
		Yes	No	Total
(A) Has HIV?	Yes	0.00029	$\boxed{0.00001}$	0.00030
	No	0.00061	$\boxed{0.99909}$	0.99970
Total		0.00090	$\boxed{0.99910}$	1.00000

(b) [4 points] Below is a generic Venn Diagram for problems with two simple events, A and B. Let A = "Has HIV" and let B = "Tests Positive". Shade in the part of the diagram corresponding to "A and not B". From the joint probability table above, write the probability of A and not B here:

P[A and not B] = 0.00001



(c) [8 points] Of the people who have HIV, what percent would test positive by this test?

$$P[B|A] = \frac{P[A \cap B]}{P[A]} = \frac{\boxed{0.00029}}{\boxed{0.00030}} = 0.97$$

Would you say this is an effective test? Explain why or why not (one sentence only).

Yes. Nice high rate of detection of cases.

(d) [8 points] Of the people who test positive by this test, what percent would actually have HIV?

$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{\boxed{0.00029}}{\boxed{0.00090}} = 0.32$$

Would you want to implement this test for everyone in the US population? Explain why or why not (one sentence only).

No. Too many (2/3) of people who test positive are not really HIV carriers.

5. [12 points; 3 for each part] The population of students at a certain college spends an average of 2 hours outside of class on homework assignments, for every hour in class lecture or lab. The SD for all students is 1.2 hours. Halfway through the introductory statistics course there, Joe begins to suspect that the time spent working on assignments for this course does not follow the distribution of the whole college. He decides to poll a random sample of 36 of his classmates, and he discovers that the average time these students spend outside of class on assignments is 2.6 hours per hour in class.

(a) The standard error of the mean in Joe's sample is (mark one box only)

☐ 1.2 hours.
☒ 0.2 hours.

(b) What makes it OK to apply the 68–95–99.7 rule to the sample average and its standard error here? (mark one box only)

☒ The Central Limit Theorem
☐ The Law of Averages

(c) How surprising is the average homework load for students in Joe's statistics class? Using the 68–95–99.7 rule, the probability of seeing a sample average equal to or greater than 2.6, from a random sample of 36 students from the *whole college*, is approximately (mark one box only)

☒ 0.15%
☐ 33.0% (Acceptable if (a) was 1.2)
☐ 30.9% (Acceptable if (a) was 1.2)

(d) This makes it (mark one box only)

☐ somewhat likely (Acceptable if (a) was 1.2)
☒ unlikely

that the students in Joe's introductory statistics course have the same homework load as the college as a whole.