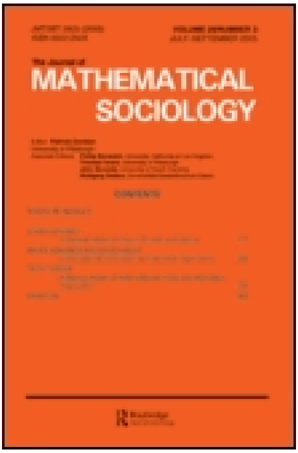

The Journal of Mathematical Sociology
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmas20

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Version of record first published: 26 Aug 2010.

Journal of Mathematical Sociology
2001, Vol. 25(1), pp. 5–41
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RANDOM GRAPH MODELS FOR TEMPORAL PROCESSES IN SOCIAL NETWORKS*

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(Received April 14, 2000; In final form September 1, 2000)

We generalize the graphical modeling approach of p^* social influence models to develop discrete time models for the temporal evolution of social networks. Plausible general processes pertaining to network evolution are broadly discussed as a basis for across-time dependence assumptions. *Systematic temporal processes* are construed as effects that are homogeneous across the network, and that reflect dynamics inherent in a particular social relation. Any one actor cannot control these dynamics, especially given that non-dyadic configurations may be implicated, for instance, tendencies for various triadic configurations to be constructed or to collapse of over time. *Non-systematic processes*, on the other hand, may pertain to the changing nature of a particular dyadic tie, or to change involving a particular *sociotemporal neighborhood* of the network. Non-systematic processes are inhomogeneous across time and across the network, and are modeled as random.

In constructing p^* dependence graphs, systematic temporal processes may be represented, in part, by the *perfect dependence assumption*, whereby network across-time dependencies "mirror" within-time dependencies. We develop temporal perfect dependence models appropriate for Markov random graphs. To disentangle non-systematic from systematic temporal processes is not straightforward, but the use of the *constant tie assumption* – whereby ephemeral ties are assumed not to have influence across time – is one possible approach. We illustrate these models with three empirical examples: first, with an analysis of the Freeman EIES data; and then with data from a newly formed small training group involving two networks, trust and friendship.

* An earlier version of this paper was presented at the Sunbelt International Social Networks meeting, Vancouver, April 2000. The authors would like to thank Tom Snijders and Peter Elliott for helpful comments on this paper.

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1 INTRODUCTION

There is a growing emphasis in the network literature on the need to model network change over time (Doreian and Stokman, 1997; Frank, 1991; Stokman and Doreian, 1997; Suitor, Wellman and Morgan, 1997). In their review of recent work on network evolution, Doreian and Stokman (1997) classified studies of network process into four main categories: studies that predicted attributes from structural information, akin to the contagion process described by Leenders (1997a); descriptive studies of network change; studies in which the network nodes, or other features of network structure, were distributed over time (Doreian and Stokman, 1997, referred to network structures "unfolding through time"); and studies where network change is seen as a transition in network structure between time points.

In this article, we describe a random graph (p^*) modeling approach to temporal network change. Robins, Pattison and Elliott (in press) and Robins, Elliott and Pattison (2000) presented p^* models for social influence and social selection, respectively. These models can be seen in the context of Doriean and Stokman's (1997) first category of studies, involving the interplay of attributes and social structure. In these models, time was not a variable, but as Robins *et al.* (in press) discussed, temporal processes are nevertheless implicit. The models in this article, on the other hand, explicitly include time. We present a random graph discrete time model of network change across two or more time points, falling within Doriean and Stokman's fourth category of network transition.

The p^* class of models has its foundation in the work on Markov random graphs by Frank and Strauss (1986) and Strauss and Ikeda (1990). The class of models was more fully explicated in three papers, detailing models for univariate networks (Wasserman and Pattison, 1996), for multivariate networks (Pattison and Wasserman, 1999), and for valued networks (Robins, Pattison and Wasserman, 1999). The models are underpinned by the Hammersley – Clifford theorem (Besag, 1974), which provides a probability description for a collection of interacting variables, assuming a given set of dependencies among the variables. The set of dependencies can be depicted in a *dependence graph*, with the nodes representing variables and an edge representing an assumed conditional dependence between two variables (i.e., conditional on all other variables). It is this approach to complex dependence structures that enables p^* models to go beyond dyadic independence network models typified by p_1 (Holland and Leinhardt, 1981).

In standard p^* models, the dependence graph represents dependencies among network variables. For social influence models, Robins *et al.* (in press) generalized the dependence graph to allow two blocks of variables with directed dependencies from one to the other, by specifying the set of network variables as predictors of a collection of mutually dependent attribute variables. Graphical modeling techniques (e.g., Lauritzen, 1996) provided the means to deal with directed dependence graphs. This approach also allows us to model network change. Rather than having attributes as predictors, we use the network measured at time t_1 to predict the network at time t_2. This generalization is relatively straightforward: the major difficulty lies in specifying appropriate cross-time dependence structures. Before we introduce details of the model, we discuss what we see as an important omission in the network change literature: there is only a limited consideration of plausible dependence structures for models of network evolution.

1.1 Dependence Structures and Sociotemporal Neighborhoods

Robins *et al.* (in press) noted that the central task in developing effective p^* models is the specification of an appropriate dependence structure. In some cases, investigators may be assisted by theoretical or empirical results, but often little such guidance is available. For non-temporal network models, there is value in investigating a nested series of increasingly complex dependence assumptions (see Pattison and Wasserman, 1999, for an example of this type of approach), commencing with a Bernoulli graph assumption (where all possible network ties are assumed independent), followed by a dyadic independence assumption, and then a Markov random graph assumption (where possible ties are conditionally dependent whenever they share an actor).

Pattison and Wasserman (1999) discussed a number of common theoretical themes in the social modeling literature and translated them into counterpart dependence assumptions. For a model to capture these widely accepted theoretical effects, a Markov random graph dependence structure at the least is required. But as Pattison and Robins (2000) argued, there are often good reasons to impose some constraints on the Markov random graph assumption, as well as to examine dependence structures that exemplify longer range effects in the network. Pattison and Robins described the techniques, based on the notion of *partial conditional independence,* that enable the exploration of these more complex dependence structures. They proposed that interdependencies occured within localized regions of a social space (*local social neighborhoods*). In p^* models, dependence graph cliques identify subsets of ties assumed mutually conditionally dependent, that is, ties (and individuals) that constitute possible sites of mutual contingency. In that sense, the cliques of the dependence graph can be identified with hypothesized social neighborhoods. Individuals, of course, may occupy many different neighborhoods, allowing processes (e.g., social influence) to be generated across the entire network. The p^* class can then be seen as models for global network structures arising from self-organizing processes in local, overlapping social neighborhoods. It is through the generalization of the notion of neighborhood to include personal characteristics of individuals as well as their interpersonal ties that the social influence and social selection models of Robins and colleagues are introduced.

As we noted earlier, p^* models are associated with an implicit temporal process. Indeed a p^* model can be seen as the equilibrium distribution of what we might term a network "birth and death" process in which, at any point in time, a network link is added or removed with probability determined by the p^* model. Such processes are assumed to be stationary, however, and the models are consequently subject to the criticism that they privilege social structural locations over temporality (Emirbayer, 1997). As Abbott (1997) argues, "One cannot understand social life without understanding the arrangements of particular social actors in particular social times and places.... Every social fact is situated, surrounded by other contextual facts and brought into being by a process relating it to past contexts" (Abbott, 1997, p. 1152). Abbott characterizes network analysis as an attempt to deal with spatial contextuality, but both he and Emirbayer observe that there have been relatively few successful attempts to incorporate aspects of social "process" or "action" into these static network accounts. Both point to Padgett and Ansell's (1993) account of Cosimo de Medici's rise to power through "robust action" as an innovative analysis that attempts to combine contextuality in both social space and time. Emirbayer argues that Padgett and Ansell, together with several others, "converge upon the idea that network dynamics are dialectically related to network structures, each of these 'moments' partially conditioning the other" (Emirbayer, 1997, p. 306). The modelling challenge associated with this insight is to build models for "recurrent patterns of action in recurrent structures" (Abbott, 1997, p. 1168), for "recurrent mechanisms, patterns, and sequences in meso-level 'occassions'" (Emirbayer, 1997, p. 296).

In this paper, we take some initial steps in formulating p^* models that attempt to characterize recurrent sociotemporal structures. We introduce *sociotemporal neighbourhoods* as the building blocks from which models are constructed. These neighbourhoods are here construed as collections of mutually contingent, temporally dependent possible network ties, in terms of which recurrent sociotemporal structures can be described. In introducing models built from sociotemporal neighbourhoods, we aim to take a first step in addressing the challenge that Abbott poses: "We require... ways of investigating complex spatial interdependence, and of making this spatial interdependence more and more temporally structured, till again we arrive at the description and measurement of interactional fields" (Abbott, 1997, p. 1166).

Despite the fact that the proposed class of models begins to address some penetrating theoretical critiques of existing network models, there is little in the theoretical or network literatures to guide detailed model formulation. Accordingly, we see the first stages of model development as somewhat tentative, but nonetheless crucial, steps in an iterative process of empirically- and theoretically-guided model development.

There is little theoretical and empirical guidance about what form sociotemporal neighborhoods might take. The theoretical themes that Pattison and Wasserman (1999) translated into a form of social neighborhood often imply a process through time (e.g., the evolution of transitive or mutual network configurations). Yet, in their standard expression, such effects do not explicitly incorporate time as a variable. Indeed, with some notable exceptions (see, for instance, Van de Bunt, Van Duijn and Snijders, 1999), the problem of dependence structures is not particularly prominent in many discussions of temporal network modeling. As Van de Bunt *et al.* point out, the two main classes of dynamic stochastic network models – loglinear models (e.g., Wasserman and Iacobucci, 1988) and continuous-time Markov chains (e.g., Wasserman, 1980; Leenders, 1997a, b) – are both restricted by a dyadic independence assumption. With this restriction, it is not surprising that discussion of dependence structures has not been central.

Banks and Carley's (1997) discussion of the types of models used in temporal network studies is similarly revealing. Most of the statistical procedures they discuss have relatively simple dependence assumptions across time. For the most part, dyadic independence is assumed. Where models go beyond that assumption, computer simulation is usually required and a non-statistical approach is often adopted. Simulation is valuable when there is firm theoretical ground from which to postulate "rules" that are hypothesized as the basis for the evolution of social structures. For instance, simulation has been used to model network evolution as an explanation of group formation (e.g., Zeggelink, 1994; 1995; Zeggelink, Stokman and Van de Bunt, 1997) and to investigate in empirical studies different substantive theories of how particular structures might develop (Stokman and Zeggelink, 1997). The rules under which simulation programs are written can be seen as embodying or implying a particular form to a sociotemporal neighborhood. Simulation approaches, however, do not permit a more exploratory investigation of empirical data to infer what these rules of social evolution might be for a particular type of relationship within a particular set of individuals.

Snijders (1995; 1997; see also Snijders and van Duijn, 1997) has gone beyond the dyadic independence assumption with continuous time Markov models that can be implemented as simulation models. Estimation is based on the method of moments. The models arise from the notion that individuals optimize a utility function given constraints determined by the network. Snijders' intention is to combine statistical and theoretical modeling, and he derives his models from a rational choice framing of social network evolution.

Given that much of the work on non-dyadic models occurs within a specific theoretical framework, little has been done on the more general forms that a sociotemporal neighborhood may take in network change. Yet, under a p^* approach, it is important that careful thought be given to dependence structures, for without an hypothesized dependence structure, a p^* model cannot be formulated. Additional complexity is presented by sociotemporal neighborhoods, as we are dealing with two different types of dependencies: within time and across time. The form of temporal dependence embodied in a sociotemporal neighborhood needs to be premised on an argument about the appropriate processes to test in the models.

In the following section, we set out the theory for a p^* temporal model of network change across two measurement points, through the graphical modeling results of Robins *et al.* (in press). In Section 3, we return to a discussion of dependence structures, prefaced by consideration of some general network change processes that we might expect to observe. Based on these processes, we develop some suggestions for plausible sociotemporal neighborhoods, using versions of Markov random graph assumptions, but incorporating certain aspects of the partial conditional dependence approach of Pattison and Robins (2000). We provide three empirical examples in Section 4. In conclusion, in Section 5 we discuss generalizations of the models to multiple time points and to the inclusion of attributes.

2 P^* MODELS FOR SOCIAL NETWORKS

2.1 The P^* Model for a Univariate Binary Network

For a set of n persons or *actors*, we represent a *relational tie* between persons i and j as a binary random variable Y_{ij} where $Y_{ij} = 1$ if person i considers person j as a partner under the relationship, and where $Y_{ij} = 0$, otherwise. The matrix $Y = [Y_{ij}]$ can also be regarded as corresponding to a random (directed) graph with the fixed node set $N = \{1, 2, \ldots, n\}$ and a (random) edge directed from node i to node j if $Y_{ij} = 1$. Let $y = [y_{ij}]$ denote the matrix of realizations of the variable Y_{ij}. Some of the cells in Y may be undefined, as relations between certain ordered pairs of individuals may be impossible (for instance, individuals are usually assumed not to have ties with themselves). An ordered pair of individuals for which a tie is possible is referred to as a *couple*.

Hypothesized dependencies among network variables are represented in a *dependence graph*. A vertex in the dependence graph can represent a couple (i,j) or equivalently a network variable, Y_{ij}. The presence of an edge in the dependence graph between Y_{ij} and Y_{st} signifies dependence between these two variables, conditional on all other network variables. For a single dichotomous network, the Hammersley–Clifford theorem (Besag, 1974) then leads to the *joint form* of the p^* model

$$p^*(y) = P(Y=y) = \frac{1}{\kappa}\exp\left(\sum_{T\subseteq C}\gamma_T \prod_{(s,t)\in T} y_{st}\right), \qquad (1)$$

where: (i) each T is a subset of C, the set of all couples;

(ii) γ_T is a parameter corresponding to T and is non-zero only if T is a clique in the dependence graph (that is, T comprises a single couple, or there is an edge between (i, j) and (s, t) for all pairs of couples in T); and

(iii) $\kappa = \sum_{Y}\exp\left(\sum_{T\subseteq C}\gamma_T \prod_{(s,t)\in T} y_{st}\right)$.

This model was introduced by Frank and Strauss (1986) and more fully explicated in three papers that dealt with a single dichotomous relation (Wasserman and Pattison, 1996), networks of multiple relations (Pattison and Wasserman, 1999) and valued relations (Robins *et al.*, 1999).

For an identifiable model, the order of terms needs to be restricted and homogeneity constraints applied (see below). Even so, maximum likelihood estimation of parameters is computationally intractable for network models with complex dependence graphs (e.g., those that are connected). Following Besag (1975; 1977), Strauss and Ikeda (1990) proposed the use of *maximum pseudo-likelihood* estimation as an approximate technique, a suggestion adopted by Wasserman and Pattison (1996). Strauss and Ikeda showed that pseudo-likelihood estimation can be conducted using standard logistic regression procedures. Wasserman and Pattison described how to set up a data set for this estimation procedure. The technique is based on converting equation (1) to a *conditional form* where the normalizing constant k is not present. In this case, with appropriate homogeneity constraints, the model parameters represent the contribution of various *network configurations* (sub-graphs of various types) to the probability of an observed network.

Pseudo-likelihood estimation is used in this article as a convenient approximate estimation procedure to illustrate the models. We interpret the models using the pseudo-likelihood estimates, but the reader needs to bear in mind that the behavior of the estimation procedure is not well understood. We see these interpretations as interim, depending on the further development of alternative estimation techniques, for example, using Monte Carlo maximum likelihood estimation procedures (Corander, Dahmström and Dahmström, 1998; Crouch and Wasserman, 1998; Snijders, 2000). As Pattison and Robins (2000) point out, much work is required to develop these techniques further if they are to be applicable to models of the level of complexity that social theory is likely to demand.

2.2 The Two Block Chain Graph

To develop social influence models, Robins *et al.* (in press) introduced a form of directionality into the dependence graph, so that certain sets of variables could become predictors of others. This approach was informed by the graphical modeling literature (Lauritzen, 1996). A directed graph may represent dependencies in probability models for one set of variables (the *child* block of variables), given the values of another set of variables (the *parent* variables).

Graphs including both directed and non-directed edges can represent a coherent probability structure when the vertex set satisfies a particular partial ordering, such that the vertices are partitioned into *blocks*, with non-directed edges within a block, and with only directed edges from one block to another, such that all arrows are pointed in the one direction. (Here, as is common, we take directed edges as depicted by arrows, and non-directed edges as depicted by lines in the graph.) A graph satisfying this condition is termed a *chain graph* (Wermuth and Lauritzen, 1990). Robins *et al.* (in press) utilized a two-block chain graph, which can be defined as a graph containing two sets of vertices – which may be termed *parent* and *child* vertices, respectively – with the only edges between the two sets being directed from parent vertices to child vertices, these being the only arrows in the graph. Lines may occur within blocks.

In general, a non-directed graph can be derived from a chain graph with equivalent conditional independence properties (in graphical modeling, these are referred to as *Markov properties* – see Lauritzen, 1996, or Whittaker, 1990, for a summary of the various results). The non-directed graph derived from a directed dependence graph is often referred to as a *moral graph* (Lauritzen and Spiegelhalter, 1988) because it involves introducing lines between parents of the same child (the so-called *marrying of the parents*).

It is straightforward to use this framework for modeling network transition between two time points, which we shall designate as $t = 1$ and 2. Let $(i,j)^t$ refer to the couple (i,j) at time t, and let C^t refer to the set of couples at time t. We assume that the set of actors and their possible ties do not change from time 1 to time 2 (although the framework below could be generalized to allow change in the couples, for instance, if a person left or joined the group. So if $(i,j)^1 \in C^1$ then $(i,j)^2 \in C^2$. (Even though the set of couples is unchanging, it is convenient at this stage to retain the superscript that labels each couple by time – we will dispense with this labeling later.)

Let the random variable $Y_{ij}^{[t]}$ with realization $y_{ij}^{[t]}$ be a binary variable denoting the presence ($y_{ij}^{[t]} = 1$) or absence ($y_{ij}^{[t]} = 0$) of a network tie on couple $(i,j)^t$. Within time (non-directed) dependencies may occur amongst the sets of variables pertaining to couples in C^t for constant t. Across-time (directed) dependencies may occur from variable $Y_{ij}^{[1]}$ to variable $Y_{st}^{[2]}$ with $Y_{ij}^{[1]}$ considered a parent of $Y_{st}^{[2]}$. It follows from the version of the Hammersley–Clifford theorem proved in Robins *et al.* (in press):

$$P\left(Y^{[2]} = y^{[2]} \middle| Y^{[1]} = y^{[1]}\right) = \frac{1}{\kappa}\exp \sum_{R \subseteq C^2} \sum_{Q \subseteq \mathrm{pa}(R)} \gamma_{R \cup Q} \prod_{(i,j)^2 \in R} y_{ij}^{[2]} \prod_{(s,t)^1 \in Q} y_{st}^{[1]}, \tag{2}$$

where: (i) R is a subset of C^2, the set of couples at time 2;

(ii) Q is a subset of C^1 with each network variable indexed by Q being a parent of a network variable indexed in R – hence the notation $Q \subseteq \mathrm{pa}(R)$;

(iii) $\gamma_{R \cup Q}$ is a parameter corresponding to $R \cup Q$ and is non-zero only if $R \cup Q$ is a clique in the moral graph, where the moral graph is derived from the directed dependence graph by replacing all arrows with lines and by adding lines between any two couples in C^1 that are both parents of the one couple in C^2; and

$$\text{(iv)}\ \kappa = \sum_{\mathbf{Y}^{[2]}} \exp \sum_{R \subseteq C^2} \sum_{Q \subseteq \text{pa}(R)} \gamma_{R \cup Q} \prod_{(i,j)^2 \in R} y_{ij}^{[2]} \prod_{(s,t)^1 \in Q} y_{st}^{[1]}.$$

Given a particular dependence graph, equation (2) describes a family of probability distributions for random graphs. It is the most general version of a discrete time temporal p^* model with two time points. Of course, as discussed immediately below, further restrictions are needed on the terms in (2) to achieve an identifiable model. In general, however, we use (2) as the basis for model development by specifying a directed dependence graph, which then determines the non-zero parameters through the cliques of the moral graph.

Equation (2) is the *joint form* of the model. There is an equivalent *conditional form* (Robins *et al.*, in press), which expresses the model in terms of the conditional log-odds of a particular tie, and removes the troublesome normalizing quantity κ.

$$\log\left[\frac{P\left(Y_{ij}^{[2]} = 1 \,\middle|\, Y_{C^2-(i,j)^2}^{[2]}, Y^{[1]}\right)}{P\left(Y_{ij}^{[2]} = 0 \,\middle|\, Y_{C^2-(i,j)^2}^{[2]}, Y^{[1]}\right)}\right]$$
$$= \sum_{J \in \zeta(i,j)^2} \sum_{R \subseteq J-(i,j)^2} \sum_{Q \subseteq \text{pa}(R)} \gamma_{R \cup Q \cup \{(i,j)^2\}} \prod_{(u,v)^2 \in R} y_{uv}^{[2]} \prod_{(s,t)^1 \in Q} y_{st}^{[1]} \qquad (3)$$

where $Y_{C^2-(i,j)^2}^{[2]}$ denotes the observed network at time 2 excluding the time 2 tie on (i,j), $\zeta(i,j)^2$ represents those maximal cliques of the moral graph that include $Y_{ij}^{[2]}$, and $J-(i,j)^2$ is a set of couples excluding $(i,j)^2$.

2.3 Homogeneity Constraints

To achieve identifiable models, suitable restrictions on the order of the terms chosen for investigation, and homogeneity constraints, may be imposed.

Frank and Strauss (1986) assumed a *Markov* condition for conditional dependence among network variables. In a *Markov directed graph*, possible ties are assumed to be conditionally dependent whenever they have an actor in common: that is, the variables Y_{ij} and Y_{st} are conditionally dependent if and only if $\{i,j\} \cap \{s,t\} \neq \phi$. By assuming that these are the only dependencies, Frank and Strauss (1986) showed that sufficient statistics for the model are confined to indicators of certain network configurations: *ties, reciprocal ties, in-stars, out-stars, mixed-stars,* and all possible triadic configurations.

Following the homogeneity strategy originally introduced by Frank and Strauss (1986), Pattison and Wasserman (1999) discussed a general strategy of assuming that parameters corresponding to certain isomorphic configurations of array entries of Y are equal. For Markov directed graphs, sufficient statistics then become counts of various stars and triads. In the models below, we take a similar approach to homogeneity. In this case, however, the isomorphism classes represent various types of configurations that can involve two types of possible tie, those at time 1 and those at time 2. We illustrate this below.

3 SOCIOTEMPORAL NEIGHBORHOODS: *PROPOSALS* FOR TEMPORAL DEPENDENCE STRUCTURES

Specification of the dependence graph is crucial in specifying a model. With the Markov graph assumption of Frank and Strauss (1986) and the various other proposals discussed by Pattison and Robins (2000), we have precedents for possible dependence structures among the couples in C^2 (although in temporal contexts within-time dependencies may take on different meanings). Proposals for within time dependencies can be seen as specifying what we term a *local social neighborhood.* Where there is little theoretical guidance, on the other hand, is in the specification of appropriate cross-time dependencies, that is, in the determination of the form of pa(R) in equation (2). It is the inclusion of couples from both C^1 and C^2 in equation (2) that leads us to express these dependencies as reflecting *sociotemporal neighborhoods.* In this section we advance some simple broad proposals for possible forms of sociotemporal neighborhoods. We begin by discussing some general processes that we might expect to see in network transition.

3.1 Different Processes of Network Change

We distinguish three types of effect that can shape network evolution over time. First, particular patterns of relational ties may be likely to emerge over time, either because patterns develop from simpler configurations or because more complicated patterns decay. We envisage this evolution as a property of the network as a whole, even though the process is instantiated across the network as change in particular relational ties. Shared cultural understandings or behavioral norms could underlie such processes. For instance, if the shared understanding of *friendship* among a group of people is that friendship is normally reciprocated, then we would expect to see the emergence of mutual friendship ties over time. Because such as evolution is a property of the system (the network) as a whole, we term it a *systematic temporal process*. As a system-wide property, a systematic temporal process is homogeneous across the social space, indifferent to the identity of the individuals whose actions nevertheless collectively generate the process.

Systematic temporal processes reflect any tendency inherent in a relationship towards some form of equilibrium structure. This notion is consistent with the idea of Doreian and Stokman (1997) that the study of network evolution involves the modeling of change via some identifiable process. As they put it, the goal is to understand the "rules" governing the sequence of changes through time, not just to observe change. A systematic temporal processes encapsulates the idea that a network may evolve according to such "rules". Systematic temporal processes create the recurrent patterns that both Abbot (1997) and Emirbayer (1997) see at the centre of the temporal modeling challenge.

Perhaps the most basic systematic process is the extent to which a network tie is present at both times 1 and 2, that is, the extent to which $Y_{ij}^{[2]}$ can be predicted from $Y_{ij}^{[1]}$. As Doreian and Stokman (1997) point out, processes can sustain structures as well as change them, and the observation of no change does not make the idea of process less relevant.

At the same time, however, *non-systematic processes* may reshape parts of a network. These are effects that are not network-wide, may be more localized in social space and may pertain to the changing nature of relations among particular individuals. If actor i has reason no longer to trust actor j, perhaps through some act of disloyalty by j, then obviously this is likely to change a tie between i and j in a trust network. But if trust ties tend to be stable (a systematic process), the change in tie between i and j will appear as noise against a background of generally stable ties. In that sense, non-systematic processes can be modeled as random, non-homogeneous occurrences. Even so, we need to recognize that this source of "randomness" is not necessarily inherent (there is no randomness necessarily arising from the norm that disloyal individuals are not to be trusted). Non-systematic processes arise because of the necessary limitations of our measurement, both across and within time. With a broader measurement focus, we can always capture more fine-grained systematicity, and what constitutes the system is determined by what we attempt to measure.

Our construal of systematic and non-systematic processes is informed by the use of homogeneity in identifying p^* models. For instance, it is standard to include a parameter for mutuality in a (non-temporal) p^* model. Typically there is usually one such parameter, although if there is some form of blocking structure, then multiple parameters might be appropriate (e.g., mutuality effects might be different for boys and girls in a school friendship network – see Wasserman and Pattison, 1996). With one mutuality parameter, the researcher is investigating the presence of a homogeneous mutuality effect across the entire network, and if such a parameter is large and positive, the inference might be that mutuality is a property of the entire social system as represented by the network. The homogeneity constraint assumes a system-wide property. Nevertheless, the only intentionality in the system is at the level of the actor, that is, locally. Accordingly, a global property of mutuality has to be generated locally. We might then argue that such a systemic property reflects shared norms or behaviors across actors, norms or behaviors that could be construed as inherent in the particular social relation for this group of people. Moreover, we might expect such norms or behaviors to shape the network dynamically, so that the system-wide effect may also reflect a form of network evolution. Accordingly, it is helpful to model such processes dynamically when we have data available.

At the same time, not all local action need have network-wide consequences. A particular subset of actors at a particular time (i.e., in effect a particular sociotemporal neighborhood) may behave in ways idiosyncratic to themselves for whatever reason. In this case, the homogeneous parameters cannot capture what is behavior peculiar to that neighborhood. If we suspect the "reason" (e.g., perhaps they have a distinctive pattern of individual attributes) we might seek to include relevant variables in the model (e.g., additional mutuality parameters for boy–boy, boy–girl and girl–girl friendship choices), thus increasing our potential to investigate the nature of "systematicity". In this sense, then, non-systematic processes can be seen as residual (or as noise) to the systematic processes that we seek to observe. The point is, however, that the models cease to be identifiable unless we impose some form of homogeneity. There is always a residue (possibly large) of non-systematic local action. (For a more extended discussion of local processes and global effects in the context of p^* models, see Mische and Pattison, 2000; Mische and Robins, 2000; Pattison and Robins, 2000.)

In studying network evolution, we also impose measurement restrictions by selecting a set of individuals as nodes of the network. This sets an implicit network boundary, leading to the possibility of external influences. We refer to *exogenous events* as those that occur beyond the network but impact on network structure. In some contexts, it makes sense to envisage some higher order social entity that imposes change. For instance, in a business organization an executive decision to give an existing member of a workgroup new responsibilities, perhaps through promotion or restructuring, has the potential to reshape the pattern of advice ties within that particular workgroup. Although in a broad sense, exogenous events are by their nature non-systematic, we restrict the term non-systematic processes to those that do not relate directly to observable external actions. Of course, an exogenous event can also lead to various non-systematic processes (in our sense of the term).

Because systematic processes are homogeneous across the network, arising from common social behaviors when individuals face similiar social circumstances, one individual cannot singly shape the overall direction of social evolution. For instance, assuming that friendship networks evolve according to certain "rules" – perhaps rules akin to the axioms postulated by Zeggelink (1994; 1995) – then individuals within a friendship network cannot control the way the network evolves. Simply by having (or not having) friends, individuals participate in an evolving group structure.

Non-systematic processes, however, relate to actions that are not homogeneous across the network. They may pertain to an individual's – or a subgroup of individuals' – attempts to reshape their immediate social environment in ways that do not follow the commonly shared "rules" of a systematic process. These local deviations from the overall trend of network evolution need not "break the rules", but merely be a change to a tie that is not the result of an underlying systematic process.

3.2 Possible Forms for Sociotemporal Neighborhoods

In a given case, there may be reasons for postulating particular directed dependencies across time, but it would be an unusual model in which $Y_{ij}^{[1]}$ was not a parent of $Y_{ij}^{[2]}$. This dependence reflects a simple proposition, that in the absence of other effects, there is a tendency for ties to persist across time. If there is little or no tie stability across time, then the implication is that non-systematic processes are playing a major role.

A simple extension of non-temporal dependence structures is to assume that the mutual dependencies postulated at time 2 *will* also be reflected in the directed dependencies across time. This assumption implies that if there is a conditional dependence between $Y_{st}^{[2]}$ and $Y_{uv}^{[2]}$, then $Y_{st}^{[1]}$ will be a predictor of $Y_{uv}^{[2]}$ and $Y_{uv}^{[1]}$ will be a predictor of $Y_{st}^{[2]}$. For instance, assume a dyadic independence model, so that $Y_{ij}^{[2]}$ and $Y_{ji}^{[2]}$ are mutually conditionally dependent. Then this across-time dependence assumption implies that the parents of $Y_{ij}^{[2]}$ will include not only $Y_{ij}^{[1]}$ but also $Y_{ji}^{[1]}$. The across-time dependence assumption becomes more plausible if we assume that the within-time dependence structure does not change from one time period to the next, so that $Y_{ij}^{[1]}$ and $Y_{ji}^{[1]}$ are mutually conditionally dependent. In this case, the parents of the same child are already connected in the time 1 block. That is, the parents are already "married" and no additional edges have to be added to the dependence graph to create the moral graph. Lauritzen (1996) termed a chain graph in which no edges have to be added to create the moral graph as *perfect*, so this assumption can be termed a *perfect dependence assumption*. Note that a perfect dependence does not have to mirror fully the time 2 dependencies. Any set of directed dependencies that are a subset of the time 2 dependencies will also be perfect. To differentiate the two situations, the situation when the directed dependencies mirror all the mutual dependencies is referred to as *complete perfect dependence*. For dyadic independence, the directed complete perfect dependence graph is presented in Figure 1.

Complete perfect dependence assumptions seem a reasonable approach to exploring network evolution when the network involves relations such as friendship or trust, where the duration of ties is

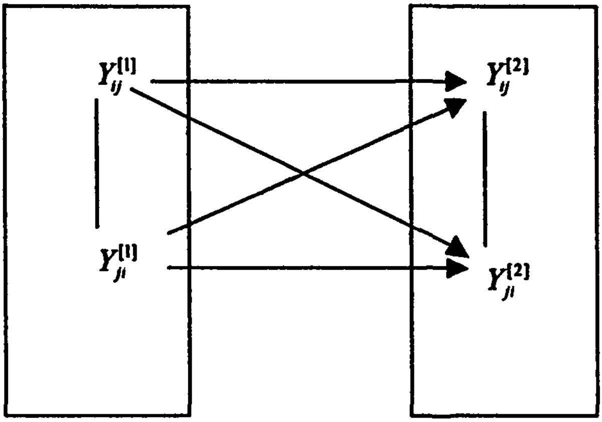

FIGURE 1 Directed dependence graph for complete perfect dyadic independence. *Note*: To create the moral graph, only arrows need to be replaced by lines; no additional lines are required.

changeable but open-ended. There may be other contexts where such assumptions are not so reasonable, for instance, when the network involves sequences of short term transactions, or perhaps when exogenous events might disrupt dependence structures from one time period to another. More generally, the assumption seems reasonable when the network is in dynamic equilibrium, with dependence structures not changing over time. As a result it seems reasonable that the time between observations should not be excessive in relation to the time scale of network change.[1]

For complete perfect dependence, the sets of parents at time 1 mirror the cliques of neighbours at time 2. If $Y_{st}^{[2]}$ and $Y_{uv}^{[2]}$ are conditionally dependent at time 2, then $Y_{st}^{[1]}$ and $Y_{uv}^{[1]}$ are also conditionally dependent and both are parents of $Y_{st}^{[2]}$ and $Y_{uv}^{[2]}$. By specifying within time dependencies (such as dyadic independence or Markov graph dependencies) with a set of maximal cliques ζ, and dropping the distinction between couples at time 1 and time 2, equation (2) then becomes:

$$P\left(Y^{[2]} = y^{[2]} \middle| Y^{[1]} = y^{[1]}\right) = \frac{1}{\kappa}\exp\sum_{R\subseteq\zeta}\sum_{Q\subseteq R}\gamma_{R,Q}\prod_{(i,j)\in R} y_{ij}^{[2]}\prod_{(s,t)\in Q} y_{st}^{[1]}. \quad (4)$$

[1] We are indebted to Tom Snijders for thoughts on perfect dependence.

So these models are now determined fully by a set of maximal cliques ζ from a dependence graph with non-temporal form. The complete perfect dependence assumption allows us to derive a temporal dependence graph. The non-temporal form can be construed as representing a dynamic equilibrium, with an underlying dependence structure that does not change across time.

3.3 Complete Perfect Bernoulli Graph Models

For Bernoulli graph models, maximal cliques are single network variables, so that ζ is simply the set of variables indexed by C. We impose homogeneity by assuming that effects in the model are equal for all ties (i.e., isomorphic configurations are a time 2 tie and a stable tie that is present across both time periods). With this homogeneity assumption, equation (4) then becomes:

$$P\left(Y^{[2]}=y^{[2]}\middle|Y^{[1]}=y^{[1]}\right)=\frac{1}{\kappa}\exp\left(\gamma_0\sum_{(i,j)\in C}y_{ij}^{[2]}+\gamma_1\sum_{(i,j)\in C}y_{ij}^{[2]}y_{ij}^{[1]}\right), \quad (5)$$

where γ_0 is a density parameter and γ_1 is a parameter expressing a tendency for ties to persist from t_1 to t_2.

Given the independence of network variables, the conditional form of this model – equation (3) – is as follows:

$$\log\left[\frac{P(Y_{ij}^{[2]}=1|Y_{ij}^{[1]})}{P(Y_{ij}^{[2]}=0|Y_{ij}^{[1]})}\right]=\gamma_0+\gamma_1 y_{ij}^{[1]},$$

so that the conditional log-odds of a tie being observed at time 2 is γ_0 if no tie is observed at time 1, and is $\gamma_0+\gamma_1$ if a tie is observed at time 1.

When we include higher order effects in the models below, particularly the Markov models, we might examine the behavior of the two Bernoulli graph parameters with a view to considering non-systematic processes. For most observed networks where ties tend to be relatively sparse, we would expect a negative γ_0 parameter in the presence of higher order parameters. In a temporal model with higher order parameters, a γ_0 parameter close to zero or even positive suggests that individuals are forming time 2 ties in ways that cannot be explained by higher order structures, or by time 1 ties. Across time, in most circumstances we would also expect a positive γ_1 parameter to reflect a tendency for ties to persist. A γ_1 parameter close to zero (or even negative) suggests that unless they are part of higher order configurations, ties are not stable. While these considerations are somewhat speculative, in particular circumstances they could be interpreted as giving some indication as to the relevance of non-systematic processes.

3.4 Complete Perfect Dyadic Independence Models

For dyadic independence models, with homogeneity applied across isomorphic cliques of the moral graph, (4) becomes:

$$
\begin{aligned}
&P\left(Y^{[2]} = y^{[2]} \,\middle|\, Y^{[1]} = y^{[1]}\right) \\
&= \frac{1}{\kappa}\exp\Bigg(\gamma_0 \sum_{(i,j)} y_{ij}^{[2]} + \gamma_1 \sum_{(i,j)} y_{ij}^{[2]} y_{ij}^{[1]} + \rho \sum_i \sum_j y_{ij}^{[2]} y_{ji}^{[2]} \\
&\quad + \rho_{12} \sum_i \sum_j y_{ij}^{[2]} y_{ji}^{[1]} + \rho_{1c} \sum_i \sum_j y_{ij}^{[2]} y_{ij}^{[1]} y_{ji}^{[1]} \\
&\quad + \rho_{2c} \sum_i \sum_j y_{ij}^{[2]} y_{ij}^{[1]} y_{ji}^{[2]} + \rho_{cc} \sum_i \sum_j y_{ij}^{[2]} y_{ij}^{[1]} y_{ji}^{[2]} y_{ji}^{[1]}\Bigg). \qquad (6)
\end{aligned}
$$

Here parameters γ_0 and γ_1 have the same interpretation as for the Bernoulli graph model. The ρ parameters reflect a variety of reciprocity effects. The parameter ρ represents a tendency for t_2 ties to be reciprocated, while ρ_{12} represents the propensity for a tie to be present at time 2 with a reciprocated tie in place at time 1. The suffix "c" has been used to label the other ρ parameters to represent a tie that persists across time (a *constant tie*). So ρ_{1c} refers to the stability of a tie when it is reciprocated at t_1. The various network configurations associated with these parameters are depicted in Figure 2.

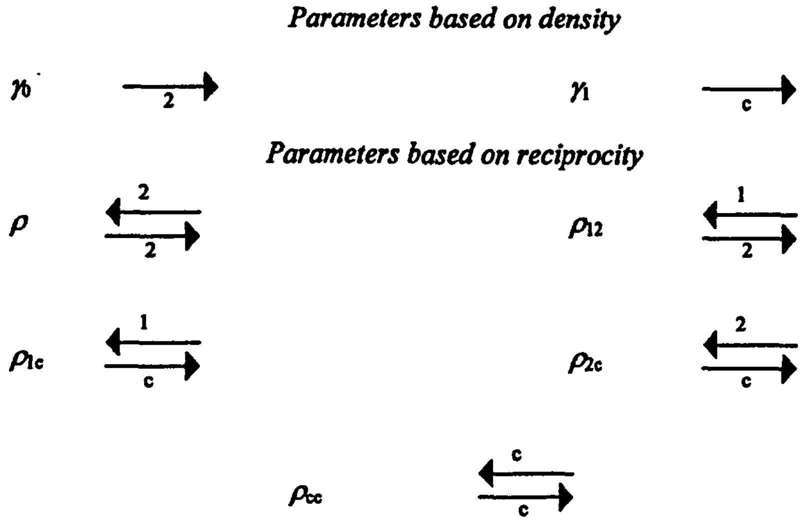

FIGURE 2 Parameters and associated configurations for complete perfect dyadic independence model.
Note: Ties are numbered depending on whether they occur at time 1 or 2; "c" refers to a *constant tie*, where both time 1 and time 2 ties are present. Under the *constant tie assumption*, parameters ρ_{12} and ρ_{1c} are removed from the model.

The conditional form expresses the model in terms of the log-odds of a tie being observed conditional on the observation of other ties in the dyad at each time point.

$$\log\left[\frac{P\left(Y_{ij}^{[2]}=1 \middle| y_{ji}^{[2]}, y_{ij}^{[1]}, y_{ji}^{[1]}\right)}{P\left(Y_{ij}^{[2]}=0 \middle| y_{ji}^{[2]}, y_{ij}^{[1]}, y_{ji}^{[1]}\right)}\right]$$
$$= \gamma_0 + \gamma_1 y_{ij}^{[1]} + \rho y_{ji}^{[2]} + \rho_{12} y_{ji}^{[1]} + \rho_{1c} y_{ji}^{[1]} y_{ij}^{[1]}$$
$$+ \rho_{2c} y_{ji}^{[2]} y_{ij}^{[1]} + \rho_{cc} y_{ji}^{[2]} y_{ji}^{[1]} y_{ij}^{[1]}$$

so, for instance, the log-odds of a new tie being observed at time 2 given reciprocal ties at both time points is $\gamma_0 + \rho + \rho_{12}$, while the log-odds of a new tie being observed at time 2 given a new reciprocal tie at time 2 is simply $\gamma_0 + \rho$. The log-odds of a tie being observed, given other states of the dyad at the two time points, can similarly be calculated.

3.5 Complete Perfect Markov Models

For Markov random graphs, the maximal cliques of the dependence graph relate to triads – that is, sets of couples of the form $\{(i,j), (j,i), (i,k), (k,i), (j,k), (k,j)\}$ – and to star configurations – that is, for each node i there is a clique of all couples involving i, $\{(i,j), (i,k), (i,l), \ldots, (j,i), (k,i), (l,i), \ldots\}$, (Robins, 1998; see also Pattison and Wasserman, 1999, where these maximal cliques are defined for multivariate networks). This results in a very large number of parameters, even allowing for the imposition of homogeneity, and the strategy typically adopted is to limit the order of the interactions, equivalent to restricting the number of edges in a configuration. For illustrative purposes, we restrict the models below to 2-stars and to transitive and cyclic triads, in addition to reciprocity and density effects.

For a complete perfect temporal model, with homogeneity imposed across isomorphic configurations, we then have parameters for 2-star, and transitive and cyclic, triadic configurations that involve at least one tie at time 2, as well as the parameters from the dyadic independence model. This is a relatively large number of parameters, many of which are difficult to interpret (Robins, 1998). In any event, there are substantive reasons for wishing to restrict the number of parameters, especially if we are primarily investigating systematic effects.

The assumption that we make below is that systematic processes come into play among ties that persist through time. The argument is that, as a systematic process concerns evolution from less stable to more stable network configurations, it will be most pronounced among ties that persist across time periods. (This argument is of course context-dependent, and has force in situations where relational ties are changeable but open-ended in duration. Friendship might be taken as an example. Where we are dealing with, say, transactions of a limited duration, the argument has less force.) For instance, if there is a tendency towards transitivity – say, that friends of friends are likely in time to become friends – then this effect will only be apparent if the original friendships persist across a sufficient time period. Effects that are based on ties that exist only at time 1 may be too ephemeral to influence ties at time 2. We refer to this assumption as the *constant tie assumption.*

What the constant tie assumption does, in effect, is to define a period of measurement for each configuration that involves three actors. For instance, suppose we are trying to predict a new tie from i to j given constant ties on (i,k) and (k,j) (that is, a constant tie transitivity effect). At time 1, the observed configuration comprises ties on (i,k) and (k,j); at time 2, the observed configuration comprises ties on (i,k) and (k,j) and possibly on (i,j). In equation (4) in this circumstance we are considering a parameter $\gamma_{R,Q}$ where $Q = \{(i,k), (k,j)\}$ and $R = \{(i,k), (k,j), (i,j)\}$.

Suppose, however, that by time 2 an earlier tie on (k,j) had disappeared. Then the constant tie assumption does not consider the time 1 configuration as a path of length two that might be completed into a transitive triad, but rather a path of length 1, that is a tie on (i,k). In other words, with the tie on (k,j) absent by time 2, we suppose that $\gamma_{R,Q} = 0$. On the other hand, the parameter $\gamma_{R,S}$, with $S = \{(i,k)\}$ is still present in the model. What we are in effect doing here is presupposing that the ephemeral (k,j) tie disappeared before it could influence a possible (i,j) tie. We are implicitly setting a period of measurement for that triad, such that the observation of a time 2 tie on (i,j) is deemed to have occurred *after* the ephemeral tie vanished. Any effects arising from ephemeral ties are construed as non-systematic effects and do not explicitly enter the model.

The constant tie assumption has been presented here as arising from substantive concerns dealing with systematic, as opposed to non-systematic, effects. Formally, it is a version of *partial conditional independence* (Pattison and Robins, 2000). In technical terms, parameters for configurations of ties from both times 2 and 1 are set to zero unless a tie from time 1 is accompanied by a time 2 tie. (Note that a constant tie dyadic independence model would not include the parameters ρ_{12} and ρ_{1c} from (6). Note also that the "constant ties" are composed of both time 2 and time 1 ties, so that it is not a simple matter that "constant ties" are predictors of other time 2 ties. Rather, the time 1 ties involved in constant ties provide the only couples at time 1 that are parents of couples at time 2.)

Figure 3 presents the parameters and associated configurations – additional to those of the dyadic independence model of Figure 2 – for the Markov model with the constant tie assumption. Here the following parameter labelling is used: σ_O, σ_I, and σ_M for parameters associated with 2-out-star, 2-in-star and 2-mixed-star effects, respectively, ψ for cyclic triad parameters, and τ for transitive triad parameters,

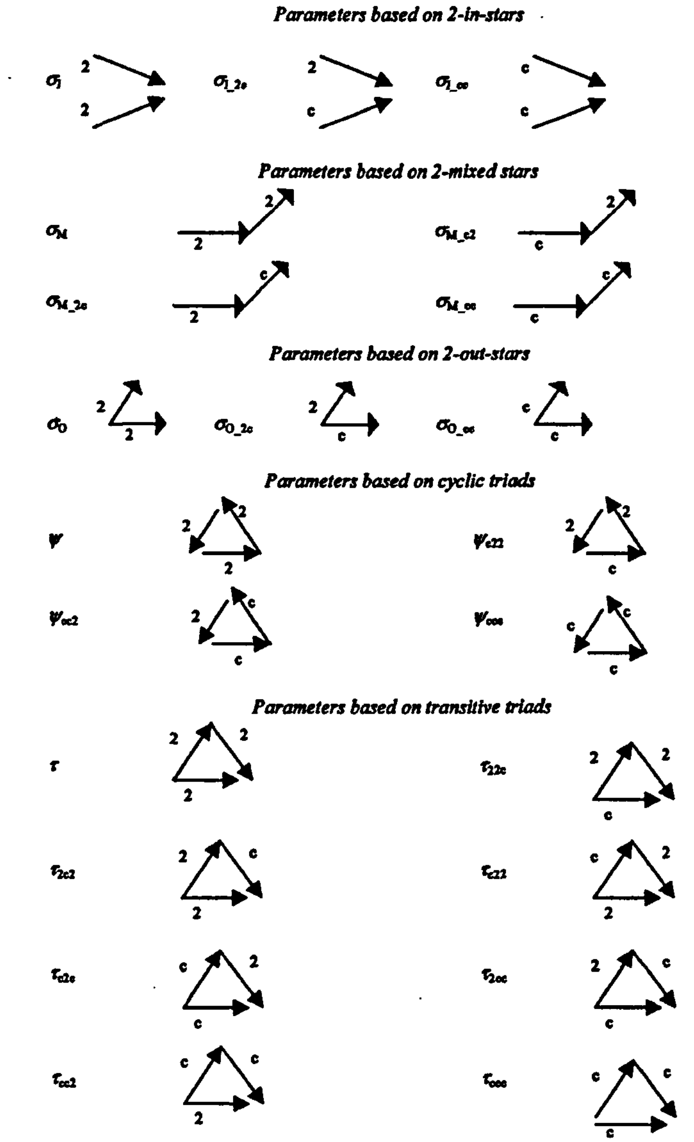

FIGURE 3 Additional parameters and associated configurations for complete perfect Markov graph model with constant tie assumption.

with the suffices describing various patterns of constant and time 2 ties in the respective configurations.

4 EMPIRICAL EXAMPLES

4.1 Freeman EIES Data

The Freeman EIES data set analyzed here involves an acquaintance relationship among 32 academics measured at two time points (Freeman and Freeman, 1979; see also Wasserman and Faust, 1994). Originally, the network data had five categories: "close personal friend", "friend"; "person I've met"; "person I've heard of, but not met", and "person unknown to me". For the purposes of this analysis, we have dichotomized the data so that the first two friendship categories are coded as "1" and the remaining three categories as "0". Accordingly, the dichotomized data represents a friendship relation.

Our strategy for fitting models follows that of Robins *et al.* (in press), in that we remove parameters if they do not contribute at least four to the pseudo-likelihood deviance. We use the pseudo-likelihood deviance and mean absolute residual as heuristic guides to comparative model fit. However, in the models fitted below, we have not insisted on hierarchical models whereby insubstantial lower order effects are retained in the presence of substantial higher order effects. Pattison and Robins (2000) show that non-hierarchical models can arise from certain partial conditional independence assumptions. Our non-hierarchical modeling approach here, however, has a somewhat simpler motivation. In fitting hierarchical models to small networks, we find that important effects can be lost to models because the contribution made by a higher order parameter may also be shared across a number of insubstantial lower order parameters. Non-hierarchical models allow identification of such effects. In interpretation, we do not suggest that small lower order effects are not present; rather, we assume that they are not of sufficient size to alter basic interpretation, and so we use the non-hierarchical model as a parsimonious interpretative device.

4.1.1 Markov Graph Model for Time 2 Network

We begin by fitting a non-temporal Markov random graph model solely to the time 2 network. The parameters fitted are those from Figures 2 and 3 relating to configurations comprising only time 2 ties. The purpose is to provide a basis for comparison to see what additional explanatory power the temporal models provide. The cyclic triad parameter dropped out of the model, resulting in a pseudo-likelihood deviance of 585.4 and a mean absolute residual of 0.18 with 6 parameters. Pseudo-likelihood parameter estimates are provided in the top panel of Table 1, together with standard errors as calculated for standard logistic regression. These standard errors are approximate at best and are provided only as a guide.

We restrict interpretation to a few broad comments. There is clearly a strong reciprocity effect; moreover, the positive transitive triad estimate, especially in conjunction with a negative mixed-star effect, suggests some clustering within the network. The absence of a cyclic triadic effect suggests there is little evidence for exchange effects that go beyond the dyadic level (i.e., beyond reciprocity).

4.1.2 Bernoulli Model

Parameter estimates for the model of equation (5) are presented in the second panel of Table 2. The two parameter model had a pseudo-likelihood deviance of 511.1 with a mean absolute residual of 0.133. The model suggests that ties are likely to persist from time 1 to time 2. Comparison of the mean absolute residual with the time 2 Markov graph model suggests that temporal effects are quite strong in this network, for even the simplest cross time dependence assumption seems to result in a better fit.

4.1.3 Dyadic Independence Model

For the model represented in Figure 2, but with a constant tie assumption, the only parameter removed was ρ_{cc}. Parameter estimates are presented in the third panel of Table 1. The four parameter model had a pseudo-likelihood deviance of 457.2 with a mean absolute residual of 0.121. The model again demonstrates the persistence of ties across time with a large γ_1 estimate.

The positive ρ parameter suggests that new ties are likely to be reciprocated by other new ties. In interpreting the cross-time ρ_{2c} parameter, however, we need to bear in mind that interpretations of higher order parameters are marginal to those of lower order parameters. Robins *et al.* (1999) presented a useful heuristic approach to detailed interpretation of p^* models. They pointed out that various summations of parameter estimates can be interpreted as the (conditional) log-odds of a tie "completing" a sub-configuration, assuming

TABLE 1
Parameter Estimates for Models for Freeman EIES Data

Parameter	Pseudo-likelihood estimate	Standard error (approx.)
Time 2 Markov random graph model		
γ_0 (density)	−3.57	0.35
ρ (reciprocity)	3.11	0.28
σ_I (in-stars)	0.15	0.03
σ_M (mixed-stars)	−0.17	0.02
σ_O (out-stars)	0.13	0.03
τ (transitive triads)	0.25	0.04
Complete perfect Bernoulli temporal model		
γ_0	−2.56	0.13
γ_1	5.21	0.35
Complete perfect dyadic independence temporal model		
γ_0	−3.07	0.18
γ_1	5.35	0.42
ρ	2.56	0.34
ρ_{2c}	−0.99	0.32

that no other effects are present (that is, that the sub-configuration is the only one completed by that tie). Of course, in relation to observed ties, such an interpretation may well be "hypothetical" in that other effects will often be present. The idea, however, is to disentangle various effects by considering the implications of the model for (possibly hypothetical) cases where particular network configurations are isolated.

For instance, we might ask what is the estimated probability that an unreciprocated tie at time 1 from j to i becomes a reciprocated tie at time 2, assuming that no other ties involve i and j. The question can be rephrased: what is the estimated probability of an (i,j) time 2 tie "completing" a ρ_{2c} configuration, given a constant tie on (j,i)? In completing a ρ_{2c} configuration in the constant tie model, however, a time 2 (i,j) tie also completes a ρ configuration and a γ_0 configuration (both of which are lower order configurations to the ρ_{2c} configuration). The conditional log-odds then is a sum of the estimates of the associated parameters, in this case $-3.07 + 2.56 - 0.99 = -1.50$. So in the absence of any other effects, we would not expect that a large number of unreciprocated ties at time 1 would become reciprocated at time 2. When a tie is reciprocated at time 1, however, the strong

γ_1 effect comes in to play, so that generally we would expect reciprocated ties to remain reciprocated.

Overall, then, the model gives a strong impression of stability, with ties tending to persist through time, and the status of reciprocation within a dyad tending to remain the same. The model seems to improve the Bernoulli model in terms of fit, with a change in pseudo-likelihood deviance of 53.9 for a difference of two parameters. There is evidence here that the reciprocation effects give additional explanatory power. What the model cannot account for, however, is the evidence of clustering suggested by the time 2 Markov graph model above.

4.1.4 Markov Graph Model

A model with all 27 constant tie parameters from Figures 2 and 3 had a pseudo-likelihood deviance of 329.3 and a mean absolute residual of 0.091. With 18 parameters removed, the resulting model had a pseudo-likelihood deviance of 351.9 and a mean absolute residual of 0.098. Parameter estimates for the reduced Markov model are presented in Table 2.

TABLE 2
Freeman EIES Data: Parameter Estimates for Complete Perfect Markov Random Graph Model with Constant Tie Assumption

Parameter	Pseudo-likelihood estimate	Standard error (approx.)
Density parameters		
γ_0	−4.60	0.39
γ_1	7.11	0.79
Reciprocity parameters		
ρ	2.10	0.35
In-star parameters		
σ_{I_2c}	0.19	0.05
σ_{I_cc}	−0.38	0.14
Mixed-star parameters		
σ_{M_2c}	−0.19	0.05
Transitive triad parameters		
τ	0.33	0.07
τ_{22c}	−0.49	0.12
τ_{2c2}	0.28	0.11

As expected, we see similar effects for ties to persist across time and for a time 2 reciprocity effect as in the dyadic dependence model. The pattern of Bernoulli parameter estimates are as expected for a network in which non-systematic processes are not playing a major role.

For interpretation of the other parameters, again it is important to consider parameter effects simultaneously when one of the related configurations includes others as a sub-graph. For instance, in considering the in-star parameter estimates, the negative σ_{I_cc} parameter, if interpreted without considering other related parameters, might suggest a tendency for in-stars with constant ties not to be present, that is, for individuals who are popular at time 1 to be less so at time 2. In this case, stable ties are less likely to form star-like structures. However, a time 2 tie that completes a (constant tie) σ_{I_cc} in-star also completes a σ_{I_2c} in-star which, on the basis of the parameter estimates, is in the opposite direction to the σ_{I_cc} effect, and hence moderates it. This moderated effect needs to be seen in relation to the strong tendency for ties to persist (γ_I) so that, for the most part, popularity at time 1 is likely to be sustained to time 2. The positive σ_{I_2c} estimate also suggests that there are propensities for new in-stars to occur at time 2, that is, for popular actors at time 1 to attract new "fans" at time 2.

Triadic effects of course also involve the 2-star effects as sub-configurations. In interpreting the triadic parameters, we investigate whether the model provides evidence for any processes involved in triadic formations over time, especially given the suggestions of clustering from the non-temporal Markov graph model.

A triad at time 2 may emerge from a two-star configuration at time 1 if the relevant new tie is observed at time 2. For transitive triads, the three possibilities are presented in the first three rows of Figure 4, whereby a time 2 transitive triad can evolve from a time 1 mixed-star, out-star and in-star. The fourth row of the Figure depicts the emergence of a cyclic triad from a mixed-star. The Figure also includes a summation of the relevant triadic and star parameter estimates. These summations represent changes to the conditional log-odds of the new tie emerging (assuming no other effects), over and above the density effect for a new tie (γ_0). For instance, a new time 2 tie that completes a triadic configuration from a time 1 mixed-star (the first line of Figure 4) completes: a new time 2 triadic configuration (τ – as the constant ties

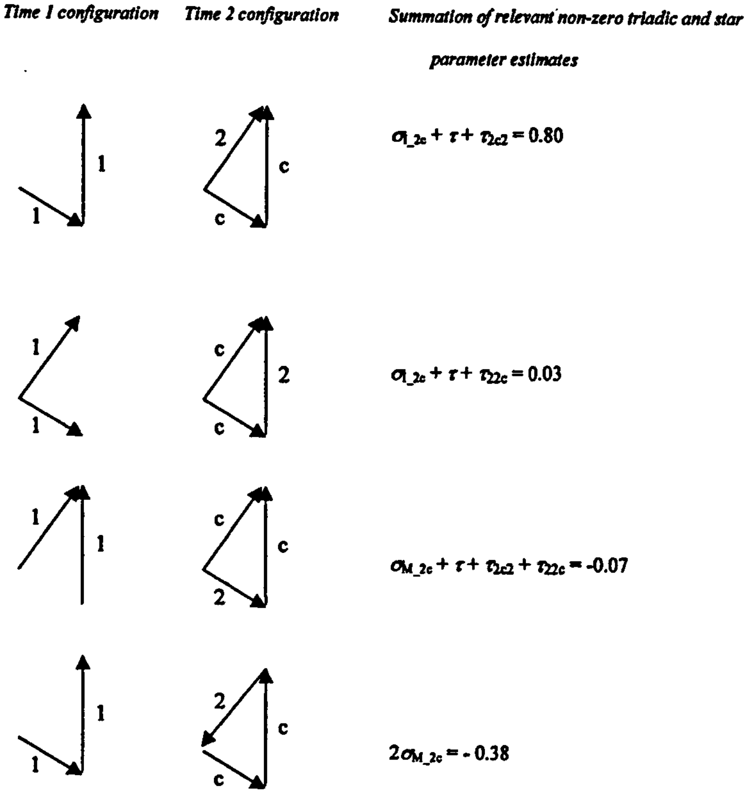

FIGURE 4 Emergence of new triads across time.
Note: Summation of parameter estimates assumes that no other configurations are involved in completion of the tie.

imply the presence of time 2 ties); a new τ_{2c2} transitive triad; a new in-star (σ_{I_2c} and σ_I but the latter parameter is not included in the summation as the parameter has been removed from the model); and a new out-star (σ_{O_2c} – not included in the summation). The total is 0.8, relatively small compared to the large new tie density effect (−4.60). Suppose, however, at time 1 there are several paths of length 2 (mixed-stars) from *i* to *j*, but no direct tie. Suppose these two-paths persist to time 2. Then, in the absence of other effects, to estimate the conditional probability of a time 2 tie emerging from *i* to *j*, the transitive triad effect needs to be multiplied by the number of two-paths. This product when added to the density effect provides an estimate of the conditional log-odds of a tie being observed. The resulting probability estimate for a new tie to emerge from only one two-path is around 0.02, but for six two-paths, the estimated probability of a new transitive triad at time 2 increases to around 0.55.

A two path in this context is a friend of a friend. The conclusion we draw is that there is some tendency for friends of friends to become friends but this effect is only of import if the two individuals are connected by multiple friends. The same effect enhances the persistence of ties. The tendency for ties to remain in place is enhanced if the two individuals are connected by a two-path. The implication is that where friends of friends are friends, the structure is particularly stable.

The above description can be contrasted with the emergence of a transitive triad from a time 1 out-star and an in-star (second and third lines of Figure 4). Here the small values of the summation suggest no particular triadic effect over and above the new tie density effect. So if an individual at time 1 nominates two others as friends, there is no notable impetus for those two others to become friends at time 2. Similarly, there is no strong effect for or against two individuals who nominate the same friend at time 1 to become friends at time 2. There appears to be a slight tendency against new cyclic triads (fourth line of Figure 4 – this effect arises from the negative two-mixed star effect, as cyclic parameters themselves do not appear in the model).

In summary, this triadic analysis provides good examples of what we have termed systematic processes that go beyond the dyadic level. The model provides evidence for an increasing clustering of individuals as friends of friends become friends, and evidence for the decay of cyclic structures.

4.2 Training Group: Trust and Friendship Networks

This data set was collected as part of a larger project examining the efficacy of in-house training by a large Australian government instrumentality. The training group of 14 participants came from different parts of the organization and did not necessarily know each other beforehand. The training course lasted for four days. At the mid-point and at the end-point of the course, participants completed a number of network questionnaires. Two networks are examined here: trust and friendship. As this group was newly formed and existed for only four days, non-systematic processes are likely to play an important part in determining the network changes from the first time point (after two days of training) to the second time point (after four days).

We present below parameter estimates for the Markov random graph temporal models for these two networks, restricting our discussion of simpler dependence models principally to matters of fit as a basis of comparison for the Markov graph model. With these relatively small networks, given the large number of parameters that are present, particularly in the Markov graph model, occasional estimation problems result. When we observe evidence of collinearity, we fit models with certain of the problematic parameters removed. For instance, in the Markov model for the trust network, the ρ_{cc} parameter was removed, given that it was collinear with a group of other parameters.

4.2.1 Trust Network

For the trust network at time 2, the star parameters were retained in a three parameter non-temporal Markov graph model, resulting in a pseudo-likelihood deviance of 111.2 and a mean absolute residual of 0.18. The dyadic independence model for the trust network, on the other hand, reduced to the two-parameter Bernoulli model with a pseudo-likelihood deviance of 174.9 and a mean absolute residual of 0.30. These results suggest that reciprocity effects do not play a major role and that dyadic effects have little explanatory power.

Parameter estimates for the Markov graph temporal model are presented in Table 3. With seven parameters, the model had a pseudo-likelihood deviance of 72.4 with a mean absolute residual of 0.11.

The first point to note is a very strong estimated effect (γ_1) for a trust tie that exists at time 1 to continue to time 2, in the absence of any higher order effects. However, in this non-hierarchical model, the time 2 density parameter (γ_0) drops out, suggesting that given the higher order effects included in the model, there are no tendencies against the emergence of new time 2 ties. We thus have evidence for some non-systematic processes, not in terms of original ties changing, but in terms of new ties emerging.

The 2-out-star effect (σ_O) is readily interpreted. The positive time 2 estimate suggests that if a person sends many trust ties, the conditional log-odds of additional trust ties is increased; in other words, some

TABLE 3
Training Group Trust Network: Parameter Estimates for Complete Perfect Markov Random Graph Model with Constant Tie Assumption

Parameter	Pseudo-likelihood estimate	Standard error (approx.)
Density parameters		
γ_I	4.05	1.48
In-star parameters		
σ_I	−0.60	0.20
σ_{I_2c}	−1.46	0.47
Mixed-star parameters		
σ_M	−0.29	0.09
σ_{M_2c}	0.73	0.25
Out-star parameters		
σ_O	0.60	0.12
Transitive triad parameters		
τ_{ccc}	1.91	1.22

people in the network are particularly trusting. But no temporal out-star parameters remain in the model, suggesting that the expansiveness of individuals at time 2 cannot be explained by expansiveness at time 1 (unless of course the 2-out-stars are completed into triads, as then τ_{ccc} comes into play). The negative in-star parameter estimates (σ_I, σ_{I_2c} suggest, on the other hand, that there is a ceiling on popularity, and in fact that those who were popular at time 1 may be somewhat less so at time 2. The negative mixed star effect at time 2 (σ_M) in the presence of a positive across time mixed star effect (σ_{M_2c}) suggests a tendency against new two-paths at time 2 unless they take the form of a new tie connecting to a constant tie. In other words, there seems to be some propensity for those who are particularly trusting across time to be themselves more trusted at time 2, but there is a tendency against those who are trusted across time to become more trusting.

As noted above, these various star interpretations assume that the star is not part of a transitive triad, for then the constant tie transitive effect (τ_{ccc}) needs to be considered. The parameter estimate is large and positive, suggesting that transitive triads, once formed, are very stable (the estimate has a large approximate standard error but nevertheless contributes substantially to the pseudo-likelihood deviance according to our criterion, so is retained in the model). A summation of relevant parameter estimates shows that this is particularly so when we consider two individuals who continue to trust the same third party: there is a particularly strong tendency for such a pair of individuals to continue to have a trust tie across time. Similarly, a summation of appropriate star parameters (along the lines of Figure 4) reveals quite strong tendencies against mixed stars and out-stars being completed into transitive triads, but – on the other hand – a tendency for in-stars to evolve into a transitive triad, that is, a propensity for those who trust the same third party to develop a trust tie. There is little tendency for or against the emergence of new cyclic triads.

In summary, we see some evidence of non-systematic processes in the possible emergence of new ties not associated with existing higher order structures. This is hardly surprising in a network of trust within a new group. Nevertheless, initial choices of trust partners tend to be sustained, particularly so, if those choices are clustered into transitive triadic structures which remain highly stable. New transitive triads are most likely to emerge from two individuals who both trust the same people. Interestingly, there is no evidence of reciprocity in trust, and no particular tendencies for cyclic triads.

4.2.2 The Friendship Network

For the friendship network at time 2, the density, out-star and reciprocity parameters were retained in a non-temporal Markov graph model, resulting in a pseudo-likelihood deviance of 128.1 and a mean absolute residual of 0.22. The dyadic independence model, as with the trust network, reduced to the two-parameter Bernoulli model with a pseudo-likelihood deviance of 177.6 and a mean absolute residual of 0.31.

Parameter estimates for the Markov graph temporal model are presented in Table 4. With five parameters, the model had a pseudo-likelihood deviance of 107.6 with a mean absolute residual of 0.17.

The striking point of the model is that friendship at time 1 is not included as a predictor of friendship at time 2. It appears that non-systematic processes are important to understanding the changes in this network, with those friendships sustained across time needing to be explained by the maintenance of higher order configurations. A friendship tie at time 1 that is not part of such a higher order configuration

TABLE 4
Training Group Friendship Network: Parameter Estimates for Complete Perfect Markov Random Graph Model with Constant Tie Assumption

Parameter	Pseudo-likelihood estimate	Standard error (approx.)
Density parameters		
γ_0	−4.81	0.79
Reciprocity parameters		
ρ_{cc}	4.31	1.49
Out-star parameters		
σ_O	0.49	0.08
Transitive triad parameters		
τ_{c2c}	1.51	0.49
τ_{cc2}	−0.86	0.38

is not very likely to survive to time 2, given the negative density (γ_0) estimate.

However, a tie is more likely to be sustained across time if it remains reciprocated across time (ρ_{cc}). This effect almost cancels out the time 2 density effect (γ_0), suggesting that, in the absence of other effects, there is no particular tendency for originally reciprocated ties to dissipate across time, in contrast to unreciprocated ties. The positive out-star effect (σ_O) suggests that some individuals have high expansiveness at time 2, although expansiveness does not seem predictable across time. The two transitive triad effects explain different aspects of triadic evolution. The strongly positive τ_{c2c} effect suggests that (similar to the trust network) there is a tendency for a friendship to develop between two individuals who select the same person as a friend. The negative τ_{cc2} effect, on the other hand, suggests a tendency against friends of friends becoming friends in this network.

5 CONCLUSIONS

It is not conceptually difficult to generalize the approach of this article to multiple time points. The generalization from a two-block to multi-block chain graph is well understood in the graphical modeling literature. The difficulties with multi-time point network models again relate in part to specifying appropriate dependence structures. It would of course be simple enough to split the time points into adjacent pairs and to utilize the procedures of this paper on a pair by pair basis. This would involve the Markov-type assumption that network observations at time t depend only on observations at time $t-1$ but not at time $t-2$, nor at earlier time points. The pairs of time points could even be regarded as multiple observations of the same cross-time process, in effect imposing homogeneity across time points. This assumption, although not atypical, would be a strong one, implying that the systematic processes are unchanged in effect size and in direction across the entire measurement period. Nevertheless, such a model could identify whether some such constant systematic change is present.

It is also possible to generalize the models by relaxing the constant tie assumption. The additional parameters for a complete perfect Markov model can be fitted if data sets are of adequate size. Nevertheless, replacement of the constant tie assumption might best be decided after a substantive consideration of the likely bases of cross-time dependencies.

An important next step is to combine these temporal models with the attribute-related p^* models of Robins *et al.* (in press) and Robins *et al.* (2000). As Robins *et al.* (in press) noted, following Leenders (1997a), a full understanding of the distribution of attributes within a social structure requires a temporal framework. The chain graph structure provides the methodological basis for this advance.

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