# ESTIMATING MODELS FOR PANEL SURVEY DATA UNDER COMPLEX SAMPLING

## MARCEL D. T. VIEIRA, CHRIS J. SKINNER

## ABSTRACT

Complex designs are often used to select the sample which is followed over time in a panel survey. We consider some parametric models for panel data and discuss methods of estimating the model parameters which allow for complex schemes. We incorporate survey weights into alternative point estimation procedures. We also consider variance estimation using linearization methods to allow for complex sampling, and indicate connections with established asymptotically distribution free (ADF) methods. The behaviour of the proposed inference procedures are assessed in a simulation study, based upon data from the British Household Panel Survey. There appear to be some advantages of using the weighted maximum likelihood (ML) point estimation method compared to the weighted ADF method. Variance estimation methods that allow for clustering tend to lead to improvements in terms of bias. However, the variance estimator for the weighted ML estimator performs better than the ADF variance estimators.

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# Estimating Models for Panel Survey Data under Complex Sampling

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*Key words*: longitudinal survey; covariance structure; multistage sampling; stratification; weighting.

## 1. Introduction

A broad class of 'regression-type' models has found a wide range of useful applications with panel survey data (Baltagi, 2001; Wooldridge, 2001; Diggle *et al*., 2002; Hsiao, 2003). Such data often consist of repeated observations on the same variables for the same individuals across equally spaced waves of data collection. The 'regression-type' models considered here are broadly concerned with representing the relationship between one of the variables, treated as dependent, and a number of the other variables, treated as covariates. A typical example of the kind of panel survey considered here is the British Household Panel Survey (BHPS), in which a sample of households was selected at wave one and then individuals in this sample were followed up repeatedly at annual intervals.

It is common for the selection of the initial panel sample at wave one to involve a complex sampling scheme. For example, stratification and multistage sampling were employed in the selection of the initial BHPS sample. In addition, sample individuals are often selected with unequal probabilities and weights are constructed to compensate for these unequal probabilities as well as for different forms of wave nonresponse and other complexities (Kalton and Brick, 2000). There is a limited consideration of the treatment of such sampling schemes in the panel data model literature, especially in relation to the clustering of individuals (Wooldridge, 2001).

Since the 1960s, there have been a number of articles in the survey sampling literature that examined regression analysis for cross section complex survey data. Some key ideas are set out in Kish and Frankel (1974), Fuller (1975), and Binder (1983). Other important contributions are Konijn (1962), Brewer and Mellor (1973), Shah, Holt and Folsom (1977), Holt, Smith and Winter (1980), Scott and Holt (1982) and DuMouchel and Duncan (1983). Skinner, Holt and Smith (1989) and Chambers and Skinner (2003) provide overviews. Methods that permit survey features, such as unequal probability selection, stratification and multistage sampling, to be handled appropriately

have therefore been developed in the cross section data model context, and include for example a pseudo maximum likelihood approach for point estimation, and linearization methods for variance estimation.

In this paper we shall extend these methods to consider estimation of panel data models parameters, allowing for complex sampling designs. We shall discuss methods of statistical inference for models with parametric assumptions about the covariance structure of errors over time. We shall incorporate survey weights into alternative point estimation procedures, including maximum likelihood, generalized least squares and asymptotically distribution free (ADF) approaches. We shall also consider standard error estimation approaches using Taylor series linearization methods to allow for complex sampling, and indicate connections with some established ADF methods. We shall adopt an aggregate modelling strategy (Skinner, Holt and Smith, 1989) rather than a multilevel covariance modelling approach. For developments of the latter approach see Müthén and Satorra (1995, Section 5).

Some previous work on estimation for panel data models under complex designs has been undertaken by Feder, Nathan and Pfeffermann (2000), who propose combining multilevel modelling, time series modelling and survey sampling methods; Sutradhar and Kovacevic (2000), where a generalised estimating equations approach is developed by considering an autocorrelation structure in a multivariate polytomous longitudinal survey data context; Skinner and Holmes (2003), who study two approaches for dealing with sampling effects, either considering the repeated observations as multivariate outcomes and adopting weighted estimators that account for the correlation structure, or considering a two-level longitudinal model and to modify weighting strategy proposed by Pfeffermann *et al.* (1998); and Skinner and Vieira (2005), who presented some empirical evidence that the variance-inflating impacts of complex sampling schemes can be higher for longitudinal analyses than for corresponding cross-sectional analyses.

This paper is organized as follows. The basic structure of the data and sample are described in Section 2. The models are given in Section 3. Point estimation methods, including weighted

estimation of covariance matrices are reviewed in Section 4. Estimation of model parameters using least squares methods and pseudo maximum likelihood estimation are also considered. The paper proceeds in Section 5 to consider variance estimation methods, by adopting Taylor series linearization methods to allow for complex sampling and also considering ADF variance estimation techniques. Two simulation studies, based upon data from the British Household Panel Survey, will be presented in Section 6 to assess the behaviour of the different estimation procedures. We make brief remarks in the concluding discussion in Section 7.

## 2. Sampling and Data

We suppose that the data consist of the values $y_{it}$ of an outcome variable and $1 \times q$ vectors of values $\mathbf{x}_{it}$ of covariates for each individual $i$ in a sample, denoted $s$, and each wave of data collection $t = 1, \ldots, T$. The sample is assumed to be selected from a specified finite population at wave 1 according to a probability design for which the inclusion probability $\pi_i$ of each individual $i$ in $s$ is known and the sample and the population are fixed thereafter. We suppose that sampling weights, $w_i$, are available for estimation and that, by default, these are the reciprocals of the sample inclusion probablities $\pi_i$. We shall sometimes write $s = \{1, \ldots, n\}$, without loss of generality, where $n$ is the sample size. For simplicity, we shall not refer to nonresponse, treating the data as complete. In practice, this will not be the case and $s$ may be interpreted as the set of individuals providing values $y_{it}$ and $\mathbf{x}_{it}$ at each occasion, where the $w_i$ include some adjustment for nonresponse.

## 3. Models

We consider standard kinds of models for the repeated measurements (Ware, 1985; Diggle *et al.*, 2002, Chapters 4 and 5; among others) in which the $y_{it}$ obey the linear model:

$$E(y_{it}) = \mathbf{x}_{it} \boldsymbol{\beta}, \tag{1}$$

where $\mathbf{x}_{it}$ is treated as fixed (or conditioned upon), $\boldsymbol{\beta}$ is a $q \times 1$ vector of unknown parameters (and we make no distinction between the realised $y_{it}$ and the underlying random variables). We allow for serial correlation in the measurements by writing the repeated measurements for individual $i$ as the $T \times 1$ vector $\mathbf{y}_i = (y_{i1}, \ldots, y_{iT})'$ and allowing for non-zero off-diagonal elements of the covariance matrix $\Sigma$ of this vector:

$$\Sigma = \mathrm{cov}(\mathbf{y}_i) = E\{[\mathbf{y}_i - X_i \boldsymbol{\beta}][\mathbf{y}_i - X_i \boldsymbol{\beta}]'\}, \tag{2}$$

where $X_i = (\mathbf{x}_{i1}', \ldots, \mathbf{x}_{iT}')'$ is the $T \times q$ matrix of covariate values.

We consider two possible structures for the matrix $\Sigma$. The first is referred to as the uniform correlation model (UCM), where all the off-diagonal elements of $\Sigma$ are $\sigma_u^2$ and all the diagonal elements are $\sigma_u^2 + \sigma_v^2$. This corresponds to the multilevel model:

$$y_{it} = \mathbf{x}_{it} \boldsymbol{\beta} + u_i + v_{it} \tag{3}$$

where $u_i$ and $v_{it}$ are random effects with zero means and variances $\sigma_u^2$ and $\sigma_v^2$ respectively, which are uncorrelated over time. In this case the correlation between $y_{it}$ and $y_{it'}$ for any two occasions $t$ and $t'$ for $t \neq t'$ is given by $\rho = \sigma_u^2 / (\sigma_u^2 + \sigma_v^2)$.

In our second structure, referred to as the AR1 model, the correlation is allowed to decay over time. We again assume that all diagonal elements are $\sigma_u^2 + \sigma_v^2$ but now suppose that the covariance between $y_{it}$ and $y_{it'}$ for occasions $t$ and $t'$ takes the form $\mathrm{cov}(y_{it}, y_{it'}) = \sigma_u^2 + \gamma^{|t-t'|} \sigma_v^2$, where $\gamma$ is an additional parameter ($|\gamma| < 1$). This model corresponds to the following first-order autoregressive process for the $v_{it}$:

$$v_{it} = \gamma v_{it-1} + \varepsilon_{it}, \tag{4}$$

where the $\varepsilon_{it}$ are mutually independent residuals with zero mean and variance $\sigma_\varepsilon^2 = (1 - \gamma)^2 \sigma_v^2$ (Crowder and Hand, 1990; Jones, 1993). Note that in both models it is assumed that $\Sigma$ does not depend upon $i$.

To emphasise the fact that the covariance matrix $\Sigma$ takes a particular parametric structure for each model, we write $\Sigma = \Sigma(\theta)$, where $\theta$ is a $b \times 1$ parameter vector. In particular, $\theta = \left( \sigma_u^2, \sigma_v^2, \gamma \right)'$ for the AR1 model and $\theta = \left( \sigma_u^2, \sigma_v^2 \right)'$ for the UCM model. Note that the UCM model is a special case of the AR1 model where $\gamma = 0$.

We have so far only made assumptions about the correlation of the $y_{it}$ between different time points $t$ but not between different individuals $i$. We shall, indeed, assume that the parameter vector $\theta$ governing the inter-temporal covariance matrix $\Sigma(\theta)$ is of scientific interest, but that any correlation between values of $y_{it}$ for different individuals is a 'nuisance'. In the UCM and AR1 models we shall assume that the correlation between $y_{it}$ and $y_{i't'}$ is zero for any two distinct individuals $i$ and $i$' and any two occasions $t$ and $t$'. We shall also consider a UCM(C) model, where C denotes cluster, for which this correlation is given by a fixed quantity, $\tau$, for any distinct individuals $i$ and $i$' in the same cluster and any two occasions $t$ and $t$' and zero otherwise, where the inter-temporal covariance structure $\Sigma(\theta)$ is the same as for the UCM model.

## 4. Point Estimation

We shall suppose that $\beta$ is estimated following an established approach for repeated survey observations, as implemented for example in the software SUDAAN (Shah et al. 1997), by:

$$\hat{\beta} = \left( \sum_{i \in s} w_i X_i' V^{-1} X_i \right)^{-1} \sum_{i \in s} w_i X_i' V^{-1} \mathbf{y}_i \qquad (5)$$

where $V$ is a specified 'working' covariance matrix of $\mathbf{y}_i$ (Diggle *et al.* 2002, p.70) and the $w_i$ are the survey weights introduced in section 2. Provided the linear model in (1) holds and $V$ is constant, $\hat{\beta}$ will be consistent for $\beta$ with respect to the joint model and sampling design if the sample size is large (c.f. Fuller, 1975; Isaki and Fuller, 1982; Liang and Zeger, 1986).

In the simulation study we shall suppose that $V$ is estimated using the UCM model as the working model. This just requires estimating the intra-individual correlation $\rho$ since $\sigma^2 = \sigma_u^2 + \sigma_v^2$ cancels out of the two places $V$ appears in (5). We shall estimate the correlation $\rho$ by iterating between GLS estimation of $\boldsymbol{\beta}$ and survey-weighted moment-based estimation of the intra-individual correlation (Liang and Zeger, 1986; Shah *et al.*, 1997). Following standard large sample arguments (Liang and Zeger, 1986) $\hat{\boldsymbol{\beta}}$ will remain consistent for $\boldsymbol{\beta}$ even though $V$ is subject to sampling variation.

As in section 3, let $\boldsymbol{\theta}$ denote the $b \times 1$ vector of parameters of interest which determine the covariance structure $\Sigma = \Sigma(\boldsymbol{\theta})$ of $\mathbf{y}_i$, as given in (2). In order to define a class of estimators $\boldsymbol{\theta}$, we first define the weighted residual covariance matrix:

$$S_w = \hat{N}^{-1} \sum_{i \in s} w_i \left(\mathbf{y}_i - X_i \hat{\boldsymbol{\beta}}\right)\left(\mathbf{y}_i - X_i \hat{\boldsymbol{\beta}}\right)' \tag{6}$$

where $\hat{N} = \sum_{i=1}^{n} w_i$ estimates the population size, $N$. The matrix $S_w$ is a consistent estimator of $\Sigma$ with respect to the joint model and sampling design, provided that the model assumptions in (1) and (2) hold (Skinner, Holt and Smith, 1989). Having defined $S_w$, we now define the class of estimators $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$ to be considered, as those that minimise different measures of 'distance' between $S_w$ and $\Sigma(\hat{\boldsymbol{\theta}})$ (Jöreskog and Goldberger, 1972; Browne, 1984; Bollen, 1989). More precisely, if $F(S_w, \Sigma)$ denotes the fitting function, which measures the distance between $S_w$ and $\Sigma$, then $\hat{\boldsymbol{\theta}}$ is defined as the value of $\boldsymbol{\theta}$ which minimises $F(S_w, \Sigma(\boldsymbol{\theta}))$ across values of $\boldsymbol{\theta}$ in a specified *b*-dimensional parameter space.

The simplest example of a fitting function is the *unweighted least squares* (ULS) function:

$$F_{ULS}(S, \Sigma) = \frac{1}{2} \cdot tr\{[S - \Sigma]^2\}. \tag{7}$$

The resulting ULS estimator $\hat{\boldsymbol{\theta}}_{ULS}$ is uniquely defined and is consistent for $\boldsymbol{\theta}$, given that $S_w$ is consistent for $\Sigma$ (Browne, 1982; Browne, 1984). However, $\hat{\boldsymbol{\theta}}_{ULS}$ is not in general an asymptotically efficient estimator of $\boldsymbol{\theta}$. Moreover, it is not scale invariant (Jöreskog and Goldberger, 1972) although this does not seem a serious problem when the elements of $\mathbf{y}_i$ are repeated measurements of the same variable. With the aim of improving efficiency, we consider also a class of *generalised least squares* fitting functions:

$$F_{GLS}(S,\Sigma) = \{vech(S) - vech(\Sigma)\}' \, U^{-1} \{vech(S) - vech(\Sigma)\}, \tag{8}$$

where *vech* is the vector of distinct elements of a symmetric matrix (Fuller, 1987). For the $T \times T$ matrices considered here, *vech* is of dimension $k \times 1$, where $k = T(T+1)/2$. The 'weight' matrix $U$ remains to be specified. For efficient estimation, we should like $U$ to correspond to (approximately) to the covariance matrix of $vech(S)$, for the relevant matrix $S$, which is $S_w$ in our setting. A traditional approach to the specification of $U$, which ignores the complex sampling scheme and is motivated by a working assumption of normality and independent and identically distributed observations, is (McDonald, 1980):

$$U = 2 \cdot K'(W \otimes W)K, \tag{9}$$

where $K$ is the so-called transition matrix, $W$ is any consistent estimator of $\Sigma$ (Bentler and Weeks, 1980; Swain, 1975), and $\otimes$ is the right Kronecker product operator. Expression (9) may alternatively be written elementwise as (Joreskog and Goldberger, 1972; Swain, 1975):

$$U_{tt',t''t'''} = W_{tt''}W_{t't'''} + W_{tt'''}W_{t't''}, \tag{10}$$

where $U_{tt',t''t'''}$ and $W_{tt'}$ represent typical elements respectively of $U$ and $W$.

Expressions (8) and (9) imply (Browne, 1977) that $F_{GLS}(S,\Sigma)$ takes the form:

$$F_{GLS-NORM}(S,\Sigma) = \left(\frac{1}{2}\right) \cdot tr\{[(S-\Sigma)W^{-1}]^2\}, \tag{11}$$

where GLS-NORM indicates that this choice of fitting function is based upon an underlying normality assumption. There are two natural choices of $W$. The first is given by $S$, since this ($S_w$ in our setting) is assumed consistent for $\Sigma$. In this case we may write:

$$F_{GLS-NORM1}(S,\Sigma) = \left(\frac{1}{2}\right) \cdot tr\{[(S-\Sigma)S^{-1}]^2\} = \left(\frac{1}{2}\right) \cdot tr\{[I-\Sigma S^{-1}]^2\}. \qquad (12)$$

An alternative choice is to set $W$ equal to $\Sigma$, leading to:

$$F_{GLS-NORM2}(S,\Sigma) = \left(\frac{1}{2}\right) \cdot tr\{[S\Sigma^{-1} - I]^2\}. \qquad (13)$$

We denote the resulting estimators of $\boldsymbol{\theta}$ as $\hat{\boldsymbol{\theta}}_{GLS-NORM1}$ and $\hat{\boldsymbol{\theta}}_{GLS-NORM2}$. An alternative approach, not based on the working assumption of normality, is to set $U$ equal to an estimator of the asymptotic covariance matrix of $vech(S)$, making no assumption about the underlying distribution. Such an approach is often called *asymptotically distribution free* (ADF). See e.g. Browne (1982, 1984). We shall consider the use of linearization methods of variance estimation for this purpose in the next section, following some earlier applications of this idea in Skinner (1989), Satorra (1992), and Müthén and Satorra (1995).

Another approach to estimation is achieved by adopting the pseudo-maximum likelihood (PML) approach (Skinner, Holt and Smith, 1989) in which a census log-likelihood (assuming independent and identically distributed observations) is replaced by a weighted log-likelihood given by (ignoring constants):

$$-N \log|\Sigma(\boldsymbol{\theta})| - \sum_{i \in s} w_i [\mathbf{y}_i - X_i \boldsymbol{\beta}]' \Sigma(\boldsymbol{\theta})^{-1} [\mathbf{y}_i - X_i \boldsymbol{\beta}] \qquad (14)$$

If this weighted likelihood is first 'concentrated' by replacing $\boldsymbol{\beta}$ by $\hat{\boldsymbol{\beta}}$, maximising expression (14) becomes equivalent to minimising the value of the following fitting function (Jöreskog,1970) :

$$F_{PML}(S,\Sigma) = tr\left[S\Sigma^{-1}\right] - \log\left|S\Sigma^{-1}\right| - T, \qquad (15)$$

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with $S$ evaluated at $S_w$ to take account of the complex design. Alternatively, if this initial concentration does not take place, $\theta$ could be estimated simultaneously with $\beta$ by maximising expression (14). If $N$ is unknown, it might be replaced in (14) by $\hat{N} = \sum_{i=1}^{n} w_i$ .

The properties of the GLS-NORM1 and PML approaches may be compared by noting first that (12) may be alternatively expressed as (see Fuller, 1987, p. 334)

$$F_{GLS-NORM1}(S_w, \Sigma) = \frac{1}{2}(n-1)\sum_{t=1}^{T}(\lambda_t - 1)^2 ,$$

where $\lambda_1, \ldots, \lambda_t$ are the eigenvalues of $S_w^{-1/2} \Sigma S_w^{-1/2}$ . Similarly, (15) may alternatively be expressed as

$$F_{PML}(S_w, \Sigma) = \sum_{t=1}^{T}(\log \lambda_t + \lambda_t^{-1}) .$$

Moreover if the model holds, i.e. if $\Sigma = \Sigma(\theta)$, both GLS-NORM1 and PML estimators are obtained by minimizing (see Fuller, 1987, p. 335) $\sum_{t=1}^{T}(\lambda_t - 1)^2$ . Thus the GLS-NORM1 and PML estimators may be considered asymptotic equivalent.

Note that the computation of estimators which minimise fitting functions or maximise a pseudo likelihood generally involves numerical solution of equations, obtained by differentiating the fitting functions. Several alternative methods for performing the numerical solution are possible. In the simulation study in section 5, we adopted an iterative Newton type algorithm, similar to that suggested by Pourahmadi (1999). Alternative methods include: (*i*) a Nelder and Mead (1965) method; (*ii*) a quasi-Newton method or variable metric algorithm, proposed simultaneously by Broyden, Fletcher, Goldfarb and Shanno in 1970 (see Nocedal and Wright, 1999); (*iii*) a conjugate gradients method (Fletcher and Reeves, 1964); (*iv*) Byrd *et al.* (1995) method, which is a modification of method (*ii*); and (*v*) a stochastic global optimization method proposed by Belisle (1992). Our experience is that methods (*i*) to (*iii*) provide virtually the same results as those given by the Newton type algorithm we used, but that methods (*iv*) and (*v*) had difficulties in yielding converged solutions even for the largest sample sizes.

## 5. Variance estimation

In this section, we consider variance estimation for two purposes: first, to determine possible matrices $U$ to use in the generalised least squares fitting function in (8) and, secondly, for the purpose of estimating standard errors of the estimators of $\boldsymbol{\theta}$ considered in the previous section.

As a preliminary step, we consider estimation of the variances and covariances of the elements of $S_w$, i.e. we seek to estimate the asymptotic covariance matrix of the vector $vech(S_w)$. To establish the asymptotic covariance matrix with respect to both the sampling design and the underlying model requires defining a sequence of populations, sampling designs and samples. We suppose that this sequence is such that there exists a non-negative definite matrix $C$ such that the limiting distribution of $\sqrt{n}\{vech(S_w) - vech(\Sigma)\}$ is normal with a mean vector consisting of zeros and covariance matrix, $C$ (c.f. Isaki and Fuller, 1982), i.e.

$$\sqrt{n}\{vech(S_w) - vech(\Sigma)\} \to_L N_k(0, C). \tag{16}$$

We seek an estimator of the asymptotic covariance matrix $n^{-1}C$. From (6), we may write

$$vech[S_w] = \left(\sum_{i-1}^{n} w_i\right)^{-1} \sum_{i-1}^{n} w_i \hat{\mathbf{c}}_i \tag{17}$$

where $\hat{\mathbf{c}}_i = vech(\hat{\boldsymbol{\varepsilon}}_i \hat{\boldsymbol{\varepsilon}}_i')$ and $\hat{\boldsymbol{\varepsilon}}_i = \mathbf{y}_i - X_i \hat{\boldsymbol{\beta}}$. In order to employ the linearization method of variance estimation (Woodruff, 1971; Wolter, 1985), we linearize expression (17) to obtain:

$$vech(S_w) \doteq \boldsymbol{\mu}_z / \mu_w + n^{-1} \sum_{i=1}^{n} \mathbf{u}_i , \tag{18}$$

where $\mathbf{u}_i = \mu_w^{-1} w_i (\mathbf{c}_i - \boldsymbol{\mu}_z / \mu_w)$, $\mathbf{c}_i = vech(\boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}_i')$, $\boldsymbol{\varepsilon}_i = \mathbf{y}_i - X_i \tilde{\boldsymbol{\beta}}$, $\boldsymbol{\mu}_z = E(n^{-1} \sum_{i=1}^{n} w_i \mathbf{c}_i)$, $\mu_w = E(n^{-1} \sum_{i=1}^{n} w_i)$

and $\tilde{\boldsymbol{\beta}} = p\lim(\hat{\boldsymbol{\beta}})$.

A linearization variance estimator of the asymptotic covariance matrix of $vech(S_w)$ is then obtained by estimating the variance of the linear statistic $n^{-1}\sum_{i=1}^{n}\mathbf{u}_i$, allowing for the complex design, and then replacing $\mathbf{u}_i$ by $\hat{\mathbf{u}}_i = \overline{w}^{-1}w_i(\hat{\mathbf{c}}_i - \overline{\mathbf{z}}/\overline{w})$ where $\overline{w} = n^{-1}\sum_{i=1}^{n}w_i$ and $\overline{\mathbf{z}} = n^{-1}\sum_{i=1}^{n}w_i\mathbf{c}_i$.

For example, consider a multistage stratified sampling scheme that involves sampling primary sampling units (PSUs) with replacement at the first stage within $H$ strata independently, and sampling with or without replacement at subsequent stages. In this case, we rewrite $n^{-1}\sum_{i=1}^{n}\mathbf{u}_i$ as

$n^{-1}\sum_{h=1}^{H}\sum_{j=1}^{m_h}\sum_{i=1}^{n_{hj}}\mathbf{u}_{hji}$, where the triple suffix refers to elements within PSUs within strata, $m_h$ is the sample number of PSUs in stratum $h$, $n_{hj}$ is the sample number of elements in PSU $j$ in stratum $h$, and $\mathbf{u}_{hji}$ is the $k \times 1$ vector for element $i$ in PSU $j$ in stratum $h$. An estimator for the covariance matrix of $n^{-1}\sum_{h=1}^{H}\sum_{j=1}^{m_h}\sum_{i=1}^{n_{hj}}\mathbf{u}_{hji}$ under this sampling scheme, assuming the $\mathbf{u}_{hji}$ are observed and ignoring finite population corrections, is given by (Shah $et\ al.$, 1995)

$$\mathrm{v}_L\left[n^{-1}\sum_{h=1}^{H}\sum_{j=1}^{m_h}\sum_{i=1}^{n_{hj}}\mathbf{u}_{hji}\right]_{v,l} = n^{-2}\sum_{h=1}^{H}m_h\left\{\left[\sum_{j=1}^{m_h}\left(\mathbf{u}_{hj+,v}-\overline{\mathbf{u}}_{h,v}\right)'\left(\mathbf{u}_{hj+,l}-\overline{\mathbf{u}}_{h,l}\right)\right]\middle/(m_h-1)\right\},$$

(19)

where $\mathbf{u}_{hj+} = \sum_{i=1}^{n_{hj}}\mathbf{u}_{hji}$, $\overline{\mathbf{u}}_h = m_h^{-1}\sum_{j=1}^{m_h}\mathbf{u}_{hj+}$ and the subscripts $v$ and $l$ denote respectively $v = (t,t')$ and $l = (t'',t''')$. Finally, to obtain a linearization estimator $\mathrm{v}_L\{vech(S_w)\}$ of $\mathrm{var}\{vech[S_w]\}$, the values $\mathbf{u}_{hji}$ in (19) need to be replaced by values $\hat{\mathbf{u}}_{hji}$, defined in the same way that $\hat{\mathbf{u}}_i$ was defined above in terms of $\mathbf{u}_i$. The asymptotic validity of this variance estimator depends on each $m_h$ being large if $H$ is regarded as fixed.

In the special case when the population consists of only one stratum and each individual $i$ is a PSU, we rewrite (19) as

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$$\mathrm{v}_L\left[n^{-1}\sum_{i=1}^{n}\mathbf{u}_i\right]_{v,l}=\left[\sum_{i=1}^{n}\left(\mathbf{u}_{i,v}-\overline{\mathbf{u}}_v\right)'\left(\mathbf{u}_{i,l}-\overline{\mathbf{u}}_l\right)\right]\Big/[n(n-1)]$$

where $\overline{\mathbf{u}}=n^{-1}\sum_{i=1}^{n}\mathbf{u}_i$. When $\mathbf{u}_i$ is replaced by $\hat{\mathbf{u}}_i$, we find $\overline{\mathbf{u}}$ reduces to zero and the linearization estimator of $\mathrm{var}\{vech[S_w]\}$ is:

$$\mathrm{v}_L\{vech(S_w)\}=\frac{n}{(n-1)\left(\sum\limits_{i=1}^{n}w_i\right)^2}\sum_{i=1}^{n}w_i^2\left(\hat{\varepsilon}_{it}\hat{\varepsilon}_{it'}-S_{w\,tt'}\right)\left(\hat{\varepsilon}_{it''}\hat{\varepsilon}_{it'''}-S_{w\,t''t'''}\right),\qquad(20)$$

corresponding to the estimator proposed by Browne (1984) when the sampling weights are constant.

Replacing $U$ by $\mathrm{v}_L\{vech(S_w)\}$ in (8) gives a fitting function and a point estimator which we denote $F_{GLS-L}(S,\Sigma)$ and $\hat{\mathbf{\theta}}_{GLS-L}$ respectively. In the classical setting of independent and identically distributed observations the latter estimator is usually referred to as the ADF estimator. The estimator may allow for the complex design both through weighting in $S_w$ and through the choice of linearization variance estimator $\mathrm{v}_L\{vech(S_w)\}$.

We now turn to the estimation of the variance of GLS estimators of $\mathbf{\theta}$. Assuming (16) and using linearization again (Skinner and Holmes, 2003), the asymptotic variance of the GLS estimator based upon the fitting function in (8) with a specified matrix $U$ is:

$$\mathrm{var}(\hat{\mathbf{\theta}})=n^{-1}(\Delta'U^{-1}\Delta)^{-1}\Delta'U^{-1}CU^{-1}\Delta(\Delta'U^{-1}\Delta)^{-1},\qquad(21)$$

where $\Delta=\dfrac{\partial\{vech[\Sigma(\mathbf{\theta})]\}}{\partial\mathbf{\theta}}$.

The linearization estimator of this variance is then obtained by replacing $\Delta$ in (21) by $\hat{\Delta}$, defined as $\Delta$ evaluated at $\mathbf{\theta}=\hat{\mathbf{\theta}}$, and by replacing $n^{-1}C$ by a variance estimator $\mathrm{v}_L\{vech(S_w)\}$ as discussed above. When there are no covariates, this approach corresponds to estimation methods proposed by Skinner (1989), Satorra (1992), Müthén and Satorra (1995) and Skinner and Holmes (2003).

If U is chosen to be consistent for $n^{-1}C$, expression (21) reduces in the limit to:

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$$\mathrm{var}(\hat{\boldsymbol{\theta}}) = n^{-1}(\Delta' U^{-1}\Delta)^{-1}.\tag{22}$$

Let us now consider estimation of the asymptotic covariance matrix of the PML point estimator

$\hat{\boldsymbol{\theta}}_{PML}$. Following Binder (1983), we may write this asymptotic covariance matrix as:

$$\mathrm{var}(\hat{\boldsymbol{\theta}}_{PML}) = [I(\boldsymbol{\theta})]^{-1}\,\mathrm{var}[\boldsymbol{\varphi}(\boldsymbol{\theta})][I(\boldsymbol{\theta})]^{-1},\tag{23}$$

where $\boldsymbol{\varphi}(\boldsymbol{\theta})$ is the $b\times 1$ pseudo-score function with j$^{\text{th}}$ element given by:

$$\phi_j(\boldsymbol{\theta}) = \frac{\partial F_{PML}(\boldsymbol{\theta})}{\partial \theta_j} = tr\left\{\Sigma(\boldsymbol{\theta})^{-1}[\Sigma(\boldsymbol{\theta}) - S_w]\Sigma(\boldsymbol{\theta})^{-1}\frac{\partial \Sigma(\boldsymbol{\theta})}{\partial \theta_j}\right\},\tag{24}$$

using (14), and $I(\boldsymbol{\theta})$ is the $b\times b$ pseudo information matrix $I(\boldsymbol{\theta}) = -\partial\boldsymbol{\varphi}(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$. To estimate the

asymptotic covariance matrix of $\hat{\boldsymbol{\theta}}_{PML}$ it is therefore necessary to estimate the covariance matrix of

$\boldsymbol{\varphi}(\boldsymbol{\theta})$. We may write:

$$\varphi_j(\boldsymbol{\theta}) = tr\left[\Sigma(\boldsymbol{\theta})^{-1}\frac{\partial \Sigma(\boldsymbol{\theta})}{\partial \theta_j}\right] + \frac{\sum\limits_{i=1}^{n} w_i z_{ij}}{\sum\limits_{i=1}^{n} w_i},\tag{25}$$

where $z_{ji} = -\boldsymbol{\varepsilon}_i'\Sigma(\boldsymbol{\theta})^{-1}\frac{\partial \Sigma(\boldsymbol{\theta})}{\partial \theta_j}\Sigma(\boldsymbol{\theta})^{-1}\boldsymbol{\varepsilon}_i$.\qquad(26)

Linearizing the ratio in (25) gives:

$$\phi_j(\boldsymbol{\theta}) = tr\left[\Sigma(\boldsymbol{\theta})^{-1}\frac{\partial \Sigma(\boldsymbol{\theta})}{\partial \theta_j}\right] + \frac{\mu_{aj}}{\mu_w} + \frac{1}{n}\sum_{i=1}^{n}\left(a_{ji} - \frac{\mu_{aj}}{\mu_w}\right)\frac{1}{\mu_w}$$

where $a_{ji} = w_i z_{ji}$, $\mu_{aj} = E(\bar{a}_j)$ and $\bar{a}_j = n^{-1}\sum\limits_{i=1}^{n} a_{ji}$.

The covariance matrix of $\boldsymbol{\varphi}(\boldsymbol{\theta})$ may thus be approximated by

$$\mathrm{var}\{\boldsymbol{\varphi}(\boldsymbol{\theta})\} \doteq \mathrm{var}\left(n^{-1}\sum_{i=1}^{n}\mathbf{u}_i\right),$$

where $\mathbf{u}_i$ is the $b\times 1$ vector with j$^{\text{th}}$ element given by:

$$\frac{1}{\mu_w} \cdot \left( a_{ji} - \frac{\mu_{aj}}{\mu_w} \right). \tag{27}$$

This covariance matrix may be estimated for a complex design as above, for example using (19), where $\mathbf{u}_i$ is, as above, replaced by $\hat{\mathbf{u}}_i$, which is obtained by replacing $\boldsymbol{\theta}$ by $\hat{\boldsymbol{\theta}}$ and $\boldsymbol{\varepsilon}_i$ by $\hat{\boldsymbol{\varepsilon}}_i$ in (26) to give $\hat{z}_{ij}$, setting $\hat{a}_{ji} = w_i \hat{z}_{ji}$ and replacing $a_{ji}$, $\mu_{aj}$ and $\mu_w$ in (27) by $\hat{a}_{ji}$, $n^{-1} \sum_{i=1}^{n} \hat{a}_{ji}$ and $\bar{w}$ respectively. The linearization estimator of the variance of $\hat{\boldsymbol{\theta}}_{PML}$ is then obtained from (23) by replacing $\text{var}[\boldsymbol{\varphi}(\boldsymbol{\theta})]$ by this estimator and by replacing $\boldsymbol{\theta}$ by $\hat{\boldsymbol{\theta}}$ in $I(\boldsymbol{\theta})$.

Notice that the evaluation of the information matrix $I(\boldsymbol{\theta})$ requires differentiating $F_{PML}(\boldsymbol{\theta})$ and hence $\Sigma(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ twice. Some simplification is achieved by assuming that the model is correct, i.e. that $E[S_w] = \Sigma(\boldsymbol{\theta})$. If we then replace the information matrix in (23) by

$$\tilde{I}(\boldsymbol{\theta}) = E\left[ -\frac{\partial \boldsymbol{\varphi}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right],$$

which is asymptotically equivalent, we find from (24) that the jk$^{\text{th}}$ element of $\tilde{I}(\boldsymbol{\theta})$ may be expressed as:

$$\tilde{I}(\boldsymbol{\theta})_{jk} = tr\left[ \Sigma(\boldsymbol{\theta})^{-1} \frac{\partial \Sigma(\boldsymbol{\theta})}{\partial \theta_j} \Sigma(\boldsymbol{\theta})^{-1} \frac{\partial \Sigma(\boldsymbol{\theta})}{\partial \theta_k} \right],$$

and we only need to differentiate $\Sigma(\boldsymbol{\theta})$ once.

## 6. Simulation with BHPS data

In this section we shall assess the properties of the point and variance estimation procedures of sections 3 and 4 using a simulation study. In order to consider realistic values for simulation parameters, e.g. $\boldsymbol{\beta}$, $\sigma_u^2$, $\sigma_v^2$, and $\sigma_\eta^2$, we shall adopt regression analysis of the form discussed in section 2, based upon a model considered by Berrington (2002), with individual women as units of primary analytic interest and a measure of attitude to gender roles as the outcome variable, $y$.

The data come from waves 1, 3, 5, 7 and 9 (collected biannually between 1991 and 1999) of the British Household Panel Survey (BHPS) and these waves will be coded $t = 1, ..., T = 5$ respectively. Respondents were asked whether they 'strongly agreed', 'agreed', 'neither agreed nor disagreed', 'disagreed' or 'strongly disagreed' with a series of statements concerning the family, women's roles, and work out of the household. Responses were scored from 1 to 5. Factor analysis was used to assess which statements could be combined into a gender role attitude measure. The attitude score, $y_{it}$, considered here is the total score for six selected statements for woman $i$ at wave $t$. Higher scores signify more egalitarian gender role attitudes. Covariates for the regression analysis were selected on the basis of discussion in Berrington (2002) and include economic activity, which distinguishes in particular between women who are at home looking after children (denoted 'family care') and women following other forms of activity in relation to the labour market. Variables reflecting age and education are also included since these have often been found to be strongly related to gender role attitudes (e.g. Fan and Marini, 2000). All these covariates may change values between waves. A year variable (scored 1, 3, …, 9) is also included. This may reflect both historical change and the general ageing of the women in the sample.

The BHPS is a household panel survey of individuals in private domiciles in Great Britain (Taylor *et al.*, 2001). Given the interest in whether women's primary labour market activity is 'caring for a family', we define our study population as women aged 16-39 in 1991. This results in a subset of data on *n* = 1340 women. This subset consists of the longitudinal sample of women in the eligible age range for whom full interview outcomes were obtained in all five waves.

The simulation study involves simulating *D* replicate samples. Each replicate is thus based upon that BHPS subset, and drawn according to a specified sampling scheme, where the values $\mathbf{x}_{it}$ are held fixed at their values in the underlying dataset, but where the values $y_{it}$ are simulated from specified models, independently for each replicate. The models considered initially are the UCM model and the UCM(C) model from Section 2, with parameters set at the values obtained from

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fitting these models to the BHPS subset and errors following either the normal distribution or a t distribution.

### 6.1 Point estimators

We first suppose the replicate samples are obtained by srs without replacement with size $n^{sim}$, which are 1340, 500, 200 and 100. For simplicity, we shall not attempt to allow for the impact of either stratification or unequal probability sampling. Clustering is thus the only complex sampling feature considered here via the UCM(C) model. In this subsection, we aim to present results based on $D = 1000$ replicates.

Five point estimators were considered: ULS, GLS-NORM1, GLS-NORM2 and PML, defined in (7), (12), (13) and (15) respectively, and GLS-L, defined by (8) with U given by the estimator in (20). It was in fact found that the ULS and PML estimation methods produced virtually identical results for the UCM model and similar results for other models, a finding corresponding to that of Bollen (1989, p. 112). We therefore do not present the ULS results and focus instead on the remaining four estimators, assessing their properties in terms of relative bias and coefficient of variation ($cv$), estimated from the replications of the simulation study.

Table 1 presents results produced when the UCM model with normal errors is used both to generate the $y_{it}$ values and as a basis for model fitting. The parameter vector $\boldsymbol{\theta} = \left( \sigma_u^2, \sigma_v^2 \right)'$ contains two parameters of interest. In this case, we might expect the estimators $\hat{\boldsymbol{\theta}}_{GLS-NORM1}$, $\hat{\boldsymbol{\theta}}_{GLS-NORM2}$ and $\hat{\boldsymbol{\theta}}_{PML}$ which exploit the normality to outperform the estimator $\hat{\boldsymbol{\theta}}_{GLS-L}$ which does not. In fact we observe little difference between the performance of this estimator and that of $\hat{\boldsymbol{\theta}}_{GLS-NORM1}$. We do observe that $\hat{\boldsymbol{\theta}}_{GLS-NORM2}$ performs consistently better than $\hat{\boldsymbol{\theta}}_{GLS-NORM1}$ (if only slightly) with respect to relative bias and possibly with respect to coefficient of variation. The estimator $\hat{\boldsymbol{\theta}}_{PML}$ has a similar performance to $\hat{\boldsymbol{\theta}}_{GLS-NORM2}$ with respect to coefficient of variation and displays different patterns of relative bias.

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We repeated the simulation in Table 1 using the AR1 model and found similar results, which are not reported here.

In terms of the asymptotic equivalence between GLS2 and PML methods, we observe that there is not a large difference between the mean square error (mse) results when comparing these two methods, in a situation with sample size 1340. That, of course, is less clear for simulations with smaller samples sizes.

We next consider the impact of clustering, with the data now generated from the UCM-C model. The UCM model continues to be the fitted model. We considered both normal and t-distributed errors and present the results for t-distributed errors in Table 2. We expect the main difference between Table 2 and Table 1 to be an increase in $cv$ from the clustering, but we also notice a modest increase in relative bias. We again find that $\hat{\theta}_{GLS-NORM2}$ performs consistently better than $\hat{\theta}_{GLS-NORM1}$ with respect to relative bias, but this is now not necessarily the case with respect to $cv$. As the sample sizes increase, we note that again $\hat{\theta}_{GLS-NORM2}$ and $\hat{\theta}_{PML}$ appear to be the preferred methods with respect to relative bias. There does not appear to be a great difference between all four methods with respect to $cv$. Simulation results produced for AR1 model fitting in the current situation, which are not presented again, generally agreed with results presented in Table 2.

We focus on the impact of clustering in Table 3, where the inflation of mean squared error (MSE) arising from the incorporation of cluster effects in the data generation process is considered, in the case when $n^{sim} = 100$ and the errors are t distributed. There are no major differences between the estimation methods in terms of the MSE inflation, although the impact appears to be least for the GLS-L method.

Overall, these simulation results produced for the ADF method GLS-L generally agree with Bollen (1989, p. 432), Satorra (1992), Yuan and Bentler (1997), and Olsson, Foss, and Troye (2003), where it is recommended that those methods should be adopted only in situations with large sample sizes (1000 or more), for dealing with situations where departures from normality conditions are evident. We may emphasize that ADF methods have in several situations had good general

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performance, even though these methods have not shown 'good' levels of bias. PML point estimators have in general produced very good performance in terms of bias and variance, even in situations where the normality assumption was violated, as reported for example by Satorra and Bentler (1986).

## 6.2 Variance estimators

We now consider the properties of the linearization variance estimators denoted $v_L$ in section 4. We restrict attention to their use in the estimation of the variance of the two point estimators: $\hat{\boldsymbol{\theta}}_{GLS-NORM1}$ and $\hat{\boldsymbol{\theta}}_{PML}$. To provide benchmarks for comparison, we also consider the variance estimator, $\text{var}_n(.)$, which is based upon the assumption of both normality and independent and identically distributed observations, and the estimator $\text{var}_{df}(.)$ which allows for non-normality but still assumes independent and identically distributed observations . The subscript $n$ denotes naïve. In the case of $\hat{\boldsymbol{\theta}}_{GLS-NORM1}$, $\text{var}_n(.)$ and $\text{var}_{df}(.)$ are obtained from (22) and (21) respectively, with $U$ given by (10) and $W = S_w$. In the case of $\hat{\boldsymbol{\theta}}_{PML}$, $\text{var}_n(.)$ is given by $\left[I(\boldsymbol{\theta})\right]^{-1}$.

To evaluate the properties of these variance estimators, we drew a new set of replicate samples in which a two-stage sampling scheme was used, with simple random sampling with replacement at each stage. The 1340 elements were divided into 47 PSUs. The number of sampled PSUs, $m^{sim}$, was varied from $m^{sim} = 47$ to $m^{sim} = 20$ and $m^{sim} = 15$. The number of selected secondary sampling units (SSUs) in the $j^{\text{th}}$ selected PSU is denoted $n_j^{sim}$.

The UCM-C model was used to generate the values of $y_{ijt}$ now using $D = 10,000$ replicates. The parameters of the UCM-C model were the same as in the simulations in section 5.1. , except that there were some different choices for $\sigma_\eta^2$: $\sigma_\eta^{2\ sim,C} \cong 0.15$, $\sigma_\eta^{2\ sim,C} \cong 0.45$, and $\sigma_\eta^{2\ sim,C} \cong 0.75$; to enable the evaluation of effects of different impacts of clustering on the variance estimation procedures. The fitted model was taken as the UCM model.

Table 4 displays results produced when considering $m^{sim} = 47$ and $n_j^{sim} = 15$. The first three variance estimators do not take the clustering into account and, as anticipated, clearly underestimate the variance. The degree of underestimation increases with $\sigma_\eta^2$, i.e. the more clustering the more downward relative bias.

Both methods that allow for clustering have improved properties in terms of relative bias, compared to the first three methods. They are still biased downwards, however, corresponding to other findings for linearization variance estimation (Wolter, 1985, Chapter 8; Kott, 1991). Furthermore, these two methods had larger variances than the first three methods, as expected (Kott, 1991; Korn and Graubard, 1995), as a result of the reduced degrees of freedom for variance estimation. Moreover, the *cv*s for both $v_L\left(\hat{\boldsymbol{\theta}}_{PML}\right)$ and $v_L\left(\hat{\boldsymbol{\theta}}_{GLS-NORM1}\right)$ appear to have a slight tendency of increasing with larger impacts of clustering. This pattern however is not observed for the first three methods, which seem to have variances which do not vary greatly with $\sigma_\eta^2$.

Table 5 includes results that were produced when considering $m^{sim} = 20$ and $n_j^{sim} = 15$, i.e. 300 cases. Under this situation, the linearization variance estimators which allow for the complex sampling again led to noticeable improvements in terms of relative bias when compared to methods that ignored the sampling scheme. The smaller number of sample clusters does, however, seem to have led to some increases in relative bias, although these are still smaller than the *cv*s. Neither the relative bias nor the *cv* were found to vary greatly with $\sigma_\eta^2$.

Table 6 includes results that were produced when $m^{sim} = 15$ and $n_j^{sim} = 10$, i.e the number of SSUs selected per cluster was further reduced, and the sample size was diminished to 150. further increases in relative bias were observed although again the relative biases were smaller than the *cv*s. As in Table 5 there was no strong relationship between either the relative bias or the *cv* with $\sigma_\eta^2$.

In summary, the linearization method which allows for clustering appears to perform reasonably well for both point estimators considered here for a range of possible clustering effects, although

there is a tendency for the variance to be underestimated if the number of sampled clusters is small, say twenty or below.

## 7. Conclusion

This paper has proposed some methods of making inference about parameters in panel data models, allowing for complex sampling schemes. Methods have been evaluated using a simulation study based upon data from the BHPS. The study indicated that: (*i*) overall most of the proposed methods perform satisfactorily under clustered designs; (*ii*) ADF methods do not always perform as we expected, although often these had the smallest variance, were generally less sensitive to clustering, and had the best performance for stronger departures from normality; and (*iii*) ML and PML estimators produced satisfactory performance in terms of bias and variance, even when the normality assumption was violated.

Methods for variance estimation for GLS and PML point estimators were considered. Under the complex survey data approach, we extend ADF variance estimation methodology developed by Skinner (1989). In addition, we proposed a method for estimating the asymptotic covariance matrix of the PML. Results of a second simulation study suggested that: (*iv*) methods that do not take the sampling scheme into account underestimate the variance, in some situations very gravely; (*v*) underestimation tend to increase rapidly with inflations in the impacts of clustering; (*vi*) ADF methods that allow for clustering and take the sampling design into account tend to lead to noticeable improvements in terms of relative bias when compared to methods that ignore the sampling scheme characteristics, in situations where the sample size is over around 200 cases; and (*vii*) the variance estimator we propose for estimating the variance of the maximum likelihood point estimator has an evidently better performance in terms of bias than those proposed estimating the variance for the GLS estimators.

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| Estimator | | $n = 100$ | | $n = 200$ | | $n = 500$ | | $n = 1340$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | rel bias | $cv$ | rel bias | $cv$ | rel bias | $cv$ | rel bias | $cv$ |
| $\hat{\theta}_{GLS-NORM1}$ | $\hat{\sigma}_u^2$ | -16.76% | 17.77% | -9.21% | 12.14% | -3.40% | 7.16% | -1.42% | 4.29% |
| | $\hat{\sigma}_v^2$ | -9.70% | 8.41% | -4.68% | 5.56% | -1.74% | 3.39% | -0.74% | 1.90% |
| $\hat{\theta}_{GLS-NORM2}$ | $\hat{\sigma}_u^2$ | -6.43% | 17.69% | -3.77% | 11.77% | -1.18% | 7.12% | -0.60% | 4.27% |
| | $\hat{\sigma}_v^2$ | 6.41% | 7.19% | 3.51% | 5.20% | 1.59% | 3.27% | 0.47% | 1.88% |
| $\hat{\theta}_{GLS-L}$ | $\hat{\sigma}_u^2$ | -15.79% | 19.44% | -9.23% | 12.76% | -3.41% | 7.19% | -1.46% | 4.33% |
| | $\hat{\sigma}_v^2$ | -9.89% | 9.04% | -4.60% | 5.83% | -1.72% | 3.44% | -0.74% | 1.93% |
| $\hat{\theta}_{PML}$ | $\hat{\sigma}_u^2$ | -9.94% | 17.18% | -5.61% | 11.68% | -1.92% | 7.08% | -0.88% | 4.26% |
| | $\hat{\sigma}_v^2$ | 0.89% | 6.84% | 0.74% | 5.09% | 0.47% | 3.25% | 0.06% | 1.87% |

Table 1 – Properties of point estimators when both fitted model and true model are UCM.

| Estimator | | $n = 100$ | | $n = 200$ | | $n = 500$ | | $n = 1340$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | rel bias | $cv$ | rel bias | $cv$ | rel bias | $cv$ | rel bias | $cv$ |
| $\hat{\theta}_{GLS-NORM1}$ | $\hat{\sigma}_u^2$ | -16.73% | 29.27% | -8.75% | 22.07% | -4.05% | 12.10% | -1.63% | 7.54% |
| | $\hat{\sigma}_v^2$ | -12.30% | 10.98% | -7.13% | 8.08% | -2.65% | 5.23% | -1.02% | 3.28% |
| $\hat{\theta}_{GLS-NORM2}$ | $\hat{\sigma}_u^2$ | -7.11% | 29.26% | -3.32% | 22.28% | -1.78% | 12.17% | -0.76% | 7.53% |
| | $\hat{\sigma}_v^2$ | 9.45% | 14.00% | 4.83% | 9.92% | 2.18% | 6.08% | 0.92% | 3.66% |
| $\hat{\theta}_{GLS-L}$ | $\hat{\sigma}_u^2$ | -21.82% | 29.11% | -13.00% | 18.55% | -6.16% | 11.72% | -2.56% | 7.44% |
| | $\hat{\sigma}_v^2$ | -17.18% | 11.74% | -11.54% | 8.23% | -5.58% | 5.16% | -2.75% | 3.21% |
| $\hat{\theta}_{PML}$ | $\hat{\sigma}_u^2$ | -10.33% | 28.91% | -5.16% | 22.00% | -2.54% | 12.10% | -1.05% | 7.53% |
| | $\hat{\sigma}_v^2$ | 1.56% | 10.84% | 0.62% | 8.62% | 0.51% | 5.55% | 0.26% | 3.47% |

Table 2 – Properties of point estimators when fitted model is UCM and true model is UCM-C with t distributed errors

| Estimator | | UCM model | AR1 model |
|---|---|---|---|
| $\hat{\theta}_{ULS}$ | $\hat{\sigma}_u^2$ | 1.44 | 1.46 |
| | $\hat{\sigma}_v^2$ | 0.89 | 0.93 |
| | $\hat{\gamma}$ | - | 1.01 |
| $\hat{\theta}_{GLS-NORM1}$ | $\hat{\sigma}_u^2$ | 1.27 | 1.27 |
| | $\hat{\sigma}_v^2$ | 0.93 | 0.92 |
| | $\hat{\gamma}$ | - | 1.01 |
| $\hat{\theta}_{GLS-NORM2}$ | $\hat{\sigma}_u^2$ | 1.52 | 1.53 |
| | $\hat{\sigma}_v^2$ | 0.95 | 1.06 |
| | $\hat{\gamma}$ | - | 1.10 |
| $\hat{\theta}_{GLS-L}$ | $\hat{\sigma}_u^2$ | 1.22 | 1.23 |
| | $\hat{\sigma}_v^2$ | 0.86 | 0.89 |
| | $\hat{\gamma}$ | - | 0.82 |
| $\hat{\theta}_{PML}$ | $\hat{\sigma}_u^2$ | 1.44 | 1.45 |
| | $\hat{\sigma}_v^2$ | 0.89 | 0.99 |
| | $\hat{\gamma}$ | - | 1.04 |

Table 3 – Ratios of MSEs of estimators with data generated from UCM-C model (numerator) and from UCM model (denominator) (n=100 and t-distributed errors).

| Variance Estimator | | rel bias | | | $cv\left(\text{var}\left(\hat{\theta}\right)\right)$ | | |
|---|---|---|---|---|---|---|---|
| | | $\sigma_\eta^2 = 0.15$ | $\sigma_\eta^2 = 0.45$ | $\sigma_\eta^2 = 0.75$ | $\sigma_\eta^2 = 0.15$ | $\sigma_\eta^2 = 0.45$ | $\sigma_\eta^2 = 0.75$ |
| $\text{var}_n\left(\hat{\theta}_{PML}\right)$ | $\text{var}(\hat{\sigma}_u^2)$ | -0.39% | -7.75% | -11.43% | 14.07% | 14.27% | 14.54% |
| | $\text{var}(\hat{\sigma}_v^2)$ | 1.78% | -2.44% | -0.30% | 8.54% | 8.54% | 8.59% |
| $\text{var}_n\left(\hat{\theta}_{GLS-NORM1}\right)$ | $\text{var}(\hat{\sigma}_u^2)$ | -1.54% | -8.96% | -12.47% | 10.71% | 11.14% | 11.37% |
| | $\text{var}(\hat{\sigma}_v^2)$ | -5.18% | -10.25% | -7.14% | 5.39% | 5.54% | 5.47% |
| $\text{var}_{df}\left(\hat{\theta}_{GLS-NORM1}\right)$ | $\text{var}(\hat{\sigma}_u^2)$ | -1.51% | -9.07% | -12.60% | 14.13% | 14.34% | 14.61% |
| | $\text{var}(\hat{\sigma}_v^2)$ | -4.14% | -9.20% | -6.01% | 8.62% | 8.70% | 8.69% |
| $v_L\left(\hat{\theta}_{PML}\right)$ | $\text{var}(\hat{\sigma}_u^2)$ | 0.27% | -4.58% | -3.55% | 24.65% | 25.41% | 26.85% |
| | $\text{var}(\hat{\sigma}_v^2)$ | 2.53% | -2.35% | 0.99% | 22.01% | 21.86% | 21.98% |
| $v_L\left(\hat{\theta}_{GLS-NORM1}\right)$ | $\text{var}(\hat{\sigma}_u^2)$ | -0.85% | -6.02% | -4.91% | 24.78% | 25.51% | 27.00% |
| | $\text{var}(\hat{\sigma}_v^2)$ | -3.48% | -9.13% | -4.80% | 22.33% | 22.24% | 22.43% |

Table 4 – Properties of variance estimators, when UCM is fitted model, UCM-C is true model, $m^{sim} = 47$ and $n_j^{sim} = 15$

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| Variance Estimator | | rel bias | | | $cv\left(\mathrm{var}\left(\hat{\theta}\right)\right)$ | | |
|---|---|---|---|---|---|---|---|
| | | $\sigma_\eta^2 = 0.15$ | $\sigma_\eta^2 = 0.45$ | $\sigma_\eta^2 = 0.75$ | $\sigma_\eta^2 = 0.15$ | $\sigma_\eta^2 = 0.45$ | $\sigma_\eta^2 = 0.75$ |
| $v_L\left(\hat{\theta}_{PML}\right)$ | $\mathrm{var}(\hat{\sigma}_u^2)$ | -5.17% | -5.25% | -4.69% | 38.07% | 39.03% | 40.75% |
| | $\mathrm{var}(\hat{\sigma}_v^2)$ | -1.54% | -0.69% | -0.49% | 33.55% | 33.79% | 34.44% |
| $v_L\left(\hat{\theta}_{GLS-NORM1}\right)$ | $\mathrm{var}(\hat{\sigma}_u^2)$ | -7.31% | -7.60% | -6.55% | 38.42% | 39.17% | 40.83% |
| | $\mathrm{var}(\hat{\sigma}_v^2)$ | -14.17% | -12.87% | -12.23% | 34.26% | 34.39% | 35.00% |

Table 5 – Properties of variance estimators, when UCM is fitted model, UCM-C is true model, $m^{sim} = 20$ and $n_j^{sim} = 15$.

| Variance Estimator | | rel bias | | | $cv\left(\mathrm{var}\left(\hat{\theta}\right)\right)$ | | |
|---|---|---|---|---|---|---|---|
| | | $\sigma_\eta^2 = 0.15$ | $\sigma_\eta^2 = 0.45$ | $\sigma_\eta^2 = 0.75$ | $\sigma_\eta^2 = 0.15$ | $\sigma_\eta^2 = 0.45$ | $\sigma_\eta^2 = 0.75$ |
| $v_L\left(\hat{\theta}_{PML}\right)$ | $\mathrm{var}(\hat{\sigma}_u^2)$ | -5.48% | -6.11% | -4.87% | 47.86% | 47.80% | 50.19% |
| | $\mathrm{var}(\hat{\sigma}_v^2)$ | -3.41% | -2.68%NS | -1.38%NS | 41.05% | 40.43% | 40.87% |
| $v_L\left(\hat{\theta}_{GLS-NORM1}\right)$ | $\mathrm{var}(\hat{\sigma}_u^2)$ | -9.26% | -9.63% | -8.64% | 48.57% | 48.09% | 50.85% |
| | $\mathrm{var}(\hat{\sigma}_v^2)$ | -23.34% | -24.21% | -21.92% | 42.07% | 41.22% | 41.86% |

Table 6 – Properties of variance estimators, when UCM is fitted model, UCM-C is true model, $m^{sim} = 15$ and $n_j^{sim} = 10$.