The Problem of Numeracy

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Lack of numeracy is due mainly to the way data are presented. Most tables of data can be improved by following a few simple rules, such as drastic rounding, ordering the rows of a table by size, and giving a brief verbal summary of the data.

KEY WORDS: Numeracy; Rounding; Ordering by size; Table layout; Short-term memory.

1. INTRODUCTION

People often feel inept when faced with numerical data. Many of us think that we lack numeracy, the ability to cope with numbers. The message of this article is that we are not to blame: The fault is not in ourselves, but in our data. Most data are badly presented and so the cure lies with the producers of the data.

To draw an analogy with literacy, we do not need to learn to read better, but writers need to be taught to write better. Luckily, numerical data have inherent structure. This makes numbers easier to communicate than ideas or verbal arguments. These few simple rules or guidelines can work wonders in communicating a table of numbers.

1. Giving marginal averages to provide a visual focus;
2. Ordering the rows or columns of the table by the marginal averages or some other measure of size (keeping to the same order if there are many similar tables);
3. Putting figures to be compared into columns rather than rows (with larger numbers on top if possible);
4. Rounding to two effective digits;
5. Using layout to guide the eye and facilitate comparisons; and
6. Giving brief verbal summaries to lead the reader to the main patterns and exceptions.

Using these rules generally produces tables that are easier to read. A fuller discussion has been given elsewhere (Ehrenberg 1977, 1978a), including a review of the literature. Here I would like to do three things: use two examples to illustrate the rules and that they work; suggest why our mental processes require such rules; and consider problems of implementation.

2. TWO EXAMPLES

I start in Figure 1 with some sales statistics for eight cities in the United Kingdom. At first glance the table in Figure 1 may seem reasonably well laid out. But our attention has probably centered only on the captions—Product X; Bolton, Edinburgh, and Hull; Quarters 1 and 2; and so on. The numbers themselves are not as easy to take in. What are their main features? How can they be summarized? How can we tell someone over the phone?

Looked at with these questions in mind, the table now seems more of a jumble. It looks as if whoever produced it either did not know what the data were saying, or was not letting on. The main difficulty is that the cities are listed alphabetically, as in a directory. There is no apparent pattern in each column.

Figure 2 therefore orders the cities by the size of their adult populations, which helps dramatically. It also uses rounding off, marginal averages, and more compact layout.

Now we can see a major pattern: the bigger the cities, the higher the sales! Exceptions are also clear, like Leeds being relatively high and Luton relatively low (averages of 270 and 25).

Trends over time are also easier to take in. Although not typical, the column averages help us see that sales in each city were mostly steady quarter by quarter, but low in QIII and high in QIV. We can also see that the QIV increases were largest in Leeds and Edinburgh.

These patterns and subpatterns are easy to see in Figure 2, especially once they have been pointed out. But in Figure 1 they are still not very apparent. This illustrates the basic criterion of a good table: In a good table, the patterns and exceptions should be obvious at a glance, at least once one knows what they are. Faced with Figure 1 we were all nonnumerate. Faced with Figure 2 we can all more or less cope.

For a second illustration I turn to data where more advanced statistical analysis is often thought to be needed (methods like factor or component analysis, cluster analysis, etc.).

Figure 3 gives a 10 × 10 correlation matrix. The variables concern television viewing. A sample of 7,000 U.K. adults were asked whether they really liked to watch a selection of 10 sports and public affairs programs (derived from Goodhardt, Ehrenberg, and Collins 1975, Ch. 8).

The patterns and exceptions are not clear. But appropriate reordering of the variables, drastic rounding (here to a single digit), plus better labeling and spac-

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Table 1 Quarterly Sales of Product X in 8 Cities

£'000	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Bolton	31.3	29.1	25.2	29.3
Edinburgh	135.1	126.9	132.1	208.3
Hull	70.3	81.3	70.9	84.0
Leeds	276.8	258.6	223.0	336.2
Luton	23.5	27.5	22.7	27.1
Plymouth	41.4	44.0	33.2	50.2
Sheffield	233.4	220.1	193.6	220.9
Swansea	62.3	66.4	61.8	76.7

Figure 1

ing lead to a dramatic improvement, as shown by Figure 4. Reading down each column (which is easier than reading across rows), we see that there is a cluster for the five sports programs (World of Sport to Rugby Special) and another for the five public affairs programs (24 Hours to Line-Up). We also can see subpatterns, like the gradients within each cluster, and exceptions, like the three locally high correlations of .2 between Panorama and some of the sports programs.

I am not concerned here with how the ordering of the variables was achieved—whether by prior knowledge, advanced statistical procedures, insight, or whatever. What concerns me is our ability to see, understand, and communicate such a pattern once it has been established. Even now that we know the pattern, it still is not clear in Figure 3.

Table 2 The Cities Ordered by Population Size

(Rounded and with Averages)

Sales in £'000	QI	QII	QIII	QIV	Av.
Sheffield	230	220	190	220	220
Leeds	280	260	220	340	270
Edinburgh	140	130	130	210	150
Hull	70	81	71	84	76
Swansea	62	66	62	77	67
Plymouth	41	44	33	50	42
Luton	23	27	23	27	25
Bolton	31	29	25	29	29
Average	110	107	94	130	110

Figure 2

3. WHY IT WORKS: SHORT-TERM MEMORY

The two examples have illustrated six guidelines for better data presentation. They seem to work. But why do they work? The answer appears to be that they allow our short-term memories to operate more easily—or at all.

When reading a table of numbers we need to remember some or all of the numbers, at least momentarily. To note that Sheffield's Quarter I sales of 230,000 were almost eight times as high as Bolton's 31,000, we have to keep both numbers in mind briefly. To divide 230 by 31 we have to remember the 230. This is easier with short numbers than with longer ones like 233,400 and 31,300. Short-term memory and immediate recall are required in any mental arithmetic and even for just scanning a set of numbers. The fewer demands we make of our fragile short-term memories, the easier we find the task.

Table 3		Adults who "Really Like to Watch": Correlations to 4 Decimal Places (Programmes Ordered Alphabetically within Channel)									
ITV	PrB	1.0000	0.1064	0.0653	0.5054	0.4741	0.0915	0.4732	0.1681	0.3091	0.1242
"	ThW	0.1064	1.0000	0.2701	0.1424	0.1321	0.1885	0.0815	0.3520	0.0637	0.3946
"	Tod	0.0653	0.2701	1.0000	0.0926	0.0704	0.1546	0.0392	0.2004	0.0512	0.2437
"	WoS	0.5054	0.1474	0.0926	1.0000	0.6217	0.0785	0.5806	0.1867	0.2963	0.1403
BBC	GrS	0.4741	0.1321	0.0704	0.6217	1.0000	0.0849	0.5932	0.1813	0.3412	0.1420
"	LnU	0.0915	0.1885	0.1546	0.0785	0.0849	1.0000	0.0487	0.1973	0.0969	0.2661
"	MoD	0.4732	0.0815	0.0392	0.5806	0.5932	0.0487	1.0000	0.1314	0.3267	0.1221
"	Pan	0.1681	0.3520	0.2004	0.1867	0.1813	0.1973	0.1314	1.0000	0.1469	0.5237
"	RgS	0.3091	0.0637	0.0512	0.2963	0.3412	0.0969	0.3261	0.1469	1.0000	0.1212
"	24H	0.1242	0.3946	0.2432	0.1403	0.1420	0.2661	0.1211	0.5237	0.1212	1.0000

Figure 3

Short-term memory and immediate recall also help in explaining the other rules:

1. We find it difficult to sort out an unordered array of numbers like those we saw in Figure 1 because the task of recalling which figures were high and which low is far too demanding.
2. It is easier to read figures down than across a page, preferably with the larger numbers on top, because we are used to doing arithmetic that way. The figures to be compared are then closer together and the eye can easily ignore later digits, such as

$$\begin{matrix}450\\270\end{matrix} \quad \text{rather than} \quad 450 \quad 270 \quad \text{or even} \quad \begin{matrix}270\\450\end{matrix}\,.$$

3. Averages can be useful even when they are not typical. They provide a common base for comparing each individual reading.
4. An ordinary typed table is less comfortable to the eye than one that has been reduced in size by about 15 to 20 percent linearly. With a smaller table the eye can take in more information without having to move so much. (Compare Figures 1 and 2, for instance, simply for *size*.) Similarly, columns of figures in single spacing with occasional deliberate gaps are easier to read than the popular double-spaced tables. We see more at one time, putting less stress on our memories.
5. A verbal summary helps because it gives us a structure to hold in our longer-term memories while looking at the detailed figures.

Strain on our short-term memory also explains something else, namely, why complex graphs are so difficult to take in: the eye has to move a great deal, calling on us to remember what we have seen. As argued elsewhere (Ehrenberg 1977, 1978a, Ch. 3, and 1978b), graphs can be outstandingly good at showing up large differences or simple *qualitative* features of the data (e.g., whether a relationship is linear or curved). But they tend to be far less effective for communicating quantitative detail. Recent empirical evidence (Wainer, Lono, and Groves 1979) supports this view.

Saying "short-term memory" does not imply that such a function really exists. But having a single theoretical explanation is satisfying, and it also means that we do not have to use the various rules of data presentation blindly. Cookbook rules are difficult to apply in unusual circumstances (e.g., what do we do with a double-yoked egg?). Rather than merely accepting rigid rules like "Order by Size" or "Round to Two Digits," it helps to know that our purpose is to aid mental arithmetic and perception by easing the memory task.

Short-term memory and immediate recall also explain more particularly the importance of rounding to two digits. The precise rule here is to round to two effective digits. This means digits that vary in the specific data. In most cases this means rounding to two digits other than zeros. (Where the numbers vary greatly, *variable* rounding can be used. For example, in Figure 2 figures in the hundreds were rounded to the nearest 10, and figures in the 10's were rounded to the nearest whole number. An extra digit can be used

Table 4 The Correlations for the 10 TV Programmes Rounded and Re-Ordered

Programmes	WoS	MoD	GrS	PrB	RgS	24H	Pan	ThW	Tod	LnU
World of Sport		.6	.6	.5	.3	.1	.2	.1	.1	.1
Match of the Day	.6		.6	.5	.3	.1	.1	.1	0	.1
Grandstand	.6	.6		.5	.3	.1	.2	.1	.1	.1
Prof.Boxing	.5	.5	.5		.3	.1	.2	.1	.1	.1
Rugby Special	.3	.3	.3	.3		.1	.1	.1	.1	.1
24 Hours	.1	.1	.1	.1	.1		.5	.4	.2	.2
Panorama	.2	.1	.2	.2	.1	.5		.4	.2	.2
This Week	.1	.1	.1	.1	.1	.4	.4		.3	.2
Today	.1	0	.1	.1	.1	.2	.2	.3		.2
Line-Up	.1	0	.1	.1	.1	.2	.2	.2	.2	

Figure 4

where a group of numbers are close to 1, 10, or 100, to avoid undue rounding errors.)

The need for rounding arises as follows. Psychologists have established that we can usually recall long numbers of seven or so digits fairly well, *as long as we are not interrupted in any way*, including by our own thoughts. If we look up a telephone number, such as 723-5473, we can usually dial it. But if we first say to ourselves "What am I going to say?," we will have forgotten the number and have to look it up again.

Scanning different figures or doing mental arithmetic are forms of mental interruption. This explains why we have such difficulty using longer numbers. Mentally subtracting 17.92 percent from 35.24 percent, or dividing one into the other to give 1.967, involves many intermediate stages where we have to try to remember other numbers as well, like the answer as it begins to emerge. Yet we may hardly remember one of the original numbers (say 35.24 percent) when we look at the other (17.92 percent).

But we can all deal with two-digit numbers: 18 percent and 35 percent. One is obviously about twice the other, and so we can scan a table of rounded figures and note how *this* ratio is about 2, that one about 3, that one 2 again, and so on. We can read the table. We can also remember the two-digit numbers much more easily over time than the longer ones.

The explanation for this appears to lie in two experiments quoted by Simon (1969, p. 40). These indicated that we can retain numbers in our immediate memory even when we are interrupted by some task, *as long as the numbers are only two digits long*.

Simon's evidence was limited and did not deal with *statistical* data. But his conclusion was no doubt right, partly because he recently got a Nobel prize, and partly because it serves to explain why we need to round to two rather than three or four digits.

There can be little compromise between rounding to two digits on the one hand and three or more on the other. This is so even when the numbers stay in front of us and we are dealing with very short term recall. Consider the following simple statement:

> The number of women teachers in training rose from 29,942 to 94,347, and that of men from 13,410 to 36,051.

Numbers like 29,942 and 94,347 are difficult to take in even if we go on looking at them. To see that the number of teachers trebled we have to round mentally. This could have been done for us as follows:

> The number of women teachers in training rose from 30,000 to 94,000, and that of men from 13,000 to 36,000.

Now we can see readily that the number of women teachers increased just over three-fold, and that of men just under three-fold.

But we cannot see it with three digits (unless we round mentally):

> The number of women teachers in training rose from 29,900 to 94,300, and that of men from 13,400 to 36,100.

This is as true for most statisticians as it is for anyone else. Faced with long numbers we all tend to be non-numerate. Faced with two-digit numbers, we can all more or less cope.

4. IMPLEMENTATION

Rules for better data presentation will not be implemented overnight. There will be upset and set-up costs. Implementation will require effort and encouragement.

Knowing the rules is important for users as well as producers of data. When faced with incomprehensible data, we must all learn to return them with constructive comments rather than do the producer's work for him or her. It makes more sense for the analyst to rearrange the figures or have them retyped than for every potential user to do so or let them pile up unstudied in a corner. We should also insist on a two- or three-line verbal summary from the analyst: that will concentrate his or her mind and our own.

Investment in better data presentation will prove worthwhile in terms of the savings in paper and printing costs for large data systems, the saving of readers' time, and fuller and better use of the data. "Better use" does not mean that the data are always important. The data may say very little, but clearer presentation can bring out their pointlessness better!

Efforts to improve data presentation will meet with some resistance. One problem is that the rules appear to be mere common sense. This implies that we need to make no effort, let alone allocate people or budgets to get things changed. But the rules reflect neither common knowledge nor common practice: they are not widely known or widely used. The rules seem obvious only after they have been stated, because no special skills are then required to implement them. Our common senses will do.

It is not enough, however, merely to say "avoid long numbers" or "use good layout." That would be no more effective than advice to "avoid unnecessary violence." We need specific rules, and we need to understand their purpose so that we can cope with exceptions, trade-offs, and controversy.

One recent writer noted that the approximate loss in accuracy when rounding a certain number to two digits was no more than 3.41 percent. Failure to implement the rules tends to occur when the reasoning behind them is not understood and their specific advantages to unnoticed: the same writer, for example, talked of "rounding to two *or three* digits."

Of all the rules, that of rounding to two effective digits raises the most heated objections. It is the only rule where information is discarded. But experience to date seems clear-cut: precision beyond two effective digits virtually never matters, either for advanced analysis or for general information. (The most that anybody usually says is that the third or fourth digits *might* matter.)

Rounding-off is already widely accepted with graphs on the one hand and model building on the other, where the degree of approximation is generally far worse than that implied by the two-digit rule. If we cannot say why the Sheffield sales in Figure 1 went down 230 to 190 and then up again to 220, we hardly need to see the data reported as 233.4, 193.6, and 220.9. There are in any case two major safeguards against losing too much information.

First, the rule can have exceptions. This does not mean that we should "always round except sometimes," but that there must be a specific reason for not doing so. For instance, more than two digits are needed for auditing and bookkeeping, but not usually for reporting the results. More digits are also needed when calculating compound interest. But we usually know when are are auditing or doing compound interest and can act accordingly.

Second, the unrounded data can always be put into an appendix or in a data bank, or in a filing cabinet—just in case anyone wants to do some fine tuning on the third or fourth digits some time. So nothing need be lost forever. (This even applies to official government statistics that are published "for the record.")

It is often argued that the way information is presented must depend on one's purpose. But layout should depend primarily on the data, not on the purpose. If we can see the data clearly, it can be adapted better to *any* purpose. Just because it is easier to read down a column than across a row, comparing the figures in each column does not have to be our main purpose. If the pattern in the data comes out more clearly when rows have been changed into columns, then general experience is that subsequent comparisons will be easier even if they have to be made across the rows. In any case, the aim is not to make a table of data perfect for any and every purpose.

Producers of numerical data in the past often have not realized how unnecessarily incomprehensible their tables usually were. They have been production oriented, showing little concern for the consumer. Many of us are still not familiar with the notion that most tables can be improved. We therefore need to see good examples set by the big battalions (e.g., official government statisticians) and by teachers.

In considering the value of these rules, we must avoid talking down, that is, worrying about the possible effects on "others"—like backward schoolchildren or apocryphal company chairmen. Let us first concentrate on what helps professional people like ourselves. Indeed, the widespread view that people lack numeracy presents no direct educational challenge. If they know basic arithmetic, few educated people lack the skills to read well-presented tables of data—especially when they are also given a brief verbal summary telling what the analyst already knows, or should know.

The fault therefore is not the user's. All he or she needs to realize is that most tables can be improved to communicate better. It is we who are to blame if we opt out of the challenge of bringing this about.

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