BOOK REVIEW

Statistical Factor Analysis and Related Methods: Theory and Applications
Alexander Basilevsky. New York: John Wiley and Sons (1994).
737+xxvi pp., \$95.00.

Ask a statistician or quantitative methods person who understands "factor models" what they are all about and you are likely to get one of two kinds of answers. A common, modern answer begins with something like the mixture model

$$P(x_{i1}, \ldots, x_{iJ}) = \int P(x_{i1}, \ldots, x_{iJ} | \theta_i, \beta_1, \ldots, \beta_J) \, f(\theta_i | \mu, \sigma^2) \, d\theta_i$$

for the distribution of the responses $(x_{i1}, x_{i2}, \ldots, x_{iJ})$ collected from each of N individuals or experimental units, $i = 1, \ldots, N$. The mixture model is analyzed directly, under various plausible simplifying assumptions such as conditional independence of the x's given the θ's, to provide estimates of or inferences about the response-specific parameters β_j, the subject-specific parameters θ_i, or the population parameters μ; for example Bartholomew's (1987) text explores this approach fully.

An older answer, under which most of the ideas of latent variable and factor models were first developed, stems from the observation that for many important problems the mixture model recipe above reduces to the linear representation

$$\underline{X} = A\underline{\Theta} + \underline{\epsilon} \qquad (*)$$

where $\underline{X} = (X_1, \ldots, X_J)^{\mathrm{T}}$ is the (random) vector of responses for a single individual, A is a matrix of "loadings", $\underline{\Theta}$ is a vector of factors or principal components, and noise and measurement error are subsumed in $\underline{\epsilon}$. This leads to the point of view that the main object of interest is the covariance matrix $\boldsymbol{\Sigma} = \mathrm{Cov}\,(\underline{X})$, and to the identification of both A and $\underline{\Theta}$ from eigenvalue/eigenvector (principal components) decompositions of $\boldsymbol{\Sigma}$ or related matrices. From this "second-order" point of view, many traditional data analytic methods related to factor analysis are seen as variations on the principal components decomposition of some Gramian (symmetric, positive-definite) matrix of similarity coefficients, which aids understanding of the pairwise association properties of $\underline{X}$, quite independently of the linear model in Equation $(*)$.

Alexander Basilevsky's *Statistical Factor Analysis and Related Methods* is an exhaustive, graduate-level text on statistical aspects of the second-order point of view of factor models, as well as the way these models play out in other areas of statistics such as time series analysis, spatial statistics, seriation problems, and regression and MANOVA. Basilevsky has extensive experience with these models, often in connection with applications in in engineering, demographics and sociology, and one of the book's strongest features is the collection of examples and real-data exercises taken from

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a wide variety of fields. On the other hand, educational and psychological measurement researchers will find the book gives limited attention to problems and applications in their fields.

Chapters 1 and 2 give an outline of the book's prerequisites: a serious calculus-based course in mathematical statistics through likelihood ratio (LR) testing and elementary multivariate statistics, and a semester-long course in linear algebra. These would be useful as review in a classroom setting, but are too sketchy for a first encounter.

For Basilevsky, all factor analytic methods begin with principal components analysis (PCA), and he spends three chapters laying out the details. Chapter 3 describes the ordinary PCA "model" in both the population and the sample. There is a nice discussion of the rationale for scree plots, in terms of isotropic variation in the residual dimensions, as well as introductory discussions of exploratory PCA, effects on the factor/component scores of non-linear relationships among the variables, and various scalings for the loadings and component scores. Chapter 4 discusses classical multivariate normal statistical testing, beginning with simple LR tests for covariance matrices, and quickly focusing on tests of equality of lower-order eigenvalues (latent roots) and tests for zero loadings. (Nevertheless, in several examples throughout the book loadings are deleted from displayed loading matrices, either without comment, or at most with the usual informal footnote, "Loadings smaller than 0.10 are omitted.") There is a brief discussion of statistical testing for the number of principal components, including the non-χ^2 behavior of LR tests of dimensionality (the null hypothesis is at the boundary of the alternative hypothesis in the parameter space), and (in-)consistent estimation of number of dimensions with AIC (Akaike's Information Criterion), vs. BIC (the Bayes Information Criterion, also known as the Schwarz criterion), and their generalizations. There is also some discussion of testing for multivariate normality and homoscedasticity; robust inference; sensitivity; and missing data. Chapter 5 discusses various extensions such as weighted PCA, rotations and clustering, R-mode (treating columns as variables, the usual practice) vs. Q-mode (treating rows as variables) analysis, biplots, canonical correlation, and PCA for proportions and mixture data.

Chapter 6 treats factor analysis (FA) proper. A nice distinction is drawn at the beginning of the chapter between the stochastic assumptions of the FA model and the PCA model, primarily in terms of the treatment of the error term $\underline{\epsilon}$ in Equation $(*)$. The usual mathematical statistics treatment of factor analysis models is given in pleasant detail, and a sketch of the development of factor analysis from the mixture model recipe is given; the emphasis, however, is firmly on second-order approaches. Some very standard objects present in the model—communalities for example—are not named or mentioned in the text (though of course they are present in the equations). Sprinkled in among the theorems there are a number of thoughtful examples and interpretations of the machinery. Discussion of psychometric applications is limited to a passing reference to alpha and image factor analysis. Multilevel analyses are omitted from the discussion, and multigroup/multimode analyses are treated only lightly.

The incorporation of repeated measures and time series analysis, Chapter 7, into a modern book on factor analysis, is welcome. The second-order point of view works extremely well, since the linear model in Equation $(*)$ has a natural analogue in the Fourier analysis usually applied to time series. In repeated measures it is usually the time-dependent functions we are interested in, and these are often revealed in the first one or more principal components of a Q-mode analysis, as prefigured by the discussion of nonlinear effects in PCA in Chapter 3. Similar methods can be used to summarize and explore large collections of continuous functions, as suggested by Ramsay and Dalzell (1991) and the references and discussion following their paper. Later sections develop the Karhunen-Loève spectral decomposition as essentially Q-mode PCA applied to continuous functions, give methodology for analysis of (necessarily discrete) sample data, and show that many time-domain problems in time series, such as estimating the dimension of a stationary ARMA process and Kalman filtering, have standard solutions which are applications of PCA or FA. The entire chapter serves as a lively and interesting introduction to time series methods from the PCA/FA perspective.

Chapters 8 and 9 treat PCA methods for discrete (ordinal, nominal and count) data. Here the second-order point of view is less successful, since the linear model of Equation ($*$) is hardly ever true for discrete data, unless errors and traits are allowed to be dependent, or it is embedded in a mixture model for product multinomial or Poisson observations (as in item response theory, and more generally random-effects generalized linear models). Chapter 8 broadly discusses the possibilities for PCA analysis of Gramian association and dispersion matrices for discrete data, more or less without regard to the validity of the linear model. The fundamental similarities among Guttman scaling, canonical correlation, optimal scoring for two-way tables, dual scaling and correspondence analysis, are also discussed. Chapter 9 treats various refinements. A nice feature of Chapter 9 is the careful attention given to the "horseshoe effect" in which, because discrete data vectors must lie on the vertices of rather simple lattices in $\mathbb{R}^n$, naive linear-model second-order factor/component scores tend to exhibit a lot of artificial nonlinear behavior. Nonlinear measures of association, from Minkowski distance to polychoric and point-biserial correlations, are sprinkled throughout the chapters. Item response data is treated under the heading "multivariate multinomial data" (Chapter 8) but the most modern analysis discussed in detail is the Bock and Lieberman threshold approach (Chapter 9), with second-order computational developments by Christofferson and Muthén. As Basilevsky's discussions of nonlinear measures of association and item response data show—not to mention the many computing advances of the 1980's and 1990's summarized by Bartholomew (1987) and Tanner (1993)—formal progress developing and fitting parametric latent variable models for discrete data greatly benefits from fully adopting the mixture model recipe.

Finally Chapter 10 considers applications of PCA to linear regression and MANOVA, and the implementation of linear regression in errors-in-variables settings using FA methods. The two uses of PCA to compensate for multicollinearity, by constructing a smaller set of regressor variables, or by performing biased regression with the original regressors, are presented very clearly. Ridge regression is mentioned only briefly, which is unfortunate since it leads naturally to estimating functions by spline smoothing (e.g. Eubank, 1988), a topic closely related to the Fourier developments of Chapter 7.

Overall the book is lengthy, but the breadth of coverage and wealth of examples within the second-order framework is valuable. I have omitted many applications and side topics from the summary above that readers should find interesting, notwithstanding the paucity of examples and literature from modern psychometrics and educational measurement. Basilevsky has also provided valuable exercise sets for each chapter, ranging from routine manipulations, to proofs, to analyzing real data sets. Unfortunately the publisher, Wiley, did not provide a data diskette with the review copy of the text. Wiley should also be taken to task for seriously inadequate copy editing. All first editions contain some typographical errors, but this book is rife with distracting spelling errors, notational glitches, missing references, and easily-correctable flaws in verbal and technical material.

Basilevsky intends in his preface to produce a unified treatment of both the theory and the practice of factor analysis and latent variable models, to set these subjects on firm statistical ground and to argue to the mainstream statistics community that these methods are broadly applicable and sensible to use in many scientific areas. His book contains a wealth of material on the second-order (principal components) approach to factor models and related methods, from a modern point of view. He does not address the larger issue of latent variable modeling on as deep a footing as Bartholomew's (1987) text, but he has addressed the statistical and practical issues of the second-order approach admirably, and produced a book to which careful users of this methodology will want to refer.

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