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Cost-benefit analysis of local decisions, with application to home radon measurement and remediation*

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Abstract

This paper examines the decision problems associated with measurement and remediation of environmental hazards, using the example of indoor radon (a carcinogen) as a case study. Innovative methods developed here include (1) the use of results from previous statistical analysis to allow recommendations to vary geographically, (2) graphical methods to display the aggregate consequences of decisions by individuals, and (3) alternative parameterizations for individual variation in the dollar value of a given reduction in risk. We perform cost-benefit analyses for a variety of decision strategies, as a function of home types and geography, so that measurement and remediation can be recommended where it is most effective. We also briefly discuss the sensitivity of policy recommendations and outcomes to uncertainty in inputs. For the home radon example, we estimate that if the recommended decision rule were applied to all houses in the United States, it would be possible to save the same number of lives as with the current official recommendations for about 40% less cost.

Keywords: Bayesian decision analysis, hierarchical models, small area decision problems, value of information

1 Introduction

1.1 Decision-making for environmental hazards

Associated with many environmental hazards is a decision problem: whether to (1) perform an expensive remediation to reduce the risk, (2) do nothing, or (3) take a relatively inexpensive measurement of the risk and use this information to decide whether to (a) remediate or (b) do nothing. This decision can often be made at the individual, household, or community level. Performing this decision analysis requires estimates for the risks. In particular, the more localized are the risk estimates, the more feasible it is to construct localized decision recommendations that allow attention and effort to be focused on the individuals, households, and communities at most risk.

In this paper, we present an analysis of the remediation/measurement decision problem in the context of a hierarchical model for estimating risk as a function of location and various covariates. We develop and illustrate our method for the problem of home radon, a recognized cancer risk, for which appropriate measurement and remediation strategies have been and continue to be the subject of debate. In addition

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to its own importance, the radon problem shares several features with other environmental hazards: (a) the risks are geographically dispersed but have strong spatial patterns; (b) information exists to identify risky areas, but one cannot easily identify individual households at high risk; (c) it is possible to perform local measurements to identify the risks of individual households, but it would be expensive to measure every household.

1.2 Our recommended approach

Our approach to hierarchical decision analysis has four steps. First, a hierarchical model is fit to available data, resulting in a posterior distribution for exposure to the environmental hazard for any given household, as a function of locality and other household information. Second, the problem of "decision making under certainty" is set up: the tradeoff between dollars and lives implies a willingness to remediate at an *action level* R_{action}, so that if the true exposure were known, one would remediate if and only if it exceeds R_{action}; depending on variations in risks and risk preferences, R_{action} can vary among households. Third, the "decision making under uncertainty" problem is solved: for any household, the measurement/remediation decision is a function of its R_{action} and its posterior distribution of exposure level. If additional information is available at the household level—for example, a previous radon measurement—this can be incorporated into the posterior distribution. The fourth step of our analysis is to evaluate the effect of various decision recommendations, in terms of expected lives saved and expected costs, if applied within a larger geographic area (for example, the entire U.S.). Results can also be expressed in terms of expected marginal and aggregate cost per life saved. As always in decision analysis, sensitivity analysis is then done to see how the estimated costs and lives saved vary when assumptions are perturbed.

With the exception of the hierarchical modeling, the above steps follow the standard paradigm of Bayesian decision analysis (see, e.g., Dakins et al., 1996, and Englehardt and Peng, 1996, for a recent review and examples). The hierarchical model changes the decision analysis in two important ways. First, the decision recommendations are no longer uniform across the U.S. (or large regions of the U.S.); rather, recommended decisions vary geographically (in our analysis, by county). This is more appealing than a single nationwide recommendation (see, in particular, Evans et al., 1988, who recognize that precise modeling of radon levels should allow targeted recommendations) but requires more care in summarizing the decision analysis. Second, the aggregate outcomes of decision strategies can no longer be trivially derived from individual recommendations. We compute expected costs and lives saved by simulation using our posterior distribution. In addition, the aggregate effects themselves have more complexity—as more information is included in the hierarchical model, we can more effectively identify the high-radon houses to target in the decision strategy.

1.3 Outline of this paper

We develop our hierarchical approach to decision analysis in the context of measurement and remediation of home radon. Sections 2 and 3 of this paper provide background on the indoor radon problem and our previous work using hierarchical modeling of radon survey data to identify the houses that are likely to have high radon levels given information at the geographical and house level. Section 4 addresses the measurement and remediation decision for individual homeowners, and Section 5 presents the estimated aggregate consequences of following the recommended strategy and various alternatives if applied throughout the United States. For both the individual decisions and aggregate consequences, we develop a series of graphical displays that are potentially useful for hierarchical decision problems in general. In Section 6 we explore the sensitivity of our results to assumptions, and we conclude in Section 7 with a discussion of the specific relevance of our methods to the indoor radon problem and the general applicability to hierarchical decision problems.

2 The radon problem and available data

2.1 Health effects

Radon is a carcinogen—a naturally occurring radioactive gas whose decay products are also radioactive—known to cause lung cancer in high concentration, and estimated to cause several thousand lung cancer deaths per year in the U.S. (see Nazaroff and Nero, 1988, for an overview of the radon problem; Cole, 1993, for a discussion of the governmental response to it; and National Research Council, 1988, for an influential official report).

The well-documented dose-dependent excess of lung cancer among underground miners exposed to radon has convincingly demonstrated that exposure to very high concentrations of radon causes lung cancer. Levels in homes are usually lower than those in mines, miners are also exposed to other carcinogens, miners are overwhelmingly smokers, and working miners generally breathe both harder and more deeply than people at home, so several assumptions and extrapolations are needed to to estimate cancer risk at typical home levels (see National Research Council, 1991). These extrapolations (including an assumed linear dose-response function) suggests that about 15,000 additional lung cancer deaths occur annually in the U.S. due to radon, mostly among smokers, though this number is based on an unrealistic comparison to the number of deaths that would occur if nobody were exposed to any radon at all.

The miner studies demonstrate statistically significant elevated cancer risk at doses equivalent to lifetime residence in a home at about 20 picoCuries per liter (pCi/L). (An alternative notation is the SI unit of Bequerels per cubic meter; 1 pCi/L = 37 Bq/m^3.) Estimates based on the miner studies, on experiments on animals, and on biological and biophysical models suggest that, at least at high levels, lifetime exposure to each additional pCi/L of radon adds a lifetime risk of about 0.0035 of lung cancer, averaging over men and women, and smokers and non-smokers. (Lubin and Steindorf, 1995). See Table 1 for the parameters that we use in this paper for each sex/smoking category.

The dose-response at low concentrations is difficult to estimate, because all case-control studies have been fairly small and because lifetime radon exposures are poorly estimated. Although a linear dose-response relation is plausible and is consistent with case-control data, current data are also consistent with a threshold model or even a small beneficial effect at low doses (Cohen, 1995, Bogen, 1997, Lagarde et al., 1997, Lubin and Boice, 1997).

Partly from necessity and partly for historical reasons, radon researchers use a fairly large and confusing assortment of units. For instance, the radiation absorbed by the body (the "dose") depends not just on the concentration of radon in the air, but also on the breathing rate and of course on the duration of exposure, so there is no simple conversion between radon concentration in indoor air and dose absorbed by a human body. Moreover, it is radon's decay products, rather than radon itself, that deliver most of the radiation dose associated with radon, and the differential removal of decay products and radon itself can lead to variation in the relative concentrations of each. For clarity and convenience, we write "the radon dose" when we mean "the dose from radon and its decay products," where standard parameter estimates have been used to make all of the necessary adjustments. See Nazaroff and Nero (1988) for an overview of many of these issues.

We present cumulative exposures in terms of pCi/L-years, rather than the historical unit (also non-SI) which is "working level months (WLM)". The direct conversion, for breathing 1 pCi/L air for a year, is 1 pCi/L-year = 0.26 WLM, but it is standard to assume that an individual is only at home about 75% of the time, so that a home concentration of 1 pCi/L for a year leads to an exposure of 0.2 WLM.

2.2 U.S. residential radon concentrations and measurements

Residential radon measurements are commonly made following a variety of protocols. The most frequently used protocol in the U.S. has been the "screening" measurement: a short-term (2–7 day) charcoal-canister measurement made on the lowest level of the home (often an unoccupied basement), at a cost of about $15 to $20. Short-term measurements made at different times during the same season have a geometric standard deviation (GSD) of roughly 1.6, primarily due to temporal variation in indoor radon concentrations. In addition, because short-term measurements are usually made on the lowest level of the home and during the season of highest indoor radon exposure, they are upwardly biased measures of annual living area average radon level. The magnitude of this bias varies by season and by region of the country and depends on whether the basement (if any) is used as living space (White et al., 1990, Klotz et al., 1993, Price and Nero, 1996); our estimated correction factors for winter-season, lowest-level measurements, known as "screening" measurements, appear in Table 2. (If the short-term measurement is not made in winter, then an additional seasonal correction factor is needed.) Due to the large temporal variability and other sources of variation, a short-term measurement can predict the long-term living-area concentration only to within a factor of 1.8 or so, even after correcting for systematic biases.

A radon measure that is far less common than the screening measurement, but is believed to be much better for evaluating radon risk, is a 12-month integrated measurement of the radon concentration. By monitoring on every living level of the home (that is, on every floor in which people spend more than a small amount of time each day), one can measure the "annual living-area average radon concentration," or ALAA. For a typical home with two stories used as living space, such monitoring costs about $50. These long-term living-area measurements are not subject to the biases and effects of day-to-day and seasonal variation that affect screening measurements. A national sample of ALAA measurements was collected in the National Residential Radon Survey (NRRS) (see Marcinowski et al., 1994).

The exact relationship between the ALAA concentrations and the occupant exposures is not known: people spend different amounts of time in different areas of the home, long-term measurements are still subject to some error, even on the same floor different rooms can have slightly different radon levels, and so on. For purposes of this paper, we assume that an ALAA measurement (that is, the arithmetic mean of long-term measurements made on each occupied level of the home) estimates each resident's exposure to within a multiplicative error with a geometric mean (GM) of unity and a GSD of 1.2 (see Price and Nero, 1996, for details).

The distribution of annual-average living area home radon concentrations in U.S. houses[1], as measured in the NRRS, is approximately lognormal with geometric mean (GM) 0.67 pCi/L and geometric standard deviation (GSD) 3.1 pCi/L (Marcinowski et al., 1994). These data suggest that between 50,000 and 100,000 homes have radon concentrations in primary living space in excess of 20 pCi/L. This level causes an annual radiation exposure roughly equal to the occupational exposure limit for uranium miners. Thirty years' occupancy of such a house would yield an added estimated risk of lung cancer of about 0.85% among nonsmokers and 7.8% among smokers. These risks are very high compared with the risks estimated for other kinds of environmental exposures regulated by the EPA (for comparison, see Sandia National Laboratory, 1994).

2.3 Radon remediation

Several radon control techniques have been developed, tested and implemented (Henschel and Scott, 1987, and Turk et al., 1989), and long-term performances of these systems were reported (Turk et al., 1991). The currently preferred remediation method for most homes, "sub-slab depressurization," costs about \$1000–\$1500 to install and requires constant use of a small electric fan; the net present value of such a system is about \$2000, including the heating and cooling costs associated with increased ventilation. Although long-term experience with these systems is lacking, for purposes of our analysis we will assume that such a system remains effective for 30 years. We are not aware of any large-scale randomized studies on the effect of remediation on radon levels, but many small non-randomized studies have been conducted and are summarized in an EPA report (Henschel, 1993). These studies suggest that almost all homes can be remediated to below 4 pCi/L, while reductions under 1 pCi/L are rarely attained with conventional methods, for homes with a very wide range of pre-remediation levels. For simplicity, we make the assumption that remediation will reduce radon concentration to 2 pCi/L. For obvious reasons, little is known about effects of remediation on houses that already have low radon levels; we will assume that if the initial annual living area average level is less than 2 pCi/L, then remediation has no effect.

Recommendations for radon remediation vary by country, with Sweden setting a recommended action

[1] Throughout, we use the term "house" to refer to owner-occupied ground-contact homes.

level for the annual living-area average (ALAA) indoor radon concentration of 10 pCi/L and Canada recommending action at 20 pCi/L, compared to the U.S. level of 4 pCi/L. The current U.S. recommendations, if fully implemented, would cost on the order of $11 billion in measurement and remediation costs, plus additional expenses of something like $1 billion per year for operation and maintenance. In Section 5, we discuss the efficiency of such a program in terms of estimated dollars per life saved.

3 Geographic modeling of indoor radon levels

Although radon is thought to cause a large number of deaths compared to other environmental hazards, the vast majority of houses in the U.S. do not have elevated radon levels that would be substantially reduced by remediation: about 84% of homes have ALAA concentrations under 2 pCi/L, and about 90% are below 3 pCi/L. A goal of some researchers has been to identify locations and predictive variables associated with high-radon homes so that monitoring and remediation programs can be focused efficiently. One such effort at the Lawrence Berkeley National Laboratory used Bayesian hierarchical modeling to analyze indoor radon measurements. These models include monitoring data, county indicators, a measure of surficial radium concentration, a climatological variable, and house construction information and were fit separately in 10 regions of the U.S. (Price, Nero, and Gelman, 1996, Price, 1997, Revzan et al., 1998). These models were used to fit data from short-term measurements, which were calibrated to long-term living-area averages as described by Price and Nero (1996). Combining short and long-term measurements allowed us to estimate the distribution of radon levels in nearly every county in the U.S., albeit with widely varying uncertainties depending primarily on the amount of monitoring data within the county.

Unfortunately (from the standpoint of radon mitigation programs), indoor radon concentrations are highly variable even within small areas. Given the predictive variables mentioned in the previous paragraph, the radon level of an individual house in a specified county can be predicted only to within a factor of at best about 1.9, with a factor of 2.3 being more typical (Price et al., 1996, Price 1996), a disappointingly large uncertainty considering the factor of 3.1 that would hold given no information on the home other than that it is in the U.S. On the other hand, this seemingly modest reduction in uncertainty is still enough to identify some areas where high-radon homes are very rare or very common. For instance, in the mid-Atlantic states, more than half the houses in some counties have long-term living area concentrations over the EPA's recommended action level of 4 pCi/L, whereas in other counties fewer than 0.5 percent exceed that level (Price, 1996).

Various monitoring efforts demonstrate that the distribution of indoor radon concentrations for an area or region of almost any scale is reasonably well represented by a lognormal distribution, or sometimes the sum of two such distributions (Nero et al., 1990). Further, a large area's distribution is effectively a mixture of the individual distributions of the composite subareas, all of which are reasonably well represented by individual lognormal distributions, with geometric means (GMs) that vary from one subarea to another (See Nero et al., 1986, and Price et al., 1996, for example).

In each region of the country, a hierarchical linear regression model was previously fit to the logarithms of home radon measurements (see Price, Nero and Gelman, 1996, and Price, 1997). We shall apply these models to perform inferences and decision analyses for previously-unmeasured houses i, using the following notation:

$$
\begin{aligned}
R_i &= \text{ALAA radon concentration in house } i \\
\theta_i &= \log(R_i) \\
X_i &= \begin{array}{l}\text{vector of explanatory variables (including county-level variables, house-level variables,} \\ \text{and county indicators) for house } i\end{array} \\
\beta &= \text{vector of regression coefficients} \\
\tau^2 &= \begin{array}{l}\text{variance component in the model corresponding to variability between houses conditional} \\ \text{on the predictors}\end{array} \\
\sigma^2 &= \text{variance component in the model corresponding to measurement variability within a house}
\end{aligned}
$$

Then the unknown θ_i has the predictive distribution,

$$\theta_i | X, \beta \sim \mathrm{N}(X_i\beta, \tau^2). \tag{1}$$

There is some uncertainty in the coefficients β (particularly for the indicators corresponding to counties with few observations) and a small amount of posterior uncertainty in the variance components of the model. For the purposes of this paper, we need only know the predictive distribution for any given θ_i, averaging over all these uncertainties; it will be approximately normal (because the variance components are so well estimated), and we label it as,

$$\theta_i \sim \mathrm{N}(M_i, S_i^2). \tag{2}$$

We write $M_i = (X\hat{\beta})_i$, where $\hat{\beta}$ is the posterior mean from the analysis in the appropriate region of the country. The variance S_i^2 includes the posterior uncertainty in the coefficients β and also the within-county variance τ^2. The GSD of the unexplained within-county variation, e^{τ}, is estimated to be in the range 1.9–2.3 (depending on the region of the country) which puts a lower limit on e^S. To be precise, the prior GSD's, e^S, vary from 2.1 to 3.0, and are in the range $[2.1, 2.5]$ for most U.S. houses (the houses with $e^S > 2.5$ lie in small-population counties for which little information was available in the radon surveys, resulting in relatively high predictive uncertainty within these counties). The prior GM's, e^M, vary from 0.1 to 14.6 pCi/L, with 95% in the range $[0.3, 3.7]$ and 50% in the range $[0.6, 1.6]$. The houses with the highest prior GM's are houses with basement living areas in high-radon counties; the houses with lowest prior GM's have no basements and lie in low-radon counties. See Price and Nero (1996) for more details on the characteristics of high- and low-radon houses.

The purpose of this paper is not modeling, but decisions. For the rest of the paper we work at the individual house level and use the posterior inference for house i from the model discussed above as our prior distribution for the subsequent analysis. Since we are considering decisions for houses individually, we suppress the subscript i for the rest of the paper.

Now suppose a measurement $y \sim \mathrm{N}(\theta, \sigma^2)$ is taken in a house. (We are assuming an unbiased measurement. If a short-term measurement is being used, it will have to be corrected for the bias shown in Tables 2, and for an addition seasonal correction factor, if the measurement was not made in winter [e.g. see Mose and Mushrush (1997) and Pinel et al. (1995)]. In our notation, y and θ are the logarithms of the measurement and the true ALAA radon level, respectively. The posterior distribution for θ is

$$\theta | M, y \sim \mathrm{N}(\Lambda, V), \tag{3}$$

where

$$\Lambda = \frac{\frac{M}{S^2} + \frac{y}{\sigma^2}}{\frac{1}{S^2} + \frac{1}{\sigma^2}} \qquad V = \frac{1}{\frac{1}{S^2} + \frac{1}{\sigma^2}} \tag{4}$$

(see, e.g., Gelman et al., 1995). We base our decision analysis of when to measure and when to remediate on the distributions (2) and (3).

4 Individual decisions on whether to monitor or remediate

The suggestion that every home should monitor is highly conservative (we might also say highly "protective"), based on the knowledge that homes with elevated radon concentrations have been found in every state, so the only way to be sure that a home does not have an elevated concentration is to test. However, if the risk is low enough (that is, if the predicted radon level $M_i = X_i\beta$ is low for house i), then even the small cost of monitoring may not be worthwhile.

We now work out the optimal decisions of measurement and remediation conditional on the predicted radon level in a home, the additional risk of lung cancer death from radon, the effects of remediation, and individual attitude toward risk. We follow a standard approach in decision analysis (see, for example, Watson and Buede, 1987) by proceeding in two steps: first, decision-making under certainty—at what level would you remediate if you knew R, your home radon level—and, second, averaging over the uncertainty in R.

4.1 Decision-making under certainty

We shall express decisions under certainty in three ways, equivalent under a linear no-threshold dose-response relationship:

1. The dollar value D_d associated with a reduction of 10^{-6} in probability of death from lung cancer (the value of a microlife).

2. The dollar value D_r associated with a reduction of 1 pCi/L in home radon level for a 30-year period (the equivalent dollar cost per unit of radon exposure).

3. The home radon level R_{action} above which you should remediate if your radon level is known.

We need to work with all three of these concepts because, depending on the context, either D_d, D_r or R_{action} will be most relevant for individual decision-making. In any case, the essence of the radon decision is a tradeoff between dollars and lives.

Initially, we make the following assumptions:

- The increase of probability of lung cancer death is a linear function of radon exposure (consistent with current concepts of dose effects in high linear-energy-transfer radiation; see Upfal et al., 1995). The added risk differs for smokers and non-smokers and for males and females; we use estimates $\gamma_{g,s}$ ($g =$ male or female, and $s =$ smoking or nonsmoking) for the additional lifetime risk per additional pCi/L exposure as derived from the Committee on the Biological Effects of Radiation (National Research Council, 1988)—see Table 1.

- Remediation takes a house's annual-average living-area radon level down to a level R_{remed} if it was above that, but leaves it unchanged if it was below that. We shall assume that R_{remed} has the value 2 pCi/L.

- Mitigation costs \$2000, including the net present value of future energy cost to run the mitigation system.

- Decisions will be made based on the consequences over the next 30 years.

- If a measurement is taken, it is a long-term measurement that is an unbiased measure of annual-average living-area exposure with a measurement GSD of 1.2, and it costs \$50.

We can now determine the equivalent cost D_r per pCi/L of home radon exposure and the action level R_{action} for remediation given the following individual information:

- The numbers of male and female smokers and nonsmokers in the house, $n_{g,s}$; see Table 1.

- The dollars D_d that would be paid to reduce the probability of lung cancer death by one-millionth. From the risk assessment literature, typical values for medical interventions are in the range of \$0.1 to \$0.3; see, e.g., Eddy, 1989, and Owens el al., 1996. Higher values are often found in other contexts (e.g. jury awards for deaths due to negligence), but we feel that the lower values are more appropriate in this case since, like medical intervention, expenditure on radon remediation is voluntary and is aimed at reducing future risk rather than compensating for past injury.

For any given household, the equivalent cost per pCi/L, D_r, can be computed as a function of the risk assumed above and the individual parameters and D_d:

$$D_r = \frac{30}{70} \left(\sum_{g,s} n_{g,s} \gamma_{g,s} \right) 10^6 D_d, \tag{5}$$

where the fraction 30/70 is the ratio of the 30-year decision period to a 70-year life expectancy per occupant. For U.S. homes, the average value of $\sum_{g,s} n_{g,s}\gamma_{g,s}$ is 0.0075 (see Table 1). We can also compute the remediation concentration R_{action}, given the equivalent cost and the above assumptions of cost and effects of remediation:

$$R_{\text{action}} = \frac{\$2000}{D_r} + R_{\text{remed}}. \tag{6}$$

4.2 Individual choice of a recommended remediation level under certainty

The U.S., English, Swedish, and Canadian recommended remediation levels are $R_{\text{action}} = 4$, 5, 10, and 20 pCi/L, which, with $R_{\text{remed}} = 2$ pCi/L, correspond to equivalent costs per pCi/L of $D_r =$ \$1000, \$670, \$250, and \$111, respectively. Setting the values of $n_{g,s}$ to the average numbers of male and female smokers and nonsmokers in a U.S. household implies dollar values per microlife of $D_d =$ \$0.31, \$0.21, \$0.08, and \$0.03, respectively. This suggests that, to the extent that we believe the standard estimates of radon risk and remediation effects, the U.S. and English recommendations are within the typical range for acceptable risk reduction expenditures, while the Canadian and Swedish recommendations are too cavalier about the radon risk. However, this calculation obscures the dramatic difference between smokers and nonsmokers, which is due entirely to the difference in risk per dose associated with the two groups. For example, a family of one male nonsmoker and one female nonsmoker that is willing to spend \$0.30 per person to reduce the probability of lung cancer by 10^{-6} should spend \$270 per pCi/L, implying an action level of $R_{\text{action}} = 9.5$ pCi/L. In contrast, if the male and female are both smokers, they should be willing to spend the much higher value of \$3150 per pCi/L, because of their higher risk per pCi/L, and thus should have an action level of $R_{\text{action}} = 2.6$ pCi/L.

Other sources of variation in R_{action}, in addition to varying risk preferences, are (a) variation in the number of smokers and nonsmokers in households, (b) variation in individual beliefs about the risks of radon and the effects of remediation, and (c) variation in the perceived dollar value associated with a given risk reduction. From a public policy standpoint, one might wish to ignore the variation attributable to (a), since over the 30-year period of assumed remediation effectiveness the household composition is likely to change and indeed the house is likely to be sold to several sets of new owners with possibly different smoking habits. However, as a practical matter the homeowners are likely to perform remediation only if they foresee major risk reductions for themselves, or if they are planning to sell their house and fear that an elevated radon concentration will reduce its value. As illustrated above, a male-female non-smoking couple might choose an action level of 9.5 pCi/L or higher, depending on their risk tolerance, whereas most smokers may be more willing to risk lung cancer than are non-smokers and would thus be unwilling to remediate at levels near 2.6 pCi/L.

Through the rest of the paper we will use 4 pCi/L as an exemplary value, but rational informed individuals might plausibly choose quite different values of R_{action}, depending on smoking habits, risk tolerance, financial resources, and the number of people in the household.

4.3 Decision-making under uncertainty

Given an action level under certainty, R_{action}, we now address the question of whether to pay for a home radon measurement and whether to remediate. The decision of whether to measure depends on the prior distribution (2) of radon level for your house, given your predictors X. The decision of whether to remediate depends on the posterior distribution (3) if a measurement has been taken or the prior distribution (2) otherwise. In our computations, we shall make use of the following results from the normal distribution: if $z \sim \mathrm{N}(\mu, s^2)$, then $\mathrm{E}(e^z) = e^{\mu + \frac{1}{2}s^2}$ and $\mathrm{E}(e^z|z > a)\Pr(z > a) = e^{\mu+\frac{1}{2}s^2}(1 - \Phi(\frac{\mu+s^2-a}{s}))$, where Φ is the standard normal cumulative distribution function.

The decision tree is set up as 3 branches. In each branch, we evaluate the expected loss in dollar terms, converting radon exposure to dollars using $D_r = \$2000/(R_{\text{action}} - R_{\text{remed}})$ as the equivalent cost per pCi/L for additional home radon exposure.

1. **Remediate without monitoring.** Expected loss is remediation cost + equivalent dollar cost of radon exposure after remediation:

$$\begin{aligned} L_1 &= \$2000 + D_r \mathrm{E}(\min(R, R_{\text{remed}})) \\ &= \$2000 + D_r \left[R_{\text{remed}} \Pr(R \geq R_{\text{remed}}) + \mathrm{E}(R|R < R_{\text{remed}})\Pr(R < R_{\text{remed}})\right] \\ &= \$2000 + D_r \left[R_{\text{remed}} \Phi(\frac{M - \log(R_{\text{remed}})}{S}) + e^{M+\frac{1}{2}S^2}\left(1 - \Phi\left(\frac{M + S^2 - \log(R_{\text{remed}})}{S}\right)\right)\right]. \end{aligned} \tag{7}$$

2. **Do not monitor or remediate.** Expected loss is the equivalent dollar cost of radon exposure:

$$L_2 = D_r \mathrm{E}(e^\theta) = D_r e^{M+\frac{1}{2}S^2}. \tag{8}$$

3. **Take a measurement** y (measured in log pCi/L). The immediate loss is measurement cost (assumed to be \$50) and, in addition, the radon exposure during the year that you are taking the measurement (which is $\frac{1}{30}$ of the 30-year exposure (8)). The inner decision has two branches:

   (a) **Remediate.** Expected loss is computed as for decision 1, but using the posterior rather than the prior distribution:

$$\begin{aligned} L_{3a} \;&=\; \$50 + D_r \frac{1}{30} e^{M+\frac{1}{2}S^2} + \$2000 + \\ &\quad + D_r \left[R_{\text{remed}} \Phi(\frac{\Lambda - \log(R_{\text{remed}})}{\sqrt{V}}) + e^{\Lambda+\frac{1}{2}V}\left(1 - \Phi\left(\frac{\Lambda + V - \log(R_{\text{remed}})}{\sqrt{V}}\right)\right)\right], \end{aligned} \tag{9}$$

   where Λ and V are the posterior mean and variance, from equation (4).

   (b) **Do not remediate.** Expected loss is:

$$L_{3b} = \$50 + D_r \frac{1}{30} e^{M+\frac{1}{2}S^2} + D_r e^{\Lambda+\frac{1}{2}V}. \tag{10}$$

4.3.1 Decision of whether to remediate given a measurement

To evaluate the decision tree, we must first consider the inner decision between 3(a) and 3(b), conditional on the measurement y. Let y_0 be the point (in log space) at which you will choose to remediate if $y > y_0$, or do nothing if $y < y_0$. (Because of measurement error, $y \neq \theta$, so $e^{y_0} \neq R_{\text{action}}$.) We shall solve for y_0 in terms of the prior mean M, the prior standard deviation S, and the measurement standard deviation σ, by solving the implicit equation

$$L_{3a} = L_{3b} \text{ at } y = y_0. \tag{11}$$

The expected losses L_{3a} and L_{3b} depend on y_0 only through $\Lambda = \frac{\frac{M}{S^2}+\frac{y}{\sigma^2}}{\frac{1}{S^2}+\frac{1}{\sigma^2}}$, and so we can solve for y_0 by first solving for Λ_0 in (11), then setting

$$y_0 = (1 + \frac{\sigma^2}{S^2})\Lambda_0 - \frac{\sigma^2}{S^2}M. \tag{12}$$

Thus the relation between y_0 and M is linear, with the slope depending only on the variance ratio σ^2/S^2.

Given σ^2/S^2 and D_r, we solve for Λ_0 numerically, using the bisection method to converge on the value of Λ that satisfies (11). Figure 1 shows the measurement action level e^{y_0} as a function of the perfect-information action level R_{action}, evaluated at values of the prior GM radon level e^M ranging from 0.5 to 4.0. For this example, we have assumed that $\sigma = \log(1.2)$, and that $S = \log(2.3)$ for all counties.

4.3.2 Deciding whether to measure

We determine the expected loss for branch 3 of the decision tree by averaging over the prior uncertainty in the measurement y:

$$L_3 = \mathrm{E}(\min(L_{3a}, L_{3b})). \tag{13}$$

Given (M, S, σ, D_r), we evaluate this expression as follows.

1. Simulate 5000 draws of $y \sim \mathrm{N}(M, S^2 + \sigma^2)$.
2. For each draw of y, compute $\min(L_{3a}, L_{3b})$ from (9) and (10).
3. Estimate L_3 as the average of these 5000 values.

Of course, this expected loss is valid only if we assume that you will make the recommended optimal decision once the measurement is taken.

We can now compare the expected losses L_1, L_2, L_3, and choose among the three decisions. Figure 2 displays the expected losses as a function of the perfect-information action level R_{action} for several values of e^M. As with Figure 1, we illustrate with $\sigma = \log(1.2)$ and $S = \log(2.3)$. For any value of M and R_{action}, the recommended decision is the one with the lowest espected loss.

For any R_{action}, we can summarize the decision recommendations as the cut-off levels M_{low} and M_{high} for which decision 1 is preferred if $M > M_{\text{high}}$, decision 2 is preferred if $M < M_{\text{low}}$, and decision 3 is preferred if $M \in [M_{\text{low}}, M_{\text{high}}]$. Figure 3 displays these cut-offs as a function of R_{action}, and thus displays the recommended decision as a function of (R_{action}, e^M), once again under the simplifying assumption that $\sigma = \log(1.2)$ and $S = \log(2.3)$ for all counties. For example, setting $R_{\text{action}} = 4$ pCi/L leads to the following recommendation based on e^M, the prior GM of your home radon based on your county and house type:

- If e^M is less than 1.0 pCi/L (which corresponds to 68% of U.S. houses), do nothing.
- If e^M is between 1.0 and 3.5 pCi/L (27% of U.S. houses), perform a long-term measurement (and then decide whether to remediate).

- If e^M is greater than 3.5 pCi/L (5% of U.S. houses), remediate immediately without measuring. Actually, in this circumstance—and only in this circumstance—short-term monitoring turns out to be barely cost-efficient: the reason for the recommendation of immediate remediation is that the excess risk associated with occupying the home for a year while a long-term measurement is made is not worth bearing, given the high likelihood that the home will eventually be remediated anyway. But if a short-term measurement is made and is sufficiently low, then the home is unlikely to have such an exceptionally high level that one additional year of exposure carries a large risk. In this case, long-term monitoring can be performed to determine whether remediation is really indicated. We will ignore this additional complexity to the decision tree, since it occurs rarely and has very little impact on the overall cost-benefit analysis.

4.4 Decision making if a short-term measurement has been taken

We do not in general recommend taking short-term measurements, because long-term measurements are much superior in terms of both bias and variance. However, short-term measurements are quite popular (partly because these are often taken as a condition of sale of a house), and so it is worth considering the decision problem in this situation.

In fact, the above decision framework is immediately adaptable to a homeowner who has already taken a short-term measurement. The only change that needs to be made is that the prior distribution (2) needs to be updated given the information from the short-term measurement. We thus replace M and S^2 in the above formulas by

$$M_{\text{new}} = \frac{\frac{M}{S^2} + \frac{y_{\text{st}} - \log b}{\sigma^2_{\text{st}}}}{\frac{1}{S^2} + \frac{1}{\sigma^2_{\text{st}}}} \qquad S^2_{\text{new}} = \frac{1}{\frac{1}{S^2} + \frac{1}{\sigma^2_{\text{st}}}}. \tag{14}$$

where y_{st} is the logarithm of the short-term measurement and b is the correction factor derived from Table 2. If the short-term measurement was not made in winter, then a seasonal correction factor will also apply; see, e.g., Mose and Mushrush (1997) and Pinel et al. (1995). At this point, we can return to the procedure described in the previous sections.

4.5 Summary of the individual decision process

Ideally, an individual homeowner in the U.S. can now make a remediation decision using the following process:

1. Determine the radon level R_{action} above which you would remediate, if you knew your home radon level exactly. This value can be chosen in its own right, or by choosing a value of D_r based on the perceived gains from lowering radon level, or by assigning a dollar value D_d to a millionth of a life and computing based on the number of smokers S and nonsmokers N in the house. As discussed in Section 4.2, current understanding of the risks of radon and the effects of remediation suggest that the EPA's recommendation of 4 pCi/L is a reasonable catch-all value, with 8 pCi/L being a more reasonable value for non-smokers.

2. Look up e^M and e^S, the GM and GSD of the posterior predictive distribution for your home's radon level, as estimated from the hierarchical model described in Section 3.

3. If a short-term measurement has been taken, update the prior distribution using (14) and the bias correction from Table 2 (and possibly an additional seasonal correction).

4. Calculate the expected losses of decisions 1, 2, and 3 from the formulas in Section 4.3 and, if decision 3 is chosen, the recommended measurement action level e^{y_0}. The recommended decision—that with the lowest expected loss—corresponds to that indicated in Figure 3 (with slight alterations depending on the exact value of S).

5. If decision 3 is chosen, perform a long-term measurement. In one year, the measurement e^y is available. Remediate if $e^y > e^{y_0}$.

We are in the process of constructing a web site at `http://www.stat.columbia.edu/~gelman/` to automate the steps listed above.

5 Aggregate consequence of decision strategy

Now that we have made idealized recommendations, we consider their aggregate effects if followed by all homeowners in the U.S. In particular, how much better are the consequences compared to other policies such as the current one, implicitly endorsed by the EPA, of taking a short-term measurement as a condition of a home sale and performing remediation if the measurement higher than 4 pCi/L?

5.1 Estimated consequences of applying the recommended decision strategy to the entire U.S.

Figures 4 and 5 display the geographic pattern of recommended measurements (and, after one year, recommended remediations), based on action levels R_{action} of 4 and 8 pCi/L, respectively. These recommendations incorporate the effects of parameter uncertainties in the models that predict radon distributions within counties, so these maps would be expected to change somewhat as better predictions become available. Note that these maps are *not* based on a single estimated parameter such as "the probability that a home's concentration exceeds 4 pCi/L." Although a discrete action level does play a role in the decision process—after all, each home must either monitor or not, and remediate or not—the benefit of remediation is a continuous function of the initial radon concentration, and that concentration is assumed to be drawn from a continuous distribution. It is the confluence of these continuous distributions and the discrete willingness-to-remediate point that give rise to the fairly complex expressions for expected loss in Section 4.3.

From a policy standpoint, perhaps the most significant feature of the maps is that even if the EPA's recommended action level of 4 pCi/L is assumed to be correct—and, as we have discussed, it does lead to a reasonable value of D_d, under standard dose-response assumptions—monitoring is still not recommended in most U.S. homes. Indeed, only 28% of U.S. homes would perform radon monitoring. A higher action level of 8 pCi/L, a reasonable value for nonsmokers under the standard assumptions, would lead to even more restricted monitoring and remediation: only about 5% of homes would perform monitoring.

5.2 Decision strategies considered and evaluation criteria

In this section, we shall consider various decision strategies.

1. Follow the recommended strategy from Section 4.3 (that is, monitor homes with prior mean estimates above a given level, and remediate those with high measurements).

2. Perform long-term measurements on all houses and then remediate those for which the measurement exceeds a specified level: $e^y > R_{\text{action}}$.

3. Perform short-term measurements on all houses and then remediate those for which the bias-corrected measurement exceeds a specified level: $e^{y_{\text{st}}}/b > R_{\text{action}}$ (with b defined as described in Section 4.4).

4. Perform short-term measurements on all houses and then remediate those for which the uncorrected measurement exceeds a specified level: $e^{y_{\text{st}}} > R_{\text{action}}$.

We evaluate each of the above strategies in terms of aggregate lives saved and dollars cost, with these outcomes parameterized by the radon action level R_{action}. Both lives saved and costs are considered for a 30-year period. For each strategy, we assume that the level R_{action} is the same for all houses (this would correspond to a uniform national recommendation) and that 0.30 male and 0.27 female smokers and 1.07 male and 1.16 female nonsmokers live in each house (or, rather, that these are the averages over the 30-year period).

We also evaluate strategies based on the estimated cost per life saved. This aggregate cost per life is different from the marginal cost per life used to set the action level R_{action} in Section 4.2. For example, as discussed previously, an action level of $R_{\text{action}} = 4$ pCi/L approximately corresponds to a value of \$0.31 per microlife, which corresponds to a marginal cost of \$310,000 per life saved. However, if the optimal recommendation is followed for the entire country, the estimated aggregate cost per life saved is only \$160,000: the aggregate cost averages over the whole population, ranging from mitigations that are barely cost-effective through mitigations that are highly efficient in terms of risk reduction for a given cost. See also Figure 10 for comparison.

5.3 Modeling the variation in the population of U.S. homes

Because we use inferences from a hierarchical model, we are able to give different recommendations for different houses in the population as characterized by location as well as continuous covariates.

Thus, aggregate effects are determined by adding up the individual decisions over all the ground-contact homes in the country. Considering 3078 counties with 3 house types within each, we have 3078×3 pairs of (M, S) obtained from the hierarchical model fit to the national and state radon survey data as described in

Section 3. Given $(M, S, R_{\text{action}})$, the decisions of whether to monitor and whether to measure are made as described in Section 4.5, and expected number of lives saved and cost spent are assessed if remediation is implemented.

For any of the decision strategies, in any given house, we evaluate the total cost:

$$\text{Cost} = \$50\,\Pr(\text{measurement}) + \$2000\,\Pr(\text{remediation}) \tag{15}$$

where

$$\Pr(\text{measurement}) = \begin{cases} 1_{M_{low}<M<M_{high}} & \text{for strategy 1} \\ 1 & \text{for strategies 2, 3, and 4} \end{cases}$$

and

$$\Pr(\text{remediation}) = \begin{cases} \Pr(M > M_{high}) + \Pr((M_{low} < M < M_{high}) \text{ and } (y > y_0)) & \\ \quad = 1_{\{M>M_{high}\}} + 1_{\{M_{low}<M<M_{high}\}}(1 - \Phi(\frac{\Lambda - y_0}{\sqrt{V}})) & \text{for strategy 1} \\ \Pr(y > \log(R_{\text{action}})) = 1 - \Phi(\frac{\Lambda - \log(R_{\text{action}})}{\sqrt{V}}) & \text{for strategy 2} \\ \Pr(y > \log(R_{\text{action}})) = 1 - \Phi(\frac{\Lambda_{new} - \log(R_{\text{action}})}{\sqrt{V_{new}}}) & \text{for strategy 3} \\ \Pr(y_{st} > \log(R_{\text{action}})) = 1 - \Phi(\frac{\Lambda - \log(R_{\text{action}})}{\sqrt{V}}) & \text{for strategy 4} \end{cases}$$

(with \$50 replaced by \$15 for strategies 3 and 4 in which short-term measurements are used), and we evaluate the expected lives saved:

$$\begin{aligned} \text{Expected lives saved} &= A\left(\mathrm{E}(\max(e^{\theta} - R_{\text{remed}}, 0)|\text{remediation})\Pr(\text{remediation})\right) \\ &= (A)(R_{\text{reduced}})\Pr(\text{remediation}), \end{aligned} \tag{16}$$

where $R_{\text{reduced}} = e^{\Lambda+V/2}\Phi(\frac{\Lambda+V-\log(R_{\text{remed}})}{\sqrt{V}}) - R_{\text{remed}}\Phi(\frac{\Lambda-\log(R_{\text{remed}})}{\sqrt{V}})$ and A is the expected lives lost in a 30-year period per pCi/L of home radon exposure, given by D_r/D_d from equation 5 for any home, and equal to 0.0075 for the "average household" of 1.07 male nonsmokers, 0.3 male smokers, 1.16 female nonsmokers and 0.27 female smokers.

We evaluate the expectations in (15) and (16) by simulation. First, we simulate 5000 draws of $y \sim \mathrm{N}(M, S^2 + \sigma^2)$, with σ^2 replaced by σ^2_{st} for strategies 3 and 4, and with M replaced by $M + b$ for strategy 4. Second, for each draw of y, we compute $A \times R_{\text{reduced}}$ under the constraints of $M > M_{\text{high}}$ or $((M_{\text{low}} < M < M_{\text{high}})$ and $(y > y_0))$ or $y > \log(R_{\text{action}})$, then estimate (16) and (15) as the average of these 5000 draws. Simulations average over uncertainties in home radon levels R and variability in measurements e^y (or $e^{y_{\text{st}}}$). For these calculations we used the actual model estimates of S, rather than setting them all equal to a single value as was done for illustrative purposes in the previous section.

We then multiply by the total number of ground contact houses for each (M, S), i.e. for each house type and for each county, and sum them up to get expected total costs and lives saved over a 30-year period in the U.S.

5.4 Results

For the present county-level radon model, within each county monitoring is recommended for some subset of homes: for all homes, for all homes with basements, for all homes with living-area basements, or for no homes. The maps in Figure 4 display, for each county, the fraction of houses that would measure, and the estimated fraction of houses that would remediate, if the recommended decision strategy were followed everywhere with $R_{\text{action}} = 4$ pCi/L. About 28% of the 70 million ground-contact houses in the U.S. would monitor. This would result in detection of and remediation of 2.8 million homes above 4 pCi/L (75% of all such homes), and 840,000 of the homes above 8 pCi/L (93% of all such homes). Some additional estimates of the program's effectiveness are presented in Tables 3 and 4, and Figure 5 displays similar maps for an 8 pCi/L action level.

In order to understand the effects of the different decision strategies on aggregate outcomes, we have developed a series of graphs. Figures 6 and 7 illustrate the efficiency of the recommended remediation strategy by showing the overall distributions of radon levels (and total radon exposures) and the distributions of homes to be monitored and remediated; as is apparent in the figures, even with the large uncertainties in individual county distributional parameters the recommended program is quite effective at focusing on the homes with the highest indoor radon concentrations.

Figure 8 displays the tradeoff between expected cost and expected lives saved over a thirty-year period for the four strategies listed in Section 5.2. The numbers on the curves are action levels R_{action}. This figure allows us to compare the effectiveness of alternative strategies of equal expected cost or equal expected lives saved. For example, the recommended strategy (the solid line on the graph) at $R_{\text{action}} = 4$ pCi/L would result in an expected 54,000 lives saved at an expected cost of \$7.4 billion. Let us compare this to the EPA's implicitly recommended strategy based on uncorrected short-term measurements (the dashed line on the figure). For the same cost of \$7.4 billion, the uncorrected short-term strategy is expected to save only 35,000 lives; to achieve the same expected savings of 54,000 lives, the uncorrected short-term strategy would cost about \$17 billion.

Figure 9 displays these results in another way, as estimated cost per life saved, as a function of expected cost, for the four strategies. Finally, Figure 10 displays the estimates for both marginal and average cost per life saved, for the recommended decision strategy, as a function of the radon action level R_{action}. The average cost per life saved is estimated as described above, and the marginal cost per life saved is simply $10^6 D_d$ (as defined in Section 4.1). Average cost per life saved is always lower than marginal cost because, for any action level, the average includes all houses at or above that level, and remediations are more efficient (in terms of lives saved per dollar) in the higher-radon houses.

6 Sensitivity to assumptions

Our results are subject to potential error in:

- estimates of annual-average living area radon exposure (and its variation) from home radon measurements and the hierarchical model (including basement information and geographic predictors);
- the magnitude of cancer risk from a given radon concentration, (including the assumed linearity of cancer risk as a function of radon level); and
- the effects of remediation.

We consider each of these in turn.

Statistical model of home radon levels. The model has been extensively validated (see Price, Nero, and Gelman, 1996, Price and Nero, 1996, and Price, 1997). In general, the model behaves well; cross-validation indicates that the uncertainty intervals are approximately correct, for example. However, it is likely that the lognormality assumption (for homes in a given county, with a given set of explanatory variables) underestimates the number of homes in the high tail of radon concentrations for some counties. For instance, Hobbes and Maeda (1997) suggest that some counties in Southern California might be better fit as a mixture of two lognormals, one with a low geometric mean for most of the homes, and one with a high geometric mean for the small fraction of homes on a particular geologic deposit. Similar high-radon pockets or exceptionally high within-county variability are known to occur in a few counties in Florida, New York, Washington State, and elsewhere.

From the standpoint of individual decisions, an underestimate of the size of the very high tail of radon concentrations would generally have a small effect: as long as the cumulative exposure for homes exceeding the action level is not seriously in error, the recommendation of whether or not to monitor will not be affected, so if the fraction of homes over 4 pCi/L or 8 pCi/L is fairly accurately estimated using the lognormal approximation, the exact distribution of a small number of very high homes is not critical. The fraction of homes over 4 or 8 pCi/L is fairly well estimated under the lognormal approximation for most of the counties with GMs over 1 or 2 pCi/L, respectively, and most counties with GMs lower than that have such low numbers of homes over the action level that even a large relative error in their prevalence would probably not change the monitoring recommendations. Given the large number of counties in the United States (over 3000) it is very likely that there are at least a few for which non-lognormality is a significant issue, but it is unlikely to seriously affect most of our results.

This whole issue becomes more important, though, if the action level is set very high (e.g., for a female nonsmoker living alone)—we would not trust the model's exact predictions when estimating the frequency of rare cases such as homes over 20 pCi/L.

On a different model-related topic, it is possible that the model can be improved by including more spatial or geological information (see, e.g., Boscardin, Price, and Gelman, 1996, Geiger and Barnes, 1994, Mose and Mushrush, 1997, Miles and Ball, 1996), which would cause predictions for individual homes to become more precise and the prior standard deviations S to decrease. For instance, radon mapping within counties would allow recommendations to discriminate more precisely among houses and thus increase expected lives saved for any given dollar expenditure. Indeed, such targeted recommendations may already be possible in localized (sub-county) areas that can be confidently identified as having disproportionate numbers of high-radon homes.

Magnitude of cancer risk from radon exposure. There is disagreement as to the estimate of lung cancer risk attributable to radon exposure, despite the efforts of the BEIR committee to thoroughly review the data available. The main issue is whether the results from the analysis of the data for miners could be generalized and applied to the progeny of radon in home (Lubin et al, 1997). Even if a linear no-threshold model is appropriate, the coefficients (the risk per unit exposure) are uncertain by at least a factor of 1.4. In addition, the model of discrete risks for smokers and nonsmokers is a simplification since smoking levels vary and many nonsmokers are exposed to second-hand smoke.

Linearity of the dose-response function. Experiments on animals, plus epidemiological studies with miners and other exposed at very high doses, suggest that at high doses the dose-response is approximately linear (see Nazaroff and Nero, 1988, Chap. 8–9). But there are really no good data at low doses. The Environmental Protection Agency assumes the function is linear all the way to zero, but others have suggested that there is a threshold (an exposure below which there is no effects) or even a protective effect at low concentrations (Cohen, 1995, Bogen, 1996).

Case-control studies suggest that, if there is a protective effect at low levels, it cannot be large, but mild protective effects, or a threshold so that levels below 5 pCi/L or so have no effect, cannot be ruled out. However, in spite of claims to the contrary by Cohen (1995), we are confident that long-term exposure to 2 pCi/L is safer than exposure to, say, 10 pCi/L (e.g. see Lubin and Boice, 1997).

Moreover, our results are less sensitive than one might suppose to nonlinearities in the dose-response function at low concentrations. This is because we assume that remediation reduces radon levels to 2 pCi/L, so the dose-response below that concentration is irrelevant. For instance, if long-term exposure at 2 pCi/L were actually safer than no exposure at all, that would have no effect on our analysis under the present assumptions.

To get some idea of the sensitivity of our results to the details of the dose-response relationship at low doses, we consider the effects of a relationship with a threshold at 4 pCi/L, so that exposure below that level has no health effect. One might examine this issue in several ways. For instance, we could ask what the optimal strategy would be under this modified dose-response relationship, and see how the recommended actions (e.g., which homes should monitor, and which should remediate) would change compared to the recommendations based on the linear dose-response. Instead, we look at how the number of lives saved would change if the strategy based on the linear dose-response were implemented—that is, if all of the same homes monitor or remediate as for the linear dose-response, but if the dose-response actually has a threshold. This seems to us to be the more relevant question, since our goal is to understand the robustness of the present analysis rather than to seriously propose analyses under alternative dose-response functions. Also, alternative recommendations would merely entail further restrictions on which homes are candidates for monitoring, so determining exactly which homes those are is not likely to be particularly instructive.

Given a threshold at 4 pCi/L, remediations in homes close to that threshold are mostly wasted (and all remediations are less beneficial), so we expect a reduction in lives saved. Some summary statistics are given in the columns labeled (b) in Table 5. As expected, the resulting number of lives saved substantially changes according to this assumption: compared to the situation with a linear dose-response, 37% fewer lives are saved for $R_{\text{action}} = 4$ pCi/L, and 18% fewer are saved for $R_{\text{action}} = 8$ pCi/L. Costs per life saved are still lowest under the recommended strategy 1.

Effect of remediation. We have assumed that remediation reduces a home radon level to 2 pCi/L. This cannot be accurate for several reasons. First, the post-remediation radon level must, in reality, vary among houses. In the context of our linear dose-response model, we can account for variation by considering the assumed post-remediation level as an expected radon level, averaging over houses. Second, the assumed reduction level of 2 pCi/L is a rough estimate from sparse data on remediation effects. Raising or lowering this post-remediation level would correspondingly raise or lower the recommended action level R_{action} and raise or lower the estimated costs per life saved. Third, the post-remediation level must certainly, in reality, depend on the initial radon level in a more complex way than simply $\text{E}(\text{post-remediation level}|R) = \min(R, R_{\text{remed}})$. In particular, we would expect that, for some houses with initially low radon levels (below 2 pCi/L), remediation might still have an effect. Unfortunately, available data on remediation effectiveness have been collected only for houses with fairly high pre-remediation levels—see Henschel, 1993, for examples.

For a sensitivity analysis, we consider a model in which the post-remediation radon level is lognormally distributed with GM equal to the square root of the pre-remediation radon level (in pCi/L) and GSD of 1.3, further constrained to not exceed the pre-remediation level. This rule is arbitrary, of course, but it behaves reasonably in that post-remediation radon levels are variable, and are sometimes above 2 pCi/L for houses originally above 4 pCi/L. Under this model, high-radon houses are typically not remediated all the way down to 2 pCi/L, so it is not surprising that the effects of the measurement/remediation strategy are less, with reductions of 13% and 15% of estimated total lives saved for $R_{\text{action}} = 4$ and 8 pCi/L, respectively (see columns (c) of Table 5).

Additional modeling and decision issues. We have made several simplifying assumptions and choices regarding what parameters to calculate. These include the following:

1. Examining benefits in terms of "lives saved" rather than, say, "quality-adjusted life-years saved";
2. Ignoring the influence of age and latency on personal risk: it takes several to many years for lung cancer to develop and to kill, once it has been initiated, so there is little benefit of remediation for,

say, a 70-year-old person—if a cancer has already been initiated then remediation is too late, whereas if they don't yet have cancer then they are likely to die of another cause before a cancer can kill them;

3. Ignoring possible interactions of radon exposure and age (e.g., children may have a different dose-response from adults).

4. Assuming risk is a function of cumulative 30-year exposure: if risk per dose is highly nonlinear then details of the temporal variation in radon exposure become important (so, for example, the effect of people moving from home to home must be considered);

5. Implicitly assigning zero cost to the hassle and stress of performing radon testing and remediation (a simplification that could be handled by adjusting the associated dollar costs).

All of these issues, and more, could in principle be addressed by adding additional parameters to the overall risk model. We chose instead to keep the model relatively simple, since our main goals are to illustrate how the geographic radon model can feed into a hierarchical cost-benefit analysis and to begin to bridge the gap between radon modeling and radon policy—for both of these goals our conceptually straightforward model seemed appropriate.

7 Discussion

We have used a Bayesian hierarchical model to analyze radon data in the U.S., thereby generating estimated distributional information, and uncertainties, for different types of homes in every state of the conterminous U.S. We used these results, along with estimates of radon risk taken from epidemiological data, to construct a formalism by which monitoring and remediation programs can be evaluated, allowing for individual variation in risk tolerance. To illustrate the use of this formalism, we examined the implications of a policy derived from the current EPA recommendation that sets 4 pCi/L as a remediation level, but that takes account of the wide variation in radon levels among counties. This sort of analysis can in principle be used by individuals trying to decide what actions to take but more importantly can be used by policy-makers to decide what actions to recommend or legislate.

As for the results themselves, under the assumptions used in this paper radon is indeed a major cause of lung cancer in the United States, associated with thousands of extra lung cancers per year. And yet, we recommend monitoring only for 28% of the population (or less, if separate action levels are to be used for smokers versus nonsmokers), and remediation is recommended for only homes in the highest few percent of all homes in the U.S. Our baseline recommended strategy (based on $D_d = 0.31$, equivalent to a marginal cost per life saved of \$310,000), would save only about 1,800 lives per year out of the estimated 15,000 radon-related deaths per year at an average cost per life saved of \$140,000. The problem is that because of the lognormality of the radon distribution, most of the total exposure (and thus, most of the expected radon deaths) is in people exposed at low levels of radon that cannot be substantially reduced by remediation (see

Figure 7). That is unfortunate from the standpoint of cancer prevention but fortunate from the standpoint of our analysis since it renders our recommendations relatively insensitive to the dose-response at very low concentrations. However, if cancer risk is a strongly nonlinear function of radon concentration for concentrations in the range of 2–10 pCi/L, then both the details of the dose-response and the effects of remediation for low-radon homes are crucial unknown quantities in the decision. Unfortunately, we see little hope for clarification of the dose-response issue for many years to come.

7.1 Policy implications

As discussed in this paper, smokers are thought to be at much higher risk of radon-induced lung cancer than are non-smokers. This makes radon a peculiar issue from the standpoint of public policy, as noted by Ford et al. (1998). Under the assumptions made in this paper a large majority of remediations *should* be performed by smokers, but smokers might be willing to accept more risk for lung cancer than are non-smokers. As Nazaroff and Teichman (1990) comment in an article that touches on many issues of radon risk reduction, "it seems unlikely that most smokers would make the necessary investment to reduce the radon-related risk of lung cancer when the dominant cause of their risk is smoking."

The results presented above incorporate uncertainties in the county radon distributions and explicitly allow for estimation using different assumptions about risk tolerance. As illustrated in the discussion of sensitivity analysis, it is also possible to tinker with the dose-response function and the assumptions about remediation effectiveness. An optimal decision strategy, within the framework of the model, can be determined for any choice of these parameters. But any such strategy is optimal only in the simplified world of the model. Reality differs from the model in many ways: not all people will act rationally or follow the recommendations of the model; it may be politically difficult to call for radon testing in some areas and not others, since doing so may lower property values; similarly, it may be difficult to call for different action levels for smokers and non-smokers, though in some sense it clearly makes sense to do so; people are impatient and may vastly prefer short-term tests to long-term ones; and so on. In the policy world, psychological, political and economic considerations can be at least as important as the scientific and statistical issues considered in this paper. And of course, even some scientific issues (most notably, uncertainty in the dose-response relation) are not fully addressed in our results.

However, this is not to say that our scientific and statistical results are useless. To the contrary, some conclusions are so clear that we think that policy can and should be changed to reflect them. Even considering possible non-lognormality within counties and variation in risk tolerance, there is no plausible scenario in which it makes sense to monitor every house in the country with a short-term measurement. The fact that high-radon homes (that is, over 4 pCi/L) have been found in every area of the country, which the EPA states when recommending universal testing, is true but irrelevant—someone living in a non-basement home in Louisiana surely has many risk-reduction options that are vastly more efficient uses of money and time than is performing a radon test (e.g., buy a smoke detector, get the car's brakes checked, visit a doctor, etc.). This is true even if we use the EPA's recommended remediation level of 4 pCi/L. As we have seen, that action level itself is not unreasonable—but it does not justify monitoring in every home. Of course, we say this with the luxury of having a great deal more information on the geographical distribution of radon than was available when the EPA's recommendations were first promulgated.

7.2 Generalizations to other decision problems

In the Bayesian approach to decision analysis, decision options are evaluated in terms of their expected outcomes, averaging over a probability distribution that is assigned jointly to all unknown quantities. The probability distribution is typically obtained by elicitation from experts, literature review, and sometimes data analysis (in which case it is identified as a posterior rather than a prior distribution). However, it is not yet common for decision analyses to use the sorts of hierarchical models that are becoming standard in Bayesian statistics (see, e.g, Carlin and Louis, 1996, and Gelman et al., 1995), as we have done in the present paper. When applied to decision analysis, such models have the desirable feature of assigning, to each hierarchical unit, a different parameter and thus different posterior probabilities and potentially different decision recommendations. This allows decision recommendations to vary, which in the radon example meant that even for a low action level of 4 pCi/L, we can restrict recommended measurements to only 28% of U.S. houses (see Figure 4).

Perhaps more importantly, having hierarchical recommendations allows us to assess the national effect as policy recommendations are varied continuously. With a nonhierarchical model, the results would simply reduce to: measure if R_{action} is in some range $[a, b]$, otherwise remediate if $R_{\text{action}} < a$ or do nothing if $R_{\text{action}} > b$, which would mean that a national standard on R_{action} would lead to uniform national recommendations, with perhaps some slight modifications for basement status and region of the country. The hierarchical model, in contrast, allows parameter estimates and uncertainties to vary by area, so that location-specific recommendations can be made and the influence of recommended actions within local areas can be assessed.

More generally, we suspect that hierarchical modeling can be combined with decision analysis in a wide variety of problems, which we hope will make the data analysis more useful and the decision-making more individually-focused. We also anticipate more sophisticated methods for computation (since, in general, the hierarchical posterior distributions that are input to these decision analyses will be summarized by simulation) and graphical display of the varying decision recommendations, continuing on the work developed in this case study.

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	Smokers		Nonsmokers	
	Female	Male	Female	Male
Lung cancer risk per pCi/L	0.0063	0.0118	0.0008	0.0013
Average numbers in US households	0.27	0.30	1.16	1.07

Table 1: Estimated additional lifetime lung cancer death risks for men and women, smokers and nonsmokers, for each additional pCi/L of lifetime exposure to radon. From National Research Council (1988). Average household populations by sex and smoking status are derived from the *Statistical Abstract of the United States* and *CDC* (1994), combining populations of children and adults.

Region	No basement	Basement is a living area	Basement is not a living area
1. New England	2.2 (1.3)	1.7 (1.1)	3.4 (1.3)
2. New York/New Jersey	1.6 (1.3)	1.6 (1.3)	3.0 (1.3)
3. Mid-Atlantic	1.6 (1.1)	1.6 (1.1)	2.8 (1.1)
4. Southeast	1.3 (1.1)	1.9 (1.1)	2.3 (1.1)
5. Midwest	1.2 (1.1)	1.6 (1.1)	2.2 (1.1)
6. South	1.3 (1.1)	1.8 (1.2)	1.7 (1.1)
7. Central Plains	1.5 (1.1)	1.7 (1.1)	3.1 (1.1)
8. Big Sky and Plains	1.2 (1.1)	2.1 (1.1)	3.1 (1.1)
9. Southwest	1.3 (1.1)	1.8 (1.2)	2.6 (1.1)
10. Northwest	1.2 (1.1)	1.9 (1.1)	4.0 (1.1)

Table 2: Correction factors by which one must divide a short-term winter radon measurement to estimate annual-average living-area level. Geometric standard errors of estimation for the correction factors are in parentheses; even if the correction factors were known perfectly, the annual-average living-area concentration would still be subject to large uncertainty due to temporal variability in the short-term measurements. From Price and Nero (1996).

	strategy			
	1	2	3	4
Fraction of all U.S. homes that measure	28%	100%	100%	100%
Fraction of all U.S. homes that remediate	5%	6%	8%	17%
Fraction of all homes over 4 pCi/L that remediate	75%	89%	74%	92%
Fraction of all homes over 8 pCi/L that remediate	93%	100%	95%	99%
Total cost ($ billion)	7.36	11.40	11.91	24.90
total cost of measuring	0.97	3.50	1.06	1.06
total cost of remediation	6.40	7.90	10.80	23.90
Expected lives saved	54,000	61,000	55,000	69,000
smokers	38,000	43,000	39,000	48,000
nonsmokers	16,000	18,000	16,000	21,000
Aggregate $ cost per life saved	137,000	186,000	214,000	358,000

Table 3: Some summary statistics on the effectiveness of various home radon measurement and remediation strategies: (1) recommended strategy based on decision analysis using the hierarchical model, (2) long-term measurements on all houses, (3) bias-corrected short-term measurements on all houses, (4) uncorrected short-term measurements on all houses. All are based on an action level of $R_{\text{action}} = 4$ pCi/L. Costs and lives saved cover 30 years.

	strategy			
	1	2	3	4
Fraction of all U.S. homes that measure	5%	100%	100%	100%
Fraction of all U.S. homes that remediate	0.7%	1.4%	2.4%	7.0%
Fraction of all homes over 4 pCi/L that remediate	13%	26%	36%	67%
Fraction of all homes over 8 pCi/L that remediate	46%	86%	71%	91%
Total cost ($ billion)	1.20	5.50	4.50	10.90
total cost of measuring	0.19	3.50	1.10	1.10
total cost of remediation	0.96	1.97	3.40	9.80
Expected lives saved	17,000	31,000	32,000	52,000
smokers	12,000	22,000	22,000	37,000
nonsmokers	5,000	9,000	10,000	15,000
Aggregate $ cost per life saved	66,000	175,000	139,000	211,000

Table 4: Some summary statistics on the effectiveness of various home radon measurement and remediation strategies: (1) recommended strategy based on decision analysis using the hierarchical model, (2) long-term measurements on all houses, (3) bias-corrected short-term measurements on all houses, (4) uncorrected short-term measurements on all houses. All are based on an action level of $R_{\text{action}} = 8$ pCi/L. Costs and lives saved cover 30 years.

Action Level	Strategy	Total lives saved (30 years) (a)	(b)	(c)	Total cost ($ billion)	Aggregate dollars per life saved (a)	(b)	(c)
2.5 pCi/L	1	74,000	37,000	71,000	18.5	249,000	495,000	265,000
	2	76,000	38,000	72,000	20.4	267,000	539,000	284,000
	3	69,000	36,000	66,000	21.7	316,000	598,000	327,000
	4	76,000	37,000	80,000	39.6	520,000	1,056,000	496,000
3 pCi/L	1	68,000	37,000	61,000	12.9	190,000	351,000	212,000
	2	72,000	38,000	65,000	16.3	226,000	431,000	250,000
	3	64,000	35,000	60,000	17.4	271,000	492,000	287,000
	4	74,000	37,000	76,000	33.4	451,000	899,000	441,000
4 pCi/L	1	54,000	34,000	47,000	7.4	137,000	216,000	157,000
	2	43,000	32,000	37,000	7.2	165,000	225,000	193,000
	3	56,000	33,000	51,000	11.9	214,000	362,000	235,000
	4	70,000	36,000	68,000	24.9	358,000	685,000	364,000
8 pCi/L	1	18,000	14,000	15,000	1.9	66,000	80,000	78,000
	2	31,000	25,000	27,000	5.5	175,000	219,000	206,000
	3	32,000	23,000	28,000	4.5	139,000	196,000	161,000
	4	51,000	31,000	47,000	10.1	211,000	349,000	233,000
12 pCi/L	1	5,800	5,200	5,000	0.3	45,000	51,000	52,000
	2	17,000	15,000	15,000	4.4	246,000	284,000	288,000
	3	20,000	15,000	17,000	2.6	129,000	167,000	151,000
	4	38,000	25,000	33,000	6.2	163,000	245,000	186,000
16 pCi/L	1	1,500	1,400	1,300	0.05	34,000	37,000	39,000
	2	11,000	9,600	9,200	3.9	362,000	403,000	421,000
	3	13,000	11,000	12,000	1.9	138,000	170,000	163,000
	4	30,000	21,000	25,000	4.2	143,000	202,000	166,000

Table 5: Sensitivity analysis: expected total lives saved and cost per life saved under four strategies for a grid of R_{action}, under three different models: (a) cancer risk without threshold and post-remediation radon level 2 pCi/L; (b) cancer risk with threshold 4 pCi/L and post-remediation radon level 2 pCi/L; (c) cancer risk without threshold and post-remediation radon level with a lognormal distribution with GM equal to the square root of the pre-remediation radon level and GSD of 1.3. The four strategies are (1) recommended strategy based on decision analysis, (2) long-term measurements on all houses, (3) short-term "screening" measurements on all houses, adjusted for bias, (4) short-term "screening" measurements on all houses, uncorrected.

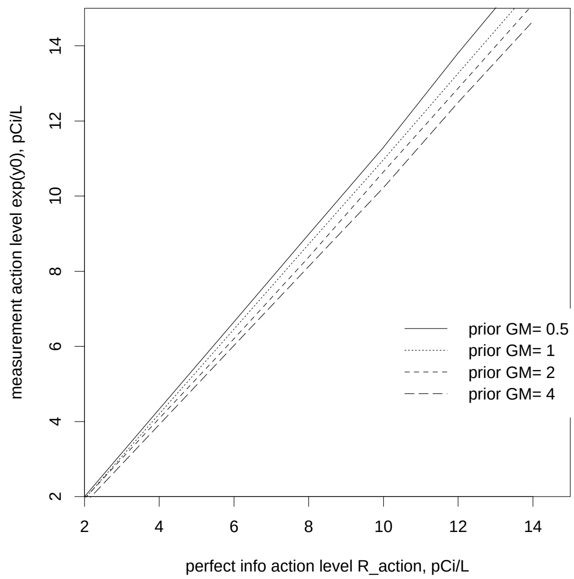


Figure 1: Measurement action levels e^{y_0} as a function of the perfect-information action level R_{action}, evaluated at values of the prior GM radon level e^M ranging from 0.5 pCi/L to 4 pCi/L.

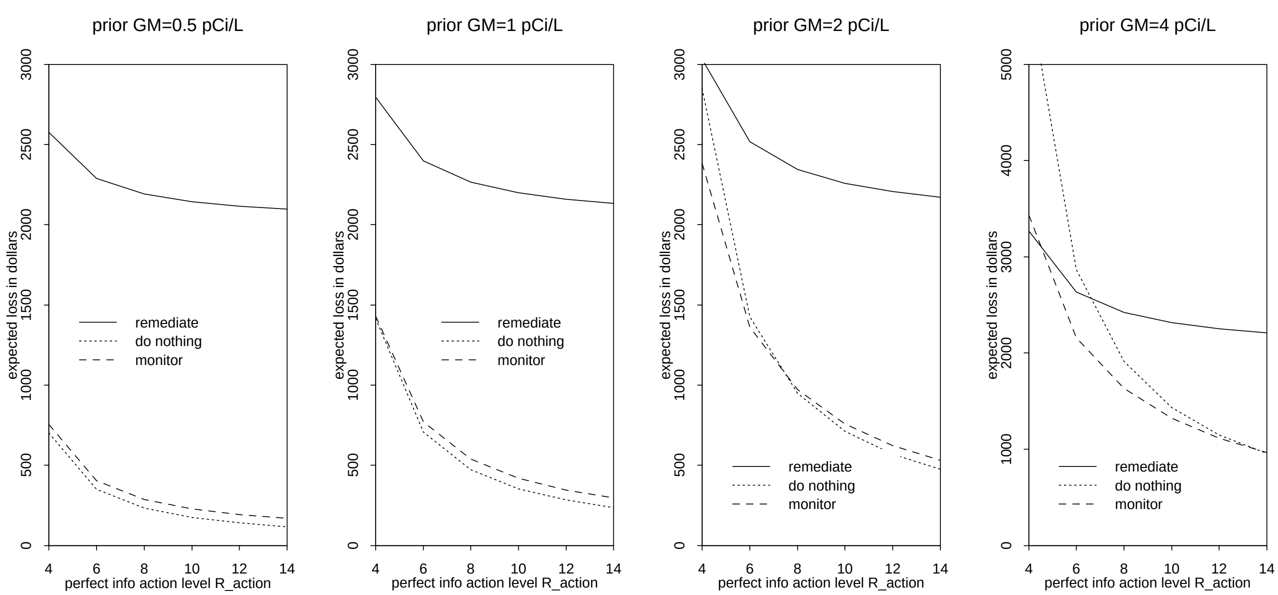


Figure 2: Expected losses in dollars (including the dollar value of the expected reductions in radon levels) of the three decisions: (1) remediate, (2) do nothing, (3) take a measurement, as a function of the perfect-information action level R_{action}. The four plots correspond to four different values of the prior geometric mean radon level e^M.

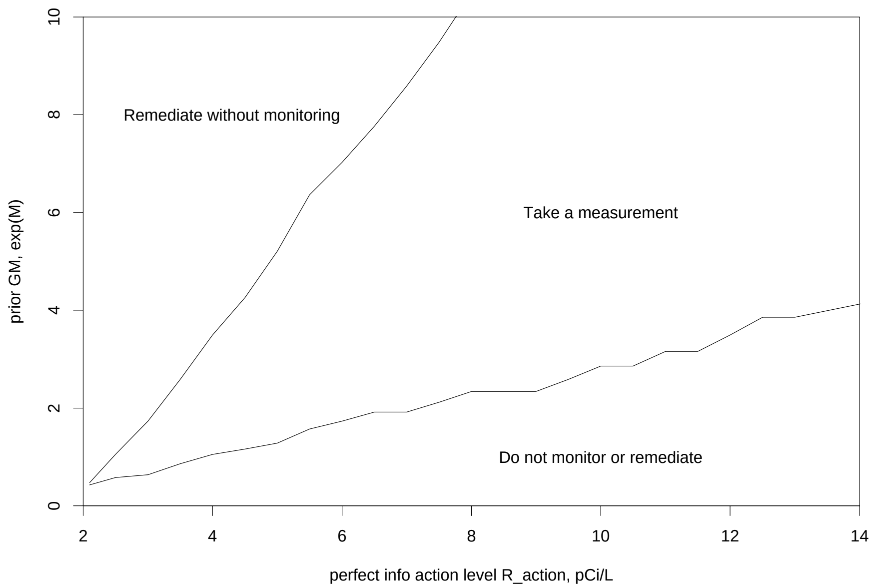
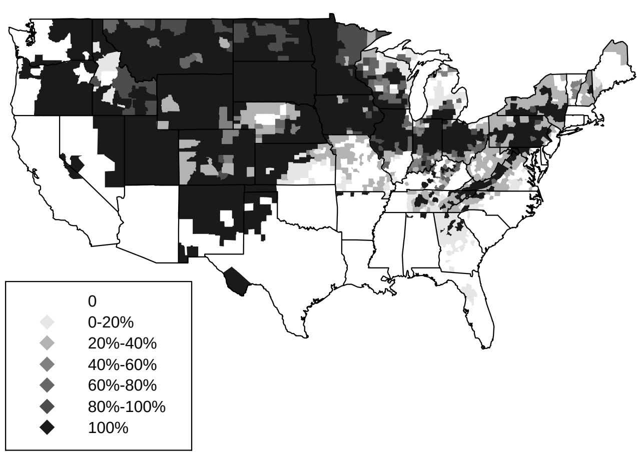


Figure 3: Recommended decisions as a function of the perfect-information action level R_{action} and the prior geometric mean radon level e^M, under the simplifying assumption that $e^S = 2.3$. You can read off your recommended decision from this graph and, if the recommendation is "take a measurement," you can do so and then use Figure 2 to tell you whether to remediate. The horizontal axis of this figure begins at 2 pCi/L because remediation is assumed to reduce ALAA radon level to 2 pCi/L, so it makes no sense for R_{action} to be lower than that value. Wiggles in the lines are due to simulation variability.



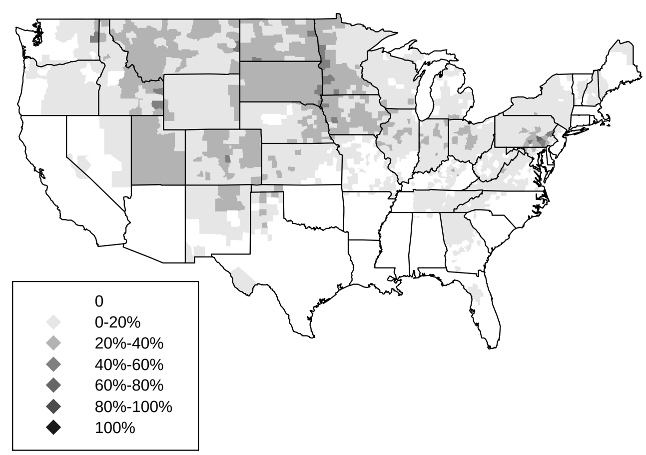


Figure 4: Map (a) showing fraction of houses in each county for which measurement is recommended, given the perfect-information action level of $R_{action} = 4$ pCi/L; (b) expected fraction of houses in each county for which remediation will be recommended, once the measurement y has been taken. For the present radon model, within any county the recommendations on whether to measure and whether to remediate depend only on the house type: whether the house has a basement and whether the basement is used as living space).

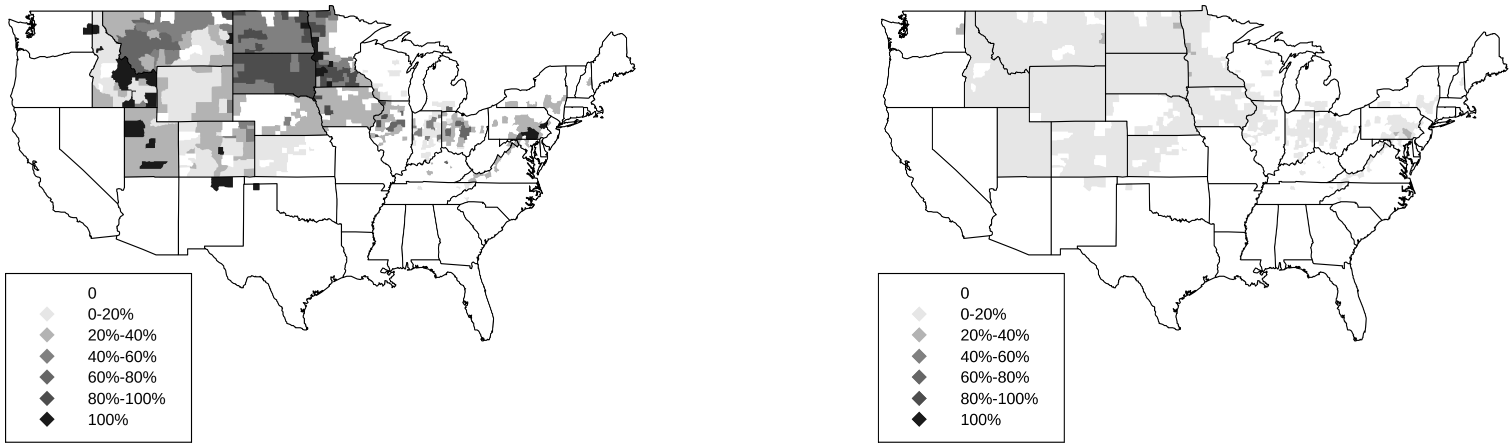


Figure 5: Map (a) showing fraction of houses in each county for which measurement is recommended, given the perfect-information action level of $R_{action} = 8$ pCi/L; (b) expected fraction of houses in each county for which remediation will be recommended, once the measurement y has been taken. As with the previous figure, the decision recommendations depend only on county and house type.

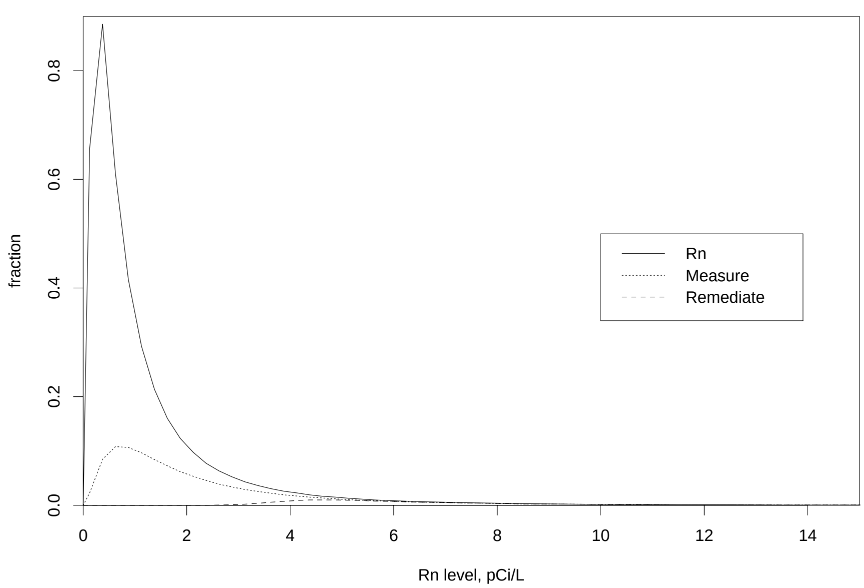


Figure 6: Estimated distributions of annual living-area average radon concentrations in (a) all U.S. houses, (b) all houses where measurement is recommended (with $R_{\text{action}} = 4$ pCi/L), and (c) all houses where remediation is recommended after measurement.

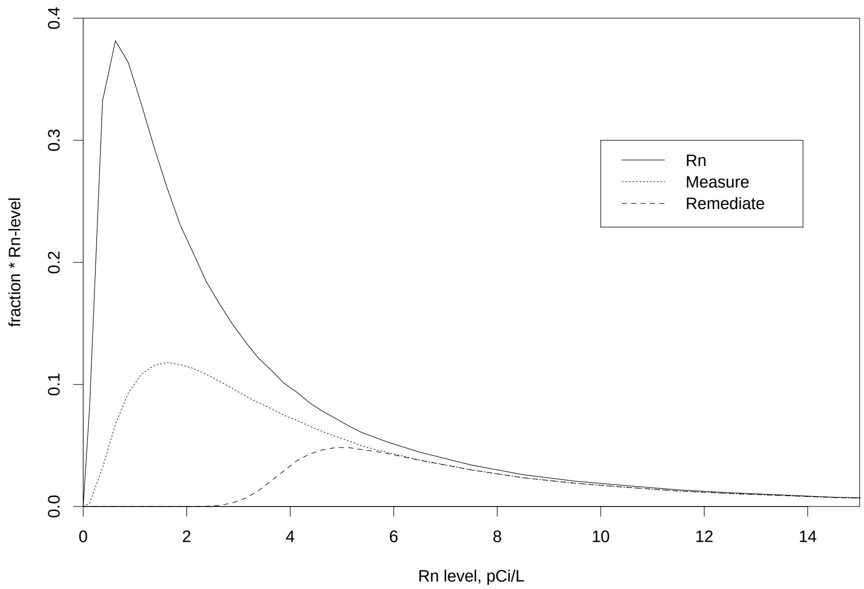


Figure 7: Fraction of total radon exposure, as a function of indoor radon concentration. Whereas the previous plot shows $f(\theta)$, this plot shows $\theta f(\theta)$. Curves are shown for: (a) all U.S. houses, (b) all houses for which measurement is recommended under the optimal strategy for $R_{\text{action}} = 4$ pCi/L), and (c) all houses for which remediation will be recommended after measurement.

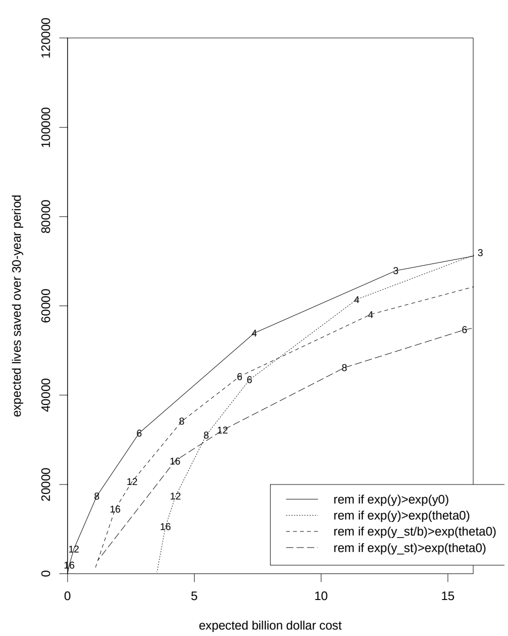


Figure 8: Expected lives saved vs. expected cost, for various radon measurement/remediation strategies. All results are estimated totals for the U.S. over a 30-year period.

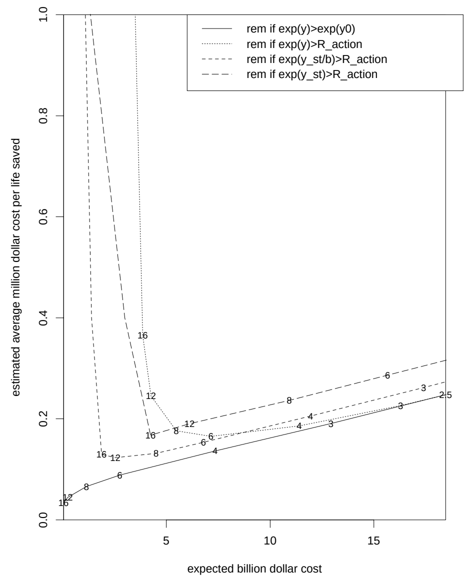


Figure 9: Estimated cost per life saved vs. expected cost (over a 30-year period), for various radon measurement/remediation strategies. By comparison, remediating every house has an estimated cost per life saved of \$3.56 million. Standard "acceptable" values for cost per life saved from risk analysis are in the range of \$300,000.

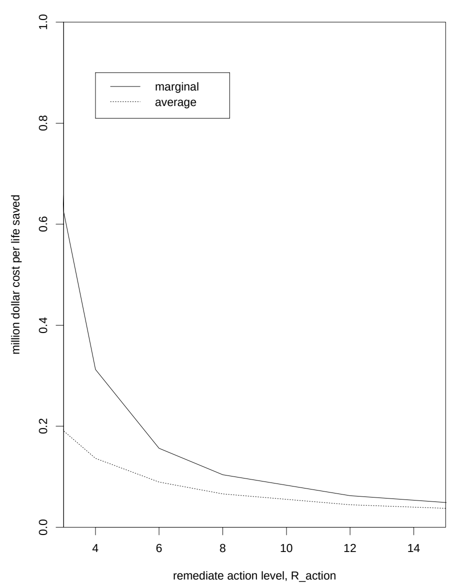


Figure 10: Estimated average and marginal costs per life saved vs. action level R_{action} for the recommended decision strategy. Average cost per life saved is computed averaging over the distribution of U.S. houses, as displayed in Figure 8. Marginal cost per life saved is $10^6 D$ (as defined in Section 4.1) based on a household with 0.3 male and 0.27 female smokers and 1.07 male and 1.16 female nonsmokers. Marginal cost is always higher than average cost because the marginal houses are those for which it is just barely cost-effective to remediate. Standard "acceptable" values for cost per life saved from risk analysis are in the range of \$300,000.