36-780: Social Network Modeling

Hierarchical Network Models

Brian Junker
132E Baker Hall
brian@stat.cmu.edu

Announcements

- Special presentations (so far):
  - Feb 4: David Krackhardt, on (I think) transitive correlation for measuring transitivity in a graph
  - Feb 13: David Choi, topic TBA (computation and inference for social network models)
  - Feb 18: Beau Dabbs on cross-validation measures of goodness of fit
- HW05 Due Friday (submit to Blackboard):
  - Propose
    - Two papers to present; or
    - One small project

Outline

- **Quick Review: LSMs, SBMs, MMSBMs**
- **Hierarchical Network Models** (HNM)
- **Example**(s)
  - (MM)SBM for considering within- and between-school professional ties
  - HLSM in teacher advice networks
- **Open Questions:**
  - Goodness of fit
  - Network size and network density
  - Power to detect interventions
  - Unification and computation (CID models)

LSM

- For adjacency matrix Y = [Y_{ij}] ,

$$\operatorname{logit} P[Y_{ij} = 1] = \beta^{\mathsf{T}} X_{ij} - \operatorname{dist}(Z_i, Z_j)$$

- Distribution for the Z_i's $\in R^d$
  - Convenient simple case:

$$Z_i \overset{iid}{\sim} N_d(\vec{\mu}, \Sigma)$$

  - Can incorporate model-based clustering as well:

$$Z_i \overset{iid}{\sim} \sum_{k=1}^{K} \pi_k N_d(\vec{\mu}_k, \Sigma_k)$$

    (e.g. Fraley & Raftery, 2002, JASA)

- Any other tractable distribution for Z is fine also.

SBM

$$\begin{aligned} \operatorname{logit} P[Y_{ij}=1] &= \beta^{\mathsf{T}} X_{ij} + \operatorname{logit} B[M_i, M_j] \\ &= \beta^{\mathsf{T}} X_{ij} + \operatorname{logit} S_i^{\mathsf{T}} B S_j \end{aligned}$$

- B is a K $\times$ K matrix, for a K-block model
- S_i is a 0/1 vector with all 0's except for a 1 in position M_i
- M_i = k if node i is in block k.
- The M_i 's (or S_i 's) are the latent variables here

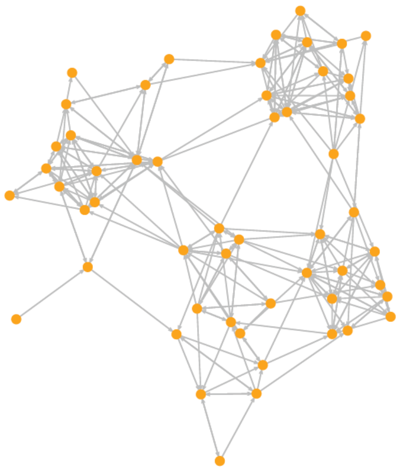
Mixed Membership SBM (MMSBM)

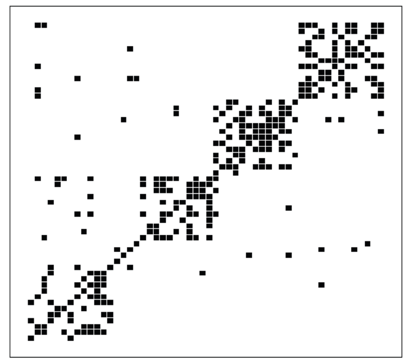
$$\operatorname{logit} P[Y_{ij}=1] = \beta^{\mathsf{T}} X_{ij} + \operatorname{logit} S_{ij}^{\mathsf{T}} B R_{ji}$$

- S_{ij} is an indicator vector, indicating which block node i is in, for sending a tie to j
  - $S_{ij} \sim$ Multinomial(1, θ_i)
- R_{ji} is an indicator vector, indicating which block node j is in, for receiving a tie from i
  - $R_{ji} \sim$ Multinomial(1, θ_j)
- Airoldi et al (2008). *J Machine Learning Research,* 1981-2014.

Mixed membership stochastic blockmodels can account for "outlier" ties

- Friendship ties among 6th graders (Sweet, 2013 NCME)





- Tie may depend on whether I am considering you from my vantage point in the swim team or the chess club

Lots of evidence that social networks matter in education

- Teacher characteristics influence ties
  - Teaching in the same grade and being in a leadership role promotes advice ties more than individual characteristics (Spillane et al., 2011)
- Social networks moderate behavior and interventions
  - Social networks play a key role in adolescent aggression (Low, Polanin & Espelage, 2013)
  - Access to expertise through social network related to changes in teacher practice (Frank et al., 2004; Peneul, 2006)
  - Dense subgroups more involved with intervention than sparse subgroups (Daly et al. 2010)
- Intervention can influence social networks
  - Teachers involved in school-wide initiatives more connected with one another (Weinbaum et al., 2008)

(MM)SBM: What affects within- and between-school professional ties?

- Spillane & Hopkins (2012) teacher advice-seeking data from 14 elementary schools in "Auburn Park"

School	Students Enrolled	Percent White	Percent African American	Percent Latina/o	Percent English Learner	Percent Free/ Reduced Lunch
Kingsley	564	89	2	4	--	7
Chamberlain	528	91	3	3	--	5
Ashton	484	74	5	12	7	40
Ashe	464	88	2	5	--	7
Warner	446	84	7	2	4	18
Abbott	441	93	1	4	--	23
Bryant	436	81	6	8	--	39
Riley	403	89	4	3	--	28
Northvale	395	86	4	5	--	14
Torres	393	76	9	8	9	44
Cisneros	353	88	3	4	--	16
Chavez	343	71	11	11	8	58
Stevenson	277	69	10	10	9	48
Easton	259	83	3	5	--	10

(MM)SBM: What affects within- and between-school professional ties?

- What drives ties between professionals (teachers, leaders) within and between schools?
- Start with basic MMSBM: $P[Y_{ij}=1] = S_{ij}^T B R_{ji}$
- But we have many covariates:
  - receiver is a formal leader
  - receiver teaches multiple grades
  - sender's career stage
  - individuals are the same sex
  - individuals teach the same grade
  - difference in over PD hours
- MMSBM with covariates: $\text{logit } P[Y_{ij}] = \text{logit } S_{ij}^T B R_{ji} + X_{ij}\beta$

(MM)SBM: What affects within- and between-school professional ties?

- **Preliminary results from Sweet (MMSBM version)**:
- *Effect on within-school ties*:

Receiver formal leader	1.79 (1.56, 1.99)*
Receiver teaches multiple grades	-1.07 (-1.32, -0.77)*
Sender Career Stage	0.21 (0.14, 0.29)*
Same Sex	1.75 (1.48, 2.04)*
Same Grade	2.11 (1.87, 2.34)*
Difference in overall PD hours	0.01 (-0.02, 0.04)

- *Effect on between-school ties*:

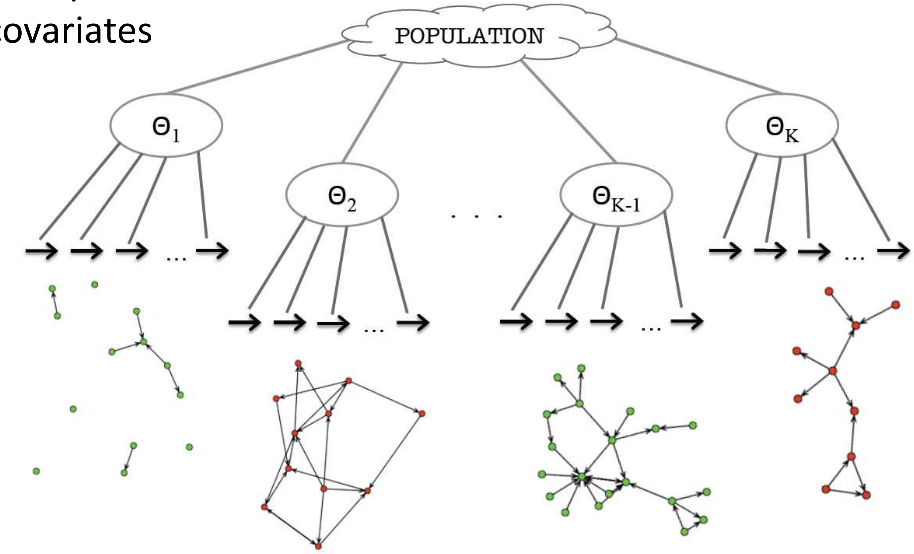
Receiver formal leader	1.09 (0.64, 1.53)*
Receiver teaches multiple grades	0.48 (-0.15, 1.06)
Sender Career Stage	-0.10 (-0.23, 0.04)
Same Sex	-0.27 (-0.72, 0.15)
Same Grade	0.38 (-0.09, 0.91)
Difference in overall PD hours	-0.01 (-0.05, 0.03)

Multiple social networks in education

- Multiple networks within buildings
  - Classes of students
  - Students or Teachers in the same grade
  - Teachers in the same department
  - Teachers geographically close (same floor, same end of hall, ...)
- Each building in a district
  - Students within the building
  - Teachers within the building
- Bldgs & other organizational units between districts

Hierarchical Network Models

- Model common elements of an ensemble of networks
- Gain power to detect effects of interventions and other covariates



An HNM for any network model

- Hierarchical Latent Space Model (HLSM, Sweet, Thomas, Junker, 2013, *JEBS*)
- Hierarchical (MM) Stochastic Blockmodel (HMMSBM, Sweet, Thomas, Junker, 2012, *MM Handbook*)
- Hierarchical ERGM (HERGM, Zijlstra et al., 2006, *Methodology*)

HLSM: Does teaching grade affect teacher advice networks?

- 15 Teacher advice networks (Pitts & Spillane, 2009)

Network Size	Density	Proportional Density	Mean In-/Out-Degree	Proportion Teaching the Same Grade
30	61	0.07	2.03	0.60
21	103	0.25	4.90	0.38
25	75	0.13	3.00	0
19	54	0.16	2.84	0.11
26	76	0.12	2.92	0.65
49	162	0.07	3.31	0.53
76	314	0.06	4.13	0.61
42	172	0.10	4.10	0.64
33	106	0.10	3.21	0.76
29	107	0.13	3.69	0.41
12	15	0.11	1.25	0.17
28	117	0.15	4.18	0.50
14	46	0.25	3.29	0.14
14	51	0.28	3.64	0.43
15	81	0.39	5.4	0.40

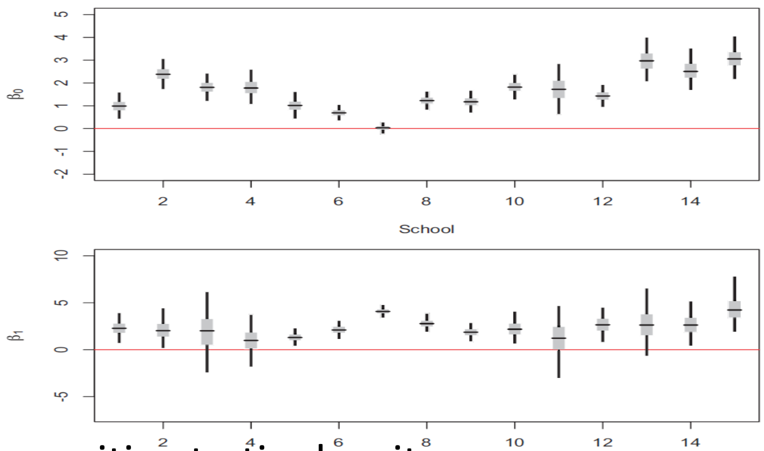
HLSM: Does teaching grade affect teacher advice networks?

- Sweet et al. (2013 JEBS) fitted the HLSM model

$$\text{logit } P[Y_{ijk} = 1] = \beta_{0k} + \beta_{1k} X_{ijk} - dist(Z_{ik}, Z_{jk})$$

- Y_{ijk} = 1 for a tie between teachers i and j in network k
- X_{ijk} = 1 if teachers i & j in network k teach the same grade
- Z_{ik} is the latent space position of teacher i in network k
  - *dist*(Z_{ik}, Z_{jk}) measures (dis)affinity due to unmodelled covariates
- β_{ok} and β_{1k} are allowed to vary across networks, but are sampled from a common population
  - Constrains the β's to be similar
  - Allows model to borrow strength from larger networks to improve estimation in smaller networks

HLSM: Does teaching grade affect teacher advice networks?



- β_0 is sensitive to tie density
- β_1 indicates the effect of teaching the same grade

Other Network Data Waiting for Analysis

- **Bully Prevention**: Which interventions are effective in reducing bullying behavior? How do they affect the friendship networks themselves? (with Dorothy Espelage, U of I; multiple school-level networks)
- **Organizational Structure**: How do org structure and individual factors shape professional advice-seeking networks? (With Jim Spillane, NW, 30 school networks)
- **Evolving Friendship Ties**: Friendship data was collected on 5th graders several times over a school year. What factors affect friendships and changes in friendship over time? (With Rebecca Madill, Penn State, 25 networks).

Some Open Questions

- **Goodness of fit**
  - Very few proven tools exist
  - Beau Dabbs (CMU Statistics) is working on cross-validation methods using prediction of missing edge statuses as a criterion
- **Effect of varying network size over the ensembles of networks**
  - In most human social networks, the number of ties one individual grows more slowly than the number of actors in the network
  - This causes the tie density to go down. How does this affect the estimation of intervention effects, etc.?
- **Power to detect treatment or covariate effects**
  - How many networks are needed, and how big should they be, to detect an intervention effect?
  - Sweet (2013) has a partial answer for HLSMs. Little is known about other models.
- **Unification and Computation**
  - (MM)SBMs, LSMs and a subset of ERGMs can be placed in a single framework, the conditionally independent dyad (CID) models
  - Thomas, Dabbs and Sweet are developing R software to estimate a general class of CID models.

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