Homework 3
Due: Thursday, September 27, 2001

1. Reading.
   - In RPD you
     - Review Ch 2 on Matrix Algebra. I will not cover much of this in class.
     - Read Ch 3 on Multivariate Regression.
   - Work through pp 1–7 of the Splus tutorial on Models, "Statistical Modeling in Splus", available in many places, for example, at `http://www.stat.cmu.edu/~brian/707/S-Tutorial`. [The remaining pages deal with extensions of linear models which we may encounter later in the course.] *You do not have to hand anything in for this question.*

2. [Related to RPD Ch 2.] Please make sure you can do question 2.1 through 2.13, pp. 69–70, in RPD. To check (or speed) your work, the following commands in Splus are helpful:

```
A <- matrix(scan(),byrow=T,ncol=4)
     Makes a matrix with 4 columns (and as many rows as needed) by
     prompting you for input at the keyboard (fills the matrix row-wise).
x <- c(3,-3,2,1,3.14159)
     Makes a vector of 5 elements (usually Splus assumes this is a
     column vector but it also usually doesn't object to it being
     treated as a row vector).
x %*% y
A %*% x
A %*% B
     Computes the dot product of two vectors, or does matrix
     multiplication between a matrix and a vector, or between
     conformable matrices.
t(x), t(A)
     Computes the transpose of a vector x or matrix A
solve(A)
     Computes the inverse of an invertible square matrix A
tmp <- eigen(A)
     Computes the eigensystem of square matrix A:
     tmp$values is the vector of eigenvalues
     tmp$vectors is the matrix of eigenvectors (in columns)
prod(eigen(A)$values)
     Computes the determinant of square matrix A.
```

   You do not have to hand anything in for this question.

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3. We can express simple regression in matrix form

$$\underline{Y} = X\underline{\beta} + \underline{\varepsilon}$$

using the matrices $\underline{Y} = (Y_1, Y_2, \ldots, Y_n)'$, $\underline{\beta} = (\beta_0, \beta_1)$, $\underline{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$, and

$$X = \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix}$$

Show that the matrix form of the least-squares estimate of $\underline{\beta}$ reduces to

$$\hat{\beta}_1 = \frac{\sum_i (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_i (X_i - \overline{X})^2}$$

and $\hat{\beta}_1 = \overline{Y} - \hat{\beta}_1 \overline{X}$ as usual for simple regression.

4. RPD, problem 3.12 (pp. 96–97).

5. RPD, problem 3.13 (pp. 97–98).

6. A collector of antique grandfather clocks knows that the price received for the clocks increases linearly with the age of the clocks. Moreover, the collector hypothesizes that the auction price of the clocks will increase linearly as the number of bidders increases. Thus the hypothesized model is:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon,$$

where Y is the auction price, X_1 is the age of the clock in years, and X_2 is the number of bidders. The file `clocks.dat` in the data area of the 707 website contains observations on a sample of 32 clocks; for each, the collector recorded the price for which it sold, its age and the number of bidders present at auction. You have been called by the collector to help him evaluate his hypotheses.

   (a) Test the hypothesis that the auction price of a clock increases as the number of bidders increases, when age is held constant.

   (b) Interpret the estimates of the β coefficients in the model.

   (c) Compare the values of R^2 and adjusted R^2. Does the adjustment change your interpretation of the usefulness of the model?

   (d) Conduct an overall F test for the usefulness of the model. What do you conclude?