INTRODUCTION TO STATISTICAL REASONING

36-201 **Lab #12 -Partial Solutions**

Question #1 The *population* is all U.S. high school students. The parameters of interest are the percentage of females and males in the population who say biology is their favorite subject.

H_0 : The percentage of female high school students who claim biology is their favorite subject is less than or equal to the percentage of male high school students who claim biology is their favorite subject.

H_a : The percentage of female high school students who claim biology is their favorite subject is greater than the percentage of male high school students who claim biology is their favorite subject.

Example #1: Faculty Salaries

Questions #5-6 A 95% confidence interval for the difference in the mean faculty salary between females and males between –$96 and $4825. This means that we are 95% confident that the true difference in salaries falls in this interval. Since the value $0 indicates no difference in the average salaries, and since this interval contains $0, we cannot rule out (at the 95% level) the possibility that there is no difference in the average salaries between the female and the male faculty.

Question #7 On the other hand a 90% confidence interval for the difference in the mean faculty salary is between $322 and $4407. Since this interval does not contain $0, we are 90% confident that there is a difference in average faculty salary between females and males, and further, the females make on the average between $4407 and $322 less money than the male faculty. This interval is narrower than the 95% interval (and therefore does not contain 0) but we are less confident that the true difference is contained in the interval.

Question #8 There are two large positive outliers in the female group; their effect on the average female faculty salary is to make it larger and therefore closer to the average male salary. This is why the 95% interval suggests that there is no difference in the average salary between the two groups, whereas the boxplot suggests that there is.

Questions #9–11 The null hypothesis is that there is no difference between the average female faculty salary and the average male faculty salary. ($H_0 : \mu_F - \mu_M = 0$ or $H_0 : \mu_F = \mu_M$.)
The alternative hypothesis is that the average salary for female faculty is different from the average salary for male faculty. ($H_a : \mu_F - \mu_M \neq 0$ or $H_a : \mu_F \neq \mu_M$.)

The p-value for this test is 0.0589. This means that the data offer mild to moderately strong evidence against the null hypothesis of no difference in average salaries between females and males. Again, the p-value is sensitive to the outliers in the data and without the outliers the evidence against H_0 would be much stronger.

Question #12 When we constructed the confidence interval, we checked to see whether a specific value, 0, was contained in the interval. The 95% interval did contain 0 but the 90% interval did not. These results are consistent with the p-value from the test which indicates that there is mild to moderately strong evidence against the null hypothesis that there is no difference in average salary.

The boxes in the boxplots, representing the middle 50% of each respective distribution of salary, overlap a little as do the 95% confidence intervals for the medians. This again suggests that there is some difference between the female faculty's distribution of salary and the male faculty's distribution but perhaps not a very strong difference. However, the major features that we notice in the boxplots are the 2 large positive outliers in the female's distribution which may have a large effect on the confidence intervals and the p-value.

Example #2: Anorexia Data

Question #13 The *response* variable is weight gain, which is a quantitative variable. The *explanatory* variable is treatment group, which is a categorical variable with two levels, behavioral counseling and psychodynamic counseling.
This study is an *experiment* since the investigator is manipulating which treatment each woman receives by randomly assigning each woman to a treatment group.

Question #14 The *null hypothesis* is that the average weight gain for women who receive behavioral counseling is the same as the average weight gain for the women who receive psychodynamic counseling. The *alternative hypothesis* is that the average weight gain for women who receive behavioral counseling is different from the average weight gain for women who receive pyschodynamic counseling.

Questions #18-19 We are 95% confident that the difference in the average weight gain for women who receive behavioral counseling is between 0.43 and 3.07 pounds greater than the women who receive psychodynamic counseling. Since this interval does not include 0 and both endpoints of the interval are positive, we would conclude with 95% confidence that the woman who receive behavioral counseling do better than the women who receive psychodynamic counseling.

Question #20 The p-value for the test of the null hypothesis of no difference in weight gain is 0.011. This means that the data provide pretty strong evidence against the null hypothesis, indicating that the data support the alternative hypothesis that the two therapies differ in average weight gain, and suggest that behavioral therapy is better.

Question #21 Even though the boxes in the boxplot overlap some, Q_3 for the psychodynamic group is still less than the median weight gain for the behavioral group. Further, the 95% confidence interval for the median in each respective group do not overlap, indicating a difference in the medians. So the evidence from the informal analysis based on the boxplots agrees with the formal analyses based on confidence intervals and p-values. We also note that the distributions of weight gain do not have any outliers and do not look terribly skewed supporting the validity of the confidence intervals and p-values. Finally, the reduced standard deviation and higher Q_1 for the behavioral group means that behavioral counseling more consistently produces a good result.