HW 11 SOLUTIONS

1. `Moore, p. 471, #8.13, and Moore, p. 481, #8.21 [but just use 2 instead of 1.96]`

8.13 Comparing confidence intervals.

(a) For a fixed sample size, to increase your confidence that you have "trapped" the true proportion in the confidence interval, you must widen the confidence interval.

(b) For a fixed confidence level, as you increase the sample size you narrow the confidence interval (this is the *square root law* in action).

8.21 We want to set $0.02 = 2\sqrt{(0.5)(0.5)/n}$ and solve for n:

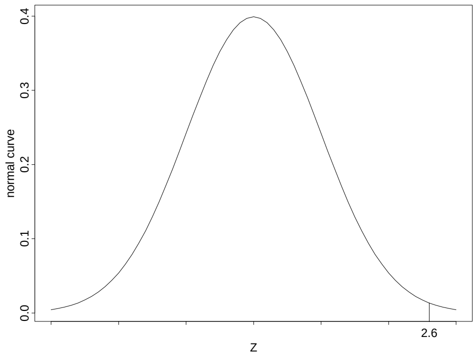
$$\begin{aligned} 0.02 &= 2\sqrt{(0.5)(0.5)/n} \;=\; 1/\sqrt{n} \\ \sqrt{n} &= 1/0.02 \;=\; 50 \\ n &= 50^2 = \boxed{2500}. \end{aligned}$$

So, to be able to report a 95% confidence interval with a margin of error of $\pm 2\%$, you should take a sample of 2500 voters. *Note: If you use 1.96 instead of 2, you will get* $n = 2401$.

2. `Moore, p. 481, #8.23 [use a Normal table (table B in the back of Moore is fine), not`
`interpolation!] and Moore pp. 481-482, #8.24.`

8.23 (a) For a 99% confidence interval we want $\$332 \pm z \cdot \$108/\sqrt{200}$. From Table B in the back of Moore (or one of the normal tables in Siegel and Morgan), we see z should be about 2.6 [reason: in the figure below, $z = 2.6$ divides the area in the normal curve into approximately 99.5% and 0.5%, so the area from -2.6 to $+2.6$ will be approximately 99%], so the interval is

$$\begin{array}{rcl} \$332 - (2.6) \cdot \$108/\sqrt{200} & \text{to} & \$332 + (2.6) \cdot \$108/\sqrt{200} \\ \$312.13 & \text{to} & \$351.86 \end{array}$$



(b) This is a one-treatment study; the increase in spending might be due to dropping the credit card fee, or it might be due to some confounding/lurking factors like season (customers generally spend more at christmas for example). A better study would be:

- Randomly divide the customers in the study into at least two groups, say, a "treatment" group that has its credit card fees waived, and another "control" group that does not have its fees waived.
- To help ensure that the the Hawthorne effect (placebo effect) affects both groups equally, subjects in both groups should be sent informational / promotional materials letting them know that they are in a study of credit card spending habits, and encouraging them to use the card "as they normally would". This will have the effect of drawing both groups' attention to their cards.

8.24 (a) At the beginning of the study, the placebo group is just a sample from the population, so we can use its data to estimate the population blood pressure. A 95% confidence interval is $114.9 \pm 2 \cdot 9.3/\sqrt{27}$, so we are 95% confident that the population blood pressure runs from about 111.32 to about 118.48

(b) We are assuming that the 27 men assigned to the placebo group are like an SRS from the population. Even though the 54 men were split randomly into placebo and treatment groups, if the origianl 54 aren't like an SRS, then the 27 in the placebo group can't be, either.

Note: In both 8.23 and 8.24 we are using z-scores from the normal distribution to construct confidence intervals, even though the SD's come from the sample and not the population. In a more advanced course (36-202) you would learn to use t-scores from the t distribution to widen the intervals slightly to ensure 95% confidence with these estimated SD's. As we've seen in class though, there is little difference between our intervals and intervals constructed with the t-distribution, as long as the sample size is fairly large.

3. `Moore, p. 491, #8.31 and #8.33`

8.31 (a) p is the true proportion of students *at that university* [not nationally] that want to be rich.

(b) $\mathbf{H_0}$**:** The null hypothesis is that p is equal to the national proportion. 0.73.

$\mathbf{H_a}$**:** The alternative hypothesis is that it's not.

(c) This is slightly tricky.

- $\hat{p} = 132/200 = 0.66$. This is the proportion of the sample of 200 students at that university that want to be rich.
- The P-value is the probability, assuming H_0 is true, of seeing a sample whose $\hat{p}$ is farther from 0.73 than the value $\hat{p} = 0.66$ that we actually observed.

(d) The P-value of 0.037 is a low number: there is only a 3.7% chance that a population whose true proportion was 0.73 could generate a sample with a $\hat{p}$ farther from 0.73 than our observed value of 0.66. This means 0.66 would be an *unusual* if H_0 were true at that university. This unusualness calls into question the assumption that the population of students at this college actually do have a proportion 0.73 who want to be rich.

8.33 Only the first sentence in each bullet below is required.

- A result whose P-value is 0.037 is "significant at the 0.05 level", since $0.037 < 0.05$. This means we can assert with 95% confidence that the null hypothesis is false for these data.
- It is *not* significant at the 0.01 level, since $0.037 > 0.01$. We cannot assert with 99% confidence that the null hypothesis is false for these data.

4. `Moore, p. 503, #8.47`

- The null hypothesis H_0 is that the mean oxygen content in this stream is 5 mg/l (or greater). The alternative hypothesis H_a is that it's less than 5 mg/l.
- Assuming that the null hypothesis H_0 is true, we calculate the P-value as

$$\begin{aligned} P[\bar{X} < 4.63] &= P\left[\frac{\bar{X} - 5}{0.92/\sqrt{45}} < \frac{4.62 - 5}{0.92/\sqrt{45}}\right] \\ &= P[Z < -2.77], \end{aligned}$$

  which is somewhere between 0.0035 and 0.0026, according to Table B in the back of Moore. These are *very small* probabilities, so it would be *very unusual* to see a mean oxygen content of 4.62 mg/l in a sample of 45 stream locations, assuming that the true mean is 5 mg/l.
- This is strong evidence against H_0, so *we do not believe that the overall mean oxygen content for this stream is as high as 5 mg/l.*

Note: Because we are using the sample SD instead of the population SD, a more sophisticated answer would use the t distribution to evaluate the P-value, rather than the normal distribution. But the answer would be virtually the same in this case.

5. Siegel and Morgan, pp. 402-403, #4. Note that all of the calculations you need have already been done and are displayed at the top of p. 403. Your job is to read and interpret the display, to answer the questions in this problem.

   I have re-created the display at the top of p. 403 here and numbered the lines so I can refer to them in my answers below:

```
Line 1:  MTB > TwoT 95.0 'ClawArc' 'Species';
Line 2:  SUBC>   Alternative 0;
Line 3:  SUBC>   Pooled.
Line 4:
Line 5:  Two Sample T-Test and Confidence Interval
Line 6:
Line 7:  Two sample T for ClawArc
Line 8:  Species       N      Mean     StDev   SE Mean
Line 9:  31           10    161.10      7.94       2.5
Line 10: 17           10     77.60      9.05       2.9
Line 11:
Line 12: 95% CI for mu (1) - mu (2): ( 75.5,  91.5)
Line 13: T-Test mu (1) = mu (2) (vs not =): T= 21.94  P=0.0000  DF=  18
Line 14: Both use Pooled StDev = 8.51
```

   (a) The average claw arc for species 31 is 161.10 (Line 9).

   (b) The average claw arc for species 19 is 77.60 (Line 10).

   (c) The difference is $161.10 - 77.60 = 83.5$.

   (d) The 95% confidence interval for the differenc is $(75.5, 91.5)$ (Line 12).

   (e) The value 0 (no difference) is not in this interval.

   (f) Yes we can reject the null hypothesis H_0 that there is no difference in the two species' claw arcs, with confidence 95% (the confidence of our confidence interval).

   (g) We already know from the previous part that the difference is statistically significant at the 0.05 level, since the 95% confidence interval didn't include 0. But from Line 13 we can see that the P-value is essentally 0. So this difference is statistically significant at practically any level.

   (h) The test statistic is $T = 21.94$ (think of comparing this to the 68–95–99.7 rule!).

   (i) Assuming H_0 is true, $P[T \geq 21.94] \approx 0$ (Line 13).

   (j) The pooled estimate of standard deviation is 8.51 (Line 14).