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Modeling Dependencies in International Relations Networks

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Despite the desire to focus on the interconnected nature of politics and economics at the global scale, most empirical studies in the field of international relations assume not only that the major actors are sovereign, but also that their relationships are portrayed in data that are modeled as independent phenomena. In contrast, this article illustrates the use of linear and bilinear random–effects models to represent statistical dependencies that often characterize dyadic data such as international relations. In particular, we show how to estimate models for dyadic data that simultaneously take into account: (a) regressor variables, (b) correlation of actions having the same actor, (c) correlation of actions having the same target, (d) correlation of actions between a pair of actors (i.e., reciprocity of actions), and (e) third-order dependencies, such as transitivity, clustering, and balance. We apply this new approach to the political relations among a wide range of political actors in Central Asia over the period 1989–1999, illustrating the presence and strength of second- and third-order statistical dependencies in these data.

1 Introduction

There is a long-standing tradition of "sovereignty" in world politics. Certainly since John Herz's (1950) association of idealism and realism in the sovereignty of the nation-state, world politics has been conceived of as relationships among sovereign entities, despite increasing evidence to the contrary. The geopolitical perspective that undergirds a realist perspective on politics goes far back into the nineteenth century. Rätzel (1879), Kjellén (1916), Haushofer et al. (1928), and Mackinder (1904), among others, promoted this geopolitical undergirding for world politics, an understanding that was quite successful in policy circles in Continental Europe (certainly in England and Germany) as well as in the

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United States.[1] The incorporation of these ideas—especially the notion of *lebensraum*—into the foreign policies of the Third Reich and especially the policy role played by Haushofer and his most famous student, Rudolph Hess, brought great, formal disregard for geopolitics in the post-World War II era.

Despite the widespread disrepute of geopolitics, its basic tenets guided much foreign policy throughout the last century. In this perspective, states were seen as irreducible objects. This meant that geopolitics, perhaps best epitomized by Henry Kissinger in the policy realm and by Hans Morgenthau in the scholarly community, implied an evaluation of world politics with a focus on the heft of monadic objects called states, a heft ascribed primarily to their "power," seen in the ability of individual states to affect the overall global geopolitical equilibrium. The main assumption of this conceptualization of politics is that world politics is (over)determined by the characteristics and actions of organizations asserting sovereign control over territory. This idea places very little importance on interactions, except as they may affect the heft or power of the individual nations in the global system.

Most investigations of world politics and studies of national security policies recognize the *interdependence* among the salient actors across the important issues but typically ignore this interdependence in empirical analysis. Traditionally, international politics has been defined as the scope and extent of the relations among independent countries, thought to be the most important elements in world politics. This means that actors as well as their actions are strategically interdependent (Signorino 1999, 2003). Ignoring the interdependence among these phenomena would appear to be a serious oversight that plagues attempts to understand, let alone predict, the course of national security policy and world politics. *Quantitative, systematic* studies of international relations and national security typically assume that the major events that comprise world politics consist of the independent actions of independent actors. An exception is analysis via game–theoretic models, but these rarely deal with more than a few actors at a time. Some beginning attempts to model the empirical interdependency in international relations have appeared in the literature (Ward and Kirby 1987; Smith 1999; Gleditsch 2002; Przeworski and Vreeland 2002).

Herein we develop a generalized regression framework for analyzing and accounting for the dependencies in valued and binary dyadic international relations data. This approach builds on the social relations model (Warner, et al. 1979; Wong 1982) that specifies random effects for the originator and recipient of a relation or action, and also allows for within–dyad correlation of relations.[2] We expand upon previous approaches by allowing for certain kinds of third-order dependence using an inner product of latent, unobserved characteristic vectors. The use of inner products to model dependencies is new and related to the recent development of "latent space" models for dyadic data (Hoff et al. 2002; Hoff 2003b; Ward et al. 2003). The idea of measuring latent characteristics or positions of political actors has a long lineage in political science, though not in international relations.[3]

[1]In Asia, the distinction between empire and state was not so clear-cut, but the omnipotence of the state was well established in both China and Japan by this point as well.

[2]This work also builds upon advances in techniques for decomposition of variances (Gill and Swartz 2001; Li 2002; Li and Loken 2002).

[3]See Martin and Quinn (2002) for recent developments as well as a summary of the canonical literature on "ideal points."

2 A Model of Dependent, Dyadic Interactions

Let Y denote an $n \times n$ matrix that contains dyadic measurements, so that the i, jth entry $y_{i,j}$ is the measurement of the relation from i to j. The matrix Y is often called a *sociomatrix.* Similarly, let X denote an $n \times n \times r$ array, so that $x_{i,j}$ is a vector of length r describing characteristics specific to dyad (i, j). It is typical to model these kinds of data with a linear regression approach:

$$y_{i,j} = \beta' x_{i,j} + \epsilon_{i,j}. \tag{1}$$

Examples of this type of model are common in the so-called democratic peace literature, where the errors $\epsilon_{i,j}$ are typically treated as independent. Dyadic data are replete in studies of international relations. Bilateral trade (Mansfield and Pollins 2003), the presence of conflicts and crises among countries (Brecher and Wilkenfeld 2000; Wilkenfeld 2001), alliances (Gartzke and Simon 1996; Leeds 2003), and joint membership in international and nongovernmental organizations (Russett et al. 2003) are examples of international phenomena that have been studied through analysis of *dyadic* data.[4] In contrast, we develop a random-effects model that can account for various second- and third-order dependencies that may be present in such dyadic data.

We begin by assuming that the errors $\{\epsilon_{i,j}, i \neq j\}$ have a covariance that is exchangeable under identical permutations of the indices i, j of the senders and receivers. With the added assumption of normality, this implies that the residuals can be represented in terms of a linear random–effects model. We decompose these effects into sender a_i, receiver b_j, and dyadic $\gamma_{i,j}$ components:

$$\begin{aligned}
&\epsilon_{i,j} = a_i + b_j + \gamma_{i,j} \\
&\begin{bmatrix} a_i \\ b_i \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{bmatrix} \right) \\
&\begin{bmatrix} \gamma_{i,j} \\ \gamma_{j,i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\gamma^2 & \rho\sigma_\gamma^2 \\ \rho\sigma_\gamma^2 & \sigma_\gamma^2 \end{bmatrix} \right).
\end{aligned} \tag{2}$$

This defines the following moments for the $\epsilon_{i,j}$'s:

$$\begin{aligned}
E(\epsilon_{i,j}^2) &= \sigma_a^2 + \sigma_b^2 + \sigma_\gamma^2 \\
E(\epsilon_{i,j}\epsilon_{j,i}) &= \rho\sigma_\gamma^2 + 2\sigma_{ab} \\
E(\epsilon_{i,j}\epsilon_{k,l}) &= 0 \\
E(\epsilon_{i,j}\epsilon_{i,k}) &= \sigma_a^2 \\
E(\epsilon_{i,j}\epsilon_{k,j}) &= \sigma_b^2 \\
E(\epsilon_{i,j}\epsilon_{k,i}) &= \sigma_{ab}.
\end{aligned}$$

where σ_a^2 represents dependence among dyadic observations having a common sender, σ_b^2 represents dependence among measurements having a common receiver, and ρ represents

[4]Among others, see Maoz and Russett (1993), Mansfield and Snyder (1995), Enterline (1996), Mousseau (1997), Beck et al. (1998), Russett et al. (1998), Bennett and Stam (2000), Russett and Oneal (2001), and Hewitt (2003). However, all of these studies ignore the network aspects of the dyadic data on which they are based.

correlation of measurements within a dyad, i.e., reciprocity. This has been called the "social relations" or "round robin" model (Warner et al. 1979; Wong 1982) and has recently been studied by Gill and Swartz (2001) and Li and Loken (2002).[5]

To accommodate other data types such as counts or binary measurements, the error structure in Eq. (2) can be incorporated into a linear predictor in the framework of a generalized linear model in which the dyadic data are conditionally independent given the random effects, but are *unconditionally dependent*:

$$\begin{aligned} \theta_{i,j} &= \beta' x_{i,j} + a_i + b_j + \gamma_{i,j} \\ E(y_{i,j} \mid \theta_{i,j}) &= g(\theta_{i,j}) \\ p(y_{1,2} \ldots y_{n,n-1} \mid \theta_{1,2} \ldots \theta_{n,n-1}) &= \prod_{i \neq j} p(y_{i,j} \mid \theta_{i,j}), \end{aligned} \tag{3}$$

where $g(\cdot)$ is the inverse-link function. For example, letting $g(\theta_{i,j}) = e^{\theta_{i,j}}$ and $y_{i,j} \mid \theta_{i,j} \sim$ Poisson $(e^{\theta_{i,j}})$ is equivalent to a mixed-effects Poisson regression model with the log-link.

The random-effects model above can capture second-order forms of dependence such as reciprocity and within-actor correlation. However, the social network literature (Wasserman and Faust 1994) suggests that third-order dependence patterns, such as transitivity and balance, are often found in dyadic data. Indeed, international relations would seem to be replete with these phenomena. For binary data, transitivity describes the dependence among three nodes i, j, k in which i and k are more likely to be linked if i and j are linked and j and k are linked. For signed measurements, such as residuals, a triad is called "balanced" if the product of the measurements among the three nodes is positive, i.e., $\epsilon_{i,j}\epsilon_{j,k}\epsilon_{k,i} > 0$. A weaker concept similar to balance is clusterability, in which a positive relation between i and j implies that $\epsilon_{i,k}$ and $\epsilon_{j,k}$ will have the same sign for each other node k. A set of nodes that are perfectly "clusterable" can be partitioned into groups that have all positive linkages within groups and negative linkages between groups.[6] In practice, dyadic data will exhibit varying degrees of transitivity, balance, and clusterability.

Building on Hoff et al. (2002), we use unobserved latent characteristic vectors to represent transitivity, balance, and clusterability among dyadic data.[7] We define an unobserved, latent K-dimensional vector z_i for each node i in the network, which can be thought of as representing a position in an unobserved latent characteristic space. Modeling the response between two nodes as an increasing function of the similarity of their latent characteristics induces a pattern of transitivity, balance, and clusterability into the network. We achieve this effect mathematically by adding the inner product $z_i'z_j$ to the linear predictor (3) to yield

$$\epsilon_{i,j} = a_i + b_j + \gamma_{i,j} + z_i' z_j, \tag{4}$$

with $\gamma_{i,j}$ portraying the dyadic error independent of the other bilinear and random effects. Note that if the vectors z_i and z_j have similar direction then $z_i'z_j$ will be positive and the effects $z_i'z_k$ and $z_j'z_k$ will be similar to each other, thus representing balance and

[5]Other models are certainly plausible. But one strong advantage of these normal assumptions is that they permit the development of a technique that falls well within the bounds of a fairly well-known and entrenched approach in the social as well as statistical sciences: a generalized linear model.

[6]These ideas were introduced into international relations in the 1960s via the balance of power literature (Zinnes 1967) but have been applied more recently by Lai (1995) to study reciprocity among superpowers.

[7]See Nowicki and Snijders (2001) for related efforts using latent approaches to modeling networks.

clusterability. Similarly, if $z_i'z_j$ and $z_j'z_k$ are both positive then $z_i'z_k$ is likely to be positive as well, thus representing transitivity.[8]

The inner product term is one way to measure a latent similarity. Hoff et al. (2002) discuss other ways of establishing the topology of latent space, including distance and projection approaches. The inner product used in the above model has the appeal of being analogous to an "error term" and facilitating substantive interpretation as a random or fixed effect. Considered as a random effect, if the z_i's are independent samples from a multivariate normal distribution with mean zero and covariance matrix $\sigma_z^2 I$, then the expectation of $z_i'z_j$ is zero. Furthermore, this inner-product term induces a third–order dependence that captures transitivity and balance via the expectation of the third–order moment $E(\epsilon_{i,j}\epsilon_{j,k}\epsilon_{k,i})$. The incorporation of $z_i'z_j$ into the linear predictor allows for additional structure on the moments of $\epsilon_{i,j}$:

$$\begin{aligned} E(\epsilon_{i,j}^2) &= \sigma_a^2 + \sigma_b^2 + \sigma_\gamma^2 + K\sigma_z^4 \\ E(\epsilon_{i,j}\epsilon_{j,i}) &= p\sigma_\gamma^2 + 2\sigma_{ab} + K\sigma_z^4 \\ E(\epsilon_{i,j}\epsilon_{j,k}\epsilon_{k,i}) &= K\sigma_z^6, \end{aligned}$$

with other second–order moments given as above and all other third–order moments equal to zero. Alternatively, as a fixed effect the inner-product term can be thought of as a reduced-rank interaction term. In this context, it has been called a "bilinear effect" or "multiplicative interaction" (Gabriel 1978, 1998).

We re-parameterize the bilinear model given above as

$$\begin{aligned} \theta_{i,j} &= \beta_d' x_{d,i,j} + s_i + r_i + \gamma_{i,j} + z_i' z_j \\ s_i &= \beta_s' x_{s,i} + a_i \\ r_i &= \beta_r' x_{r,i} + b_i, \end{aligned} \tag{5}$$

separating regressors ($x_{d,i,j}$) that are specific to a dyad from those specific to a sender ($x_{s,i}$) or receiver ($x_{r,j}$). Bayesian estimation of model parameters is made feasible by the use of conjugate priors and a Markov chain Monte Carlo algorithm, which samples values of the parameters from their posterior distributions: We construct a Markov chain in $\{\beta_d, \beta_s, \beta_r, \Sigma_{ab}, Z, \sigma_z^2, \Sigma_\gamma\}$ (where Z is the $K \times n$ matrix of latent vectors) that eventually samples from the desired target posterior distribution $p(\beta_d, \beta_s, \beta_r, \Sigma_{ab}, Z, \sigma_z^2, \Sigma_\gamma \mid Y)$. The algorithm proceeds by iterating three basic steps:

1. Resampling of linear effects:
   (a) sample $\beta_d, s, r \mid \beta_s, \beta_r, \Sigma_{ab}, \Sigma_\gamma, \theta, Z$ (similar to a linear regression);
   (b) sample $\beta_s, \beta_r \mid s, r, \Sigma_{ab}, \Sigma_\gamma, \theta, Z$ (similar to a linear regression);
   (c) sample $\Sigma_{ab}, \Sigma_\gamma$ from their full conditional distributions.
2. Resampling of bilinear effects:
   (a) for each i, sample $z_i \mid \{z_j\colon j \neq i\}, \theta, \beta, s, r, \Sigma_z, \Sigma_\gamma$ (similar to linear regression);
   (b) sample Σ_z from its full conditional distribution.

[8]If ϕ_i is the angle of z_i from a given fixed axis, then the inner product $z_i'z_j = \cos(\phi_i - \phi_j)$.

3. Resampling of dyad-specific parameters: update $\{\theta_{i,j}, \theta_{j,i}\}$ using a Metropolis-Hastings step as follows:

$$\text{propose}: \begin{bmatrix} \theta^*_{i,j} \\ \theta^*_{j,i} \end{bmatrix} \sim N\left(\begin{bmatrix} \beta' x_{i,j} + a_i + b_j + z_i' z_j \\ \beta' x_{j,i} + a_j + b_i + z_j' z_i \end{bmatrix}, \Sigma_\gamma \right)$$

$$\text{accept}: \begin{bmatrix} \theta^*_{i,j} \\ \theta^*_{j,i} \end{bmatrix} \text{ with probability } \frac{p(y_{i,j}|\theta^*_{i,j})p(y_{j,i}|\theta^*_{j,i})}{p(y_{i,j}|\theta_{i,j})p(y_{j,i}|\theta_{j,i})} \wedge 1.$$

There is no analytic solution. In any case, we have a large number (>200) of quantities to estimate. In this case, the full conditionals for the regression terms (β_d, s, r, β_s, β_r, Z) are multivariate normal, and the covariance terms have inverse-Wishart conditional distributions.[9]

This estimation procedure essentially provides a decomposition of $\theta_{i,j}$ into regressor, sender, receiver, and latent position effects in a generalized regression framework. This allows for the modeling of binomial, Poisson, and Gaussian network data in the presence of second– and third–order dependence. The model outlined also differs from the popular p^* approach (Wasserman and Pattison 1996) to modeling social networks. For one thing, the p^* models are specific to binary data and lack a natural extension to continuous or count data. Additionally, accurate parameter estimation for such models is difficult, and the most commonly used models often display a significant lack of fit.

The standard approach to analyzing the second-order dependencies in dyadic data in international relations relies on a set of mainly nonparametric approaches to "correcting" the empirical variance estimates, such as "panel corrected standard errors," "feasible generalized least squares," and "Newey-West" variance estimators. These typically focus on ways to ensure consistency of standard errors in large samples, even when the variance may be inefficient owing to dependence over time or cross section (Beck and Katz 2001). Broadly speaking, these are sandwich estimators—though they are known by different names in many different disciplines—and depend on asymptotic results for large samples. Heagerty et al. (2002) provide one example of this approach in the context of spatial dependencies; they also show that bias in empirical variance estimates can be upward or downward, depending on the extent of dependencies in sampled clusters. The bilinear approach takes an entirely different tack by modeling the dependencies themselves.[10] It differs from the nonparametric approaches in that it directly models the putative dependencies and permits statistical inferences. Further, the bilinear, latent space approach goes beyond second–order dependencies and assesses third–order properties, such as clustering. In the subsequent section we employ this approach to estimate the dependencies in dyadic data on the political interactions in Central Asia.[11]

3 Estimation of Network Links in Central Asia

We use the bilinear mixed-effects model to estimate the network structure of the political interactions among the primary actors in Central Asian politics over the period 1989–1999.

[9]More details on the full conditionals are available in Hoff (2003a).
[10]As developed it has not yet been extended to deal with time dependence.
[11]R routines, documentation, and sample data (including the data described below) to implement this approach are available at http//www.stat.washington.edu/hoff.

This region has a great deal of conflict and spotty coverage in English-language media, despite its contemporary salience. Event data collection has provided one way to examine the politics of such regions. Event data are nominal or ordinal codings of the recorded interactions of international actors.[12] Berelson (1952) introduced the concept of content analysis to the social sciences, but it was North et al. (1963) who pioneered its use in studies of world politics. Event data have been widely used in quantitative international relations research and in policy research for four decades (North 1967; McClelland and Hoggard 1969; Azar 1980). Until the development of machine coding, the World Event Interaction Survey (WEIS) and Conflict on Peace Databank (COPDAB) were the two dominant schema. The contemporary, state-of-the-art method is found in the Kansas Event Data System (KEDS), which uses automated coding of English-language news reports to generate political event data (Schrodt et al. 1994; Schrodt 2000).

While most event data analyses in the field of international relations have focused on the interactions of countries, there is no reason to presume that national governments are the only actors in world politics. We include both countries and noncountries as actors and targets. The data were taken from the Kansas Event Data Survey, an automated textually oriented data-generating process (Gerner et al. 1994; Schrodt 1994), available from http://www.ku.edu/~keds/data.html. The Central ASIA (CASIA) database records approximately 30,000 events concerning Central Asian politics over the period 1989–1999. We emphasize that these data were not constructed as a sample but resulted from an attempt to get the population of events in Central Asian politics during the period in question. We use these data as given, as a snapshot of the ebb and flow of political events concerned with Central Asia, a snapshot that contains a rich set of dyadic data. Based on the CASIA database, there are 106 actors with substantial interactions that have been deemed by substantive experts to be significant. Of these, there are 66 countries and 40 noncountry actors.[13] For the purposes of most of our analyses, we sum the paired interactions among all actors across the 11-year period so that $y_{i,j}$ is the total number of directed interactions from actor i to actor j, resulting in a 106×106 sociomatrix Y.

We separately examine the conflictual and cooperative interactions among these 106 actors. Conflictual interactions are defined as those having a negative Goldstein scale score (1992); cooperation is defined as events that have been assigned a positive score on this scale. Goldstein scale values, ranging from -10 (extreme conflict) to 8.3 (extreme cooperation), are psychometrically determined weights, where a positive weight means that the event has positive affect; conversely, a negative Goldstein score indicates negative affect.

We model the count of cooperative or conflictual interactions among these 106 actors with the generalized bilinear model, using a Poisson distribution with a log-link. We

[12]A good introduction to event data collections, as well as the data we use in this study, can be found at http://www.ku.edu/~keds/intro.html.

[13]The countries are Afghanistan, Angola, Armenia, Australia, Austria, Azerbaijan, Belgium, Belarus, Cambodia, Canada, Sri Lanka, China, Cuba, Cyprus, Czech Republic, France, Georgia, Germany, Ghana, India, Indonesia, Iran, Iraq, Israel, Italy, Japan, Kazakhstan, Kenya, North Korea, South Korea, Kuwait, Kyrgyzstan, Libya, Lebanon, Mexico, Mali, New Zealand, Norway, Netherlands, Pakistan, Palestine, Peru, Poland, Portugal, Qatar, Romania, South Africa, Saudi Arabia, Senegal, Singapore, Slovakia, Sudan, Switzerland, Tajikistan, Tanzania, Turkmenistan, Turkey, UAE, Egypt, Ukraine, UK, USA, Russia, Uzbekistan, Yemen, and Yugoslavia. Noncountries include groups such as peacekeeping forces, Islamic militants, Azerbaijan rebels, the Afghan military, Afghan opposition, Afghan politicians, and Afghan rebels; individuals such as Abdul Rashid Dostum and Usama Bin Laden; and international organizations, including the Commission on Security and Cooperation in Europe, the European Union, the North Atlantic Treaty Organization (NATO), the United Nations, and the Vatican.

Table 1 Fixed effects for dyads involving countries and noncountries

i	j	*Fixed–effects components*
$\in c$	$\in c$	$\beta_0 + \beta_s + \beta_r + \beta_1 + \beta_2 x_{i,j}$
$\in c$	$\notin c$	$\beta_0 + \beta_s$
$\notin c$	$\in c$	$\beta_0 + \beta_r$
$\notin c$	$\notin c$	β_0

Note. $i \in c$ indicates that actor i is a country; $i \notin c$ indicates that actor i is not a country but an individual, organization, or other group.

include sender and receiver random effects, as well as a two-dimensional latent space or bilinear effect.[14] In addition, we employ the so-called Tobler law of geography (Tobler 1979) to reflect the fact that actors that are close geographically have a higher rate of interaction. Although it is impossible to determine with certainty the geographical "location" of most nonstate actors—where, for example, "international negotiators" are located—we can with some confidence determine the nearest neighbor distances of the state actors. To gauge intercountry distances, we use nearest neighbor distance (Gleditsch and Ward 2001) up to 950 kilometers complemented by the distance in thousands of kilometers between the capital city of each of the countries for larger distances.[15] We also allow for the possibility that countries will have a different propensity for dyadic interaction than noncountry actors, on both the cooperative and conflictual scales. Therefore, we model the rate of interaction between two actors as depending on the country/noncountry status of both the sender and receiver.[16] The hierarchical model specifying these ideas is given by

$$\begin{aligned}\theta_{i,j} &= \beta_0 + \beta_s \times (i \in c) + \beta_r \times (j \in c) + \beta_1 \times (i,j \in c) \\ &\quad + \beta_2 x_{i,j} \times (i,j \in c) + a_i + b_j + z_i' z_j + \gamma_{i,j}, \\ y_{i,j} \mid \theta_{i,j} &\sim \text{Poisson}(e^{\theta_{i,j}}),\end{aligned} \tag{6}$$

where $x_{i,j}$ is the distance between countries i and j (in thousands of kilometers), and c is the set of countries. The fixed effects, shown above in the first line of Eq. (6), are further detailed in Table 1.[17]

[14]The choice of K is not obvious. For descriptive purposes $K \in \{1, 2, 3\}$ allows for straightforward graphical presentation of results. Based on cross-validation experiments for similar data (Hoff 2003a), using the log probability of the data given the parameters indicates that $K \in \{2, 3\}$ provides roughly equivalent predictive model performance. As a result, we employ two latent dimensions.

[15]These are scaled by 1000.

[16]There are undoubtedly many other specifications that could be explored as well as other variables that scholars may wish to include in models of dyadic conflict. Our point in this article is not to provide the best model of international conflict or cooperation but to demonstrate the importance of second- and third-order dependencies. Moreover, the random sender, receiver, and dyad effects may be thought to include other linear factors that have been excluded from the specific model examined here.

[17]The random effects are each taken to be distributed as multivariate normal: $(a_i, b_i)' \sim N(0, \Sigma_{ab})$; $(\gamma_{i,j}, \gamma_{j,i})' \sim N(0, \Sigma_\gamma)$; and $z_i \sim N(0, \sigma_z^2 I_{K\times K})$. The prior distributions of β are given multivariate normal, where q indexes β's dimension: $\beta \sim N(0, 1000 \times I_{q\times q})$. The variance of the sender and receiver effects, Σ_{ab} is modeled as inverse Wishart($I_{2\times 2}$, 4). Hyperparameters σ_u^2, σ_v^2 are taken to be i.i.d. inverse gamma (1, 1) and $\sigma_\gamma^2 = \frac{\sigma_u^2+\sigma_v^2}{4}$ and $\rho = \frac{\sigma_u^2-\sigma_v^2}{\sigma_u^2+\sigma_v^2}$.

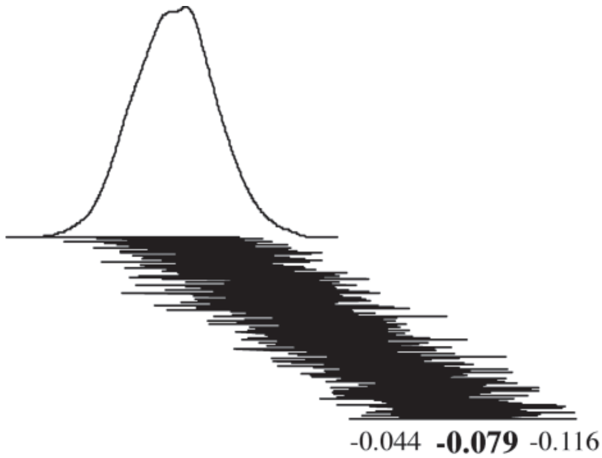

Fig. 1 The posterior density and a time series plot of the Markov chain for the distance parameter for the cooperation data. The mean response is greater for actors that are closer together geographically, and the estimated coefficient is -0.079; 95% empirical confidence interval is $[-0.044, -0.116]$. See Table 2 for posterior means and confidence intervals for all estimated parameters. Note that this image is rotated $-50°$ to provide perspective.

Separate Markov chain Monte Carlo algorithms were run for the conflict and cooperation data. Each chain was run for 200, 000 iterations, with output saved every 50th iteration.[18] Figure 1 illustrates the posterior densities and a time series plot of the Markov chain for the distance parameter β_2 is shown in Fig. 1 for the cooperation data. This mixing is representative of the mixing of the other parameters.

The parameter estimates for conflict and cooperation are presented in Table 2. These estimates illustrate that the differences between networks of conflict and cooperation are not immense, but they are recognizably different. There is a strong distance effect for both cooperation and conflict: the rate of political interaction is inversely related to distance, as expected, because both of the parameters are negative. The distance effect is about 30% greater for conflict than cooperation. This particular effect applies only to dyads for which the distance measurement makes sense, i.e., for dyads in which both nodes are countries (see Table 1). Thus the effects of pooling countries and noncountries are separated out by the model specification and shown to be empirically important. The effect of being a country seems about 40% stronger for conflict than for cooperation, suggesting that countries are more likely to share conflict linkages. The effect of countries being separately senders and receivers illustrates that countries are twice as likely to send cooperation but more likely to receive conflict. The estimates of ρ indicate a large degree of reciprocity in reported actions, but it is substantially higher in cooperative social relations than in conflictual ones, indicating a large degree of within-dyad dependence.

The model also detects a large degree of sender- and receiver-specific variance, as well as third–order dependence, because σ_a^2, σ_b^2 and the variance of the inner products $\sigma_{z'z}^2$ are all substantially larger than σ_γ^2. The standard approaches currently found in the literature assume that these effects are individually and jointly nonexistent. Finally, Fig. 2 presents posterior means of random effects for senders and receivers for conflict. The actors have

[18]In principle, every iteration contains some information and it is not necessary to thin the chain. However, due to the large number of parameters in our model, we store only every 50th iteration in order to keep the size of the output file reasonable (MacEachern and Berliner 1994).

Table 2 Posterior means and quantile-based 95% confidence intervals (above and below) for the major parameters (in bold) of the bilinear-effects model

		Posterior means	
		Conflict	*Cooperation*
		−0.067	−0.044
Distance effect	β_2	**−0.105**	**−0.079**
		−0.144	−0.116
		0.914	0.693
Country effect	β_1	**0.550**	**0.328**
		0.182	−0.006
		−3.542	−3.082
Intercept	β_0	**−4.751**	**−4.363**
		−6.015	−5.669
		0.993	1.129
Sender effect	β_s	**0.151**	**0.292**
		−0.642	−0.580
		1.337	1.201
Receiver effect	β_r	**0.499**	**0.365**
		−0.345	−0.457
		4.913	5.520
Common sender variance	σ_a^2	**3.598**	**4.082**
		2.604	3.045
		4.640	5.204
Sender-receiver covariance	$\sigma_{a,b}$	**3.398**	**3.853**
		2.450	2.842
		4.876	5.127
Common receiver variance	σ_b^2	**3.569**	**3.790**
		2.592	2.785
		1.631	1.556
Error variance	σ_γ^2	**1.439**	**1.380**
		1.265	1.216
		0.851	0.978
Reciprocity	ρ	**0.805**	**0.968**
		0.749	0.957
		1.428	1.214
Variance of latent dimensions	σ_z^2	**1.145**	**0.972**
		0.918	0.774
		3.231	2.336
Variance of inner product	$\sigma_{z'z}^2$	**2.623**	**1.892**
		2.077	1.488
Log likelihood of $Y_{i,j}$, $Y_{j,i}\,|\,$modeled effects		**−4685**	**−6005**

Note. Applied to both conflict and cooperation among 106 major political actors in Central Asia, 1989–1999. Data are available from http://www.ku.edu/~keds/data.html and are briefly described above. Statistics presented are from runs of 200K iterations with output sampled every 50 iterations; the first 50K iterations were discarded as "burn-in," although results are not appreciably different if they are included. Two latent dimensions were estimated for both domains, i.e., $K = 2$.

Broad comparison shows that there is a similar structure to cooperation and conflict. Countries that are distant geographically have lower interaction rates in both domains. Cooperative events are more highly reciprocated within dyads than conflictual ones. Strong second- and third-order dependence is evident in both cooperative and conflictual dyadic interactions.

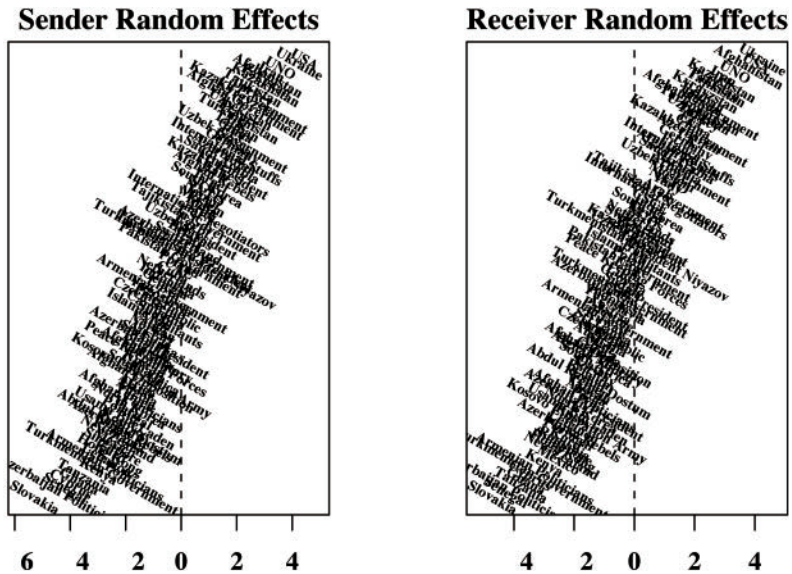

Fig. 2 Orderings of sender-specific and receiver-specific random effects are similar for the conflict data. The United States, the United Nations, Ukraine, Afghanistan, Iran, Kazakhstan, and Pakistan are actors with large, positive random effects sending and receiving both conflict and cooperation (not shown).

similar random effects orderings across both domains of dyadic interaction, though only conflict orderings are presented herein.

Figure 3 plots the posterior means and marginal distributions of the latent positions for the analysis of cooperation and conflict in Central Asia. The actor names are located at the posterior means of the latent positions. This illustrates considerable clustering in the estimated dyadic relations. Though not presented, there are similar patterns to both conflict and cooperation in the political interactions in Central Asia. Several clusters are identifiable. The lower left-hand portion is dominated by actors involved in the Armenian and Azerbaijan conflict that dominated the later part of the 1980s in Central Asia. Central European involvement in Kosovo with its implications for Central Asia cluster above this, with the Vatican, Commission on Security and Cooperation in Europe (CSCE), NATO, and other European actors being visible. At the same time, the ebb and flow of politics in southern Central Asia, particularly Afghanistan and Pakistan, present a visible cluster in the lower right of the latent space. Immediately above that one finds the cluster of Middle Eastern, Moslem, and Arab actors. The bottom panel presents sampled positions for three important actors in central Asia (which appear in quadrant I of the top panel): Usama Bin Laden, Saudi Arabia, and Abdul Rashid Dostum. This shows that the latent positions also have uncertainty. In this instance, the posterior distribution of these positions overlaps considerably for Saudi Arabia and Usama Bin Laden, which are close to one another in latent space. Abdul Rashid Dostum is relatively distant from each of these two actors.[19]

In terms of prediction, the bilinear effects approach is quite accurate. For an arbitrary draw—the 200,000th—from the posterior distribution, we compare the predicted number

[19]The complete graphics, including a color visualization of the uncertainty of each of these latent positions, are available in the auxiliary materials for this article on the *Political Analysis* Web site.

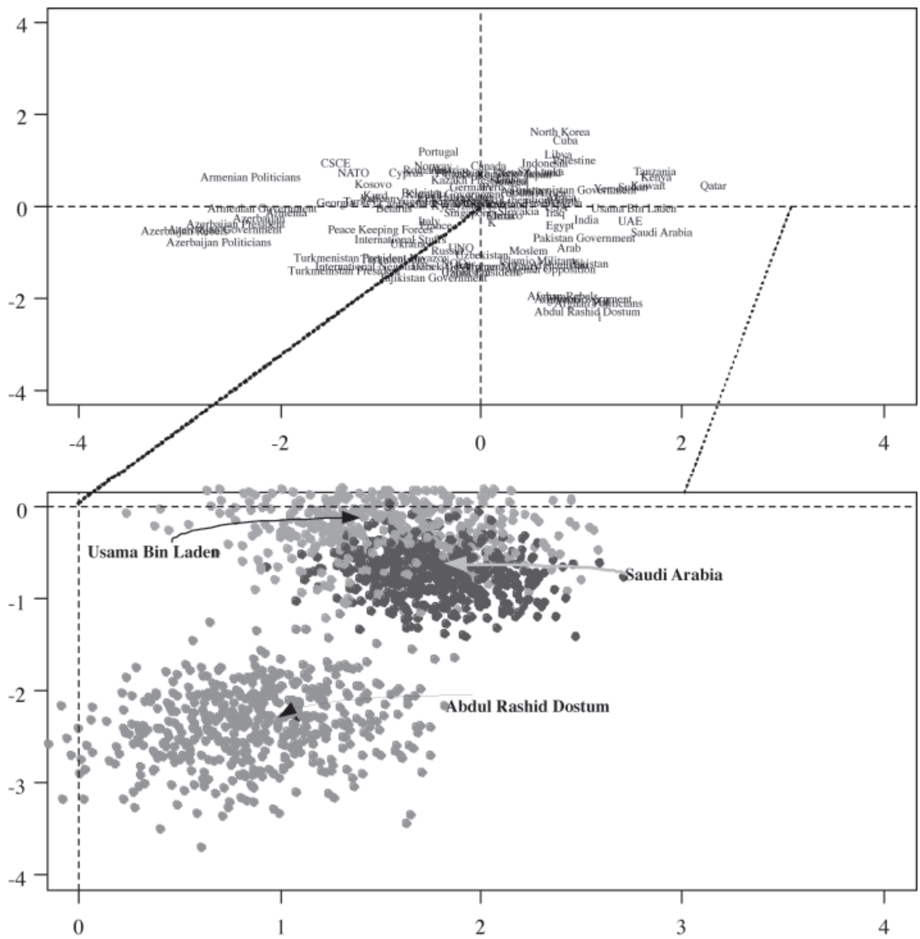

Fig. 3 A sample of the estimated posterior latent positions in two dimensions for conflict interactions identifies nodes that are similar in their patterns of interaction. The posterior mean position for each actor is located at the center of the text string identifying it. Only the last 100,000 iterations of the Markov chain were used in the sampled latent positions presented here. The top panel presents the entire set of latent positions for the conflict data; the bottom panel expands the quadrant IV positions.

of linkages between actors *i* and *j*. The correlation of the predicted number of linkages and the actual number of linkages is 0.85 for conflict and 0.86 for cooperation.[20] If we compare the predicted fit for the fixed-effects model estimated with no latent positions (i.e., with $K = 0$) with this same model using two latent dimensions, it is apparent that the fit is improved by the inclusion of the latent positions in the model. The correlation between predicted and actual number of cooperative events in the latter case is 0.85, compared with 0.75 when no latent positions are estimated.

While the in-sample fit of the model is strong, it is also important to gauge its out-of-sample predictive performance. This provides an important heuristic to determine the predictive value added for the random effects specified in the model. To examine this we

[20]These are an order of magnitude larger than for a simple model that ignores dependencies.

Table 3 Correlation between observed values and out-of-sample predictions

	Fixed effects	*Fixed effects + random intercepts*	*Fixed effects + random intercepts + latent positions*
Cooperation	0.13	0.68	0.86
Cooperation (log scale)	0.17	0.56	0.61
Conflict	0.07	0.60	0.91
Conflict (log scale)	0.11	0.53	0.60

Note. The out-of-sample predictions show the marginal benefits of modeling the sender and receiver random effects, as well as the additional predictive gain of the inner product of latent positions. For cooperation a generalized linear model produces out-of-sample predictions that have a correlation of 0.13 with out-of-sample measurements, which rises to 0.68 when random effects for senders and receivers are included and to 0.86 when the inner product of latent positions is also modeled. The gain is even stronger for conflict, rising from 0.07 in the first instance to 0.91 in the last. These patterns are less striking when measured on the log scale but are still evident.

undertook a small comparative study of out-of-sample predictive performance. For both the cooperation and conflict data, one-third of the data was randomly replaced with missing values, and three models were fit using the remaining "in-sample" data. The first uses only the fixed effects (i.e., only the β coefficients); a second model additionally includes random intercepts having country-specific sender and receiver effects; and the third model also includes the inner product effect $z_i'z_j$, with $K = 2$. The posterior mean parameter values were then used to predict the one-third of the data reserved as "out-of-sample" and the correlations of predicted and actual responses were computed in a raw, untransformed as well as a logarithmic scale.[21] Table 3 presents the results of this out-of-sample experiment.

The addition of the random effects—spanning the random intercepts for senders and receivers as well as the inner product of the latent positions—increases predictive performance substantially, viewed in terms of the correlation between actual and predicted responses. The effect is particularly dramatic on the raw scale of the data, for which the inner product model is able to predict some very high responses, whereas the other models cannot.

4 Conclusion

This article presents a generalizable way to account for types of second- and third-order dependencies in regression models for dyadic data. Most current analyses in political science—and especially international relations—ignore all of these dependencies. The latent space, bilinear regression approach is a major step forward for analysts interested in the interdependencies of dyadic data that are often used to characterize world politics. This provides a practical framework that can be used to empirically estimate and display a range of important dependence patterns in dyadic data.

This alone is an important breakthrough. Moreover, the approach facilitates the presentation of latent positions in an intuitively satisfying way, mapped into a small number of dimensions. Confidence regions for these latent positions are also available via the Markov chain Monte Carlo procedure, allowing for prediction of unmeasured relations,

[21] A small constant was added so that the correlation on the log scale was computed as cor[$log(.1 + y_{i,j})$, $\hat{\theta}_{i,j}$].

as well as confidence statements about such predictions. Perhaps most important, this approach is quite general, since it encapsulates a broad class of models. Specifically, a variety of discrete and continuous specifications can easily be adapted, depending upon the data-generating process. This allows scholars to embrace interdependence in an empirical framework that is not only rich in description at the subnational, national, multi- and transnational, and systemic levels, but one that is also firmly rooted in well-understood statistical methods such as generalized linear regression and random-effects modeling.

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