36-463/663: Multilevel & Hierarchical Models
Fall 2016
HW07 – Due Tue 18 Oct 2016

Announcements

- Please submit hw online at blackboard as usual.
- Reading: (no new reading this week)

Exercises

1. The data mn-radon.txt is in the hw07 area of the class website. Load it and set up variables and MLM's as follows:

```
library(lme4)
mn.radon <- read.table("mn-radon.txt")
attach(mn.radon)
n <- length(radon)
J <- length(unique(county))
y <- log.radon
x <- floor
u.full <- log.uranium
M0 <- lmer(y ~ 1 + (1 | county) )
M1 <- lmer(y ~ x + (1 | county) )
M2 <- lmer(y ~ x + u.full + (1 | county) )
M3 <- lmer(y ~  x + u.full + (1 + x | county) )
```

   (a) Compute AIC, BIC, and DIC for all four models (Note: AIC and BIC are computed when you compare models using the `anova()` function[1]; the `display()` function[2] computes another version of AIC as well as DIC). For each method (AIC, BIC, and DIC) indicate which model is best, using the rule of thumb that reductions of at least 3 are needed to prefer a model.

   (b) You should have come up with more than one model in part (1a). For each model you came up with plot
   - marginal residuals vs marginal fitted values
   - conditional residuals vs conditional fitted values
   - random-effect residuals vs random effect fitted values
   - qqnorm plot of any residuals which are supposed to be approximately normal *(Hint:* `qqnorm(mydata); qqline(x)` *will produce a qq plot with a guideline).*

[1] . . . as long as `lme4` is loaded. . .
[2] . . . as long as `arm` is loaded. . .

- lattice plots whenever they are helpful (feel free to use either xyplot from library(lattice) or ggplot from library(ggplot2)).

and try to determine which model seems to be best.

Note: To help with this, the file `lmer-residuals.r` *defines functions that extract all three kinds of residuals and fitted values from a fitted lmer object. This file is in the hw07 area of the class website. For examples of how to plot with them, see lecture 13, week 07.*

(c) Is there anything else you can think of to determine which model seems to be best? If so, try it.

2. This exercise is for practice in moving between the three different representations for multilevel models: the *multilevel linear model* representation, the *variance components* represenatation, and the *hierarchical (Bayes) representation*.

For each of the four models below,

(i) state which representation it is in;

(ii) write the model in the other two representations; and

(iii) write the `lmer` model notation for the model.

Be careful about notation, especially indexing subscripts like i and j and $j[i]$. State assumptions about distributions carefully, as well.

(a)
$$\begin{aligned} y_i &= \beta_0 + \eta_{j[i]} + \varepsilon_i, \\ \varepsilon_i &\sim N(0, \sigma^2) \\ \eta_j &\sim N(0, \tau^2) \\ & \quad i = 1, \ldots, n; \; j = 1, \ldots, m \end{aligned}$$

(b)
$$\begin{aligned} y_i &\sim N(\alpha_{0j[i]} + \alpha_1 x_i, \sigma^2) \\ \alpha_{0j} &\sim N(\beta_0 + \beta_1 z_j, \tau^2) \\ & \quad i = 1, \ldots, n; \; j = 1, \ldots, m \end{aligned}$$

(c)
$$\begin{aligned} y_i &= \beta_{00} + \beta_{10} x_i + \beta_{11} x_i z_{j[i]} + \eta_{0j[i]} + \eta_{1j[i]} x_i + \varepsilon_i \\ \varepsilon_i &\sim N(0, \sigma^2) \\ \eta_{0j} &\sim N(0, \tau_0^2) \\ \eta_{1j} &\sim N(0, \tau_1^2) \\ & \quad i = 1, \ldots, n; \; j = 1, \ldots, m \end{aligned}$$

(d)

$$
\begin{aligned}
y_i &= \alpha_{0j[i]} + \alpha_{1j[i]} x_i + \varepsilon_i \\
\alpha_{0j} &= \beta_{00} + \beta_{01} z_j + \eta_{0j} \\
\alpha_{1j} &= \beta_{10} + \beta_{11} w_j + \eta_{1j} \\
\varepsilon_i &\sim N(0, \sigma^2) \\
\eta_{0j} &\sim N(0, \tau_0^2) \\
\eta_{1j} &\sim N(0, \tau_1^2) \\
& \quad i = 1, \ldots, n; \; j = 1, \ldots, m
\end{aligned}
$$

Note: When going from the variance components representation to the hierarchical linear models representation, please put variables that only vary from group to group in the group level (level 2) regression (mathematically there is nothing wrong with putting them, appropriately indexed, in the individual level (level 1) regression, but it is less confusing to keep them in the group level regression).